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LIQUID SQUEEZE FILM DAMPING IN MICROSYSTEMS APPLICATIONS

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Squeeze film damping (SFD) in microscale systems employing plates parallel to a substrate and operating in a liquid environment is theoretically investigated. Previous analytical or numerical studies of SFD in microsystems are mainly focused on devices working in a compressible fluid, such as air [1-3], where fluid inertia is negligible. However, liquid inertia appears to significantly affect the dynamic response of a microsystem operating in liquid environment. This paper outlines a theoretical framework that takes into account the inertia effects on dynamic response and illustrates this effect presenting the magnification factor for media with high and low density.

The schematic of the system under consideration is shown in Fig. 1. The analysis starts from Newton's second law, which is solved under the assumption of small amplitude vibrations. The system consists of a rectangular, rigid plate of length/width *L/W* and an infinite substrate. The plate of effective mass *m* is connected to the substrate by a structure of elastic constant *k*. A thin film of fluid is confined between the plate and the substrate. The flow is considered to be two-dimensional (no variation in the z-direction, the direction perpendicular to the plane of the paper), which is strictly true for $W << L$, but a reasonable approximation for $W \sim L$. The plate has a single degree of freedom in the *y*-direction of squeezing. In a laboratory reference system the position of the plate and substrate are y_p and y_s, respectively.

Assuming that under an external excitation this system undergoes small vertical vibrations, the displacement of the substrate can be expressed as:

$$y_s = Y_{s0} + \delta H_n \sin(\omega t) \quad (1)$$

where Y_{s0} is the initial position of the substrate, δ is a small dimensionless parameter (typically ≤ 0.1), H_n is the nominal gap (the gap in the absence of a driving force and the plate effective mass), ω is the excitation frequency, and t is time. The displacement of the plate, y_p, is expected to have in-phase and out-of-phase components and is given by:

$$y_p = Y_{p0} + \delta H_n \left[C_s \sin(\omega t) + C_c \cos(\omega t) \right] \quad (2)$$

where Y_{p0} is the static displacement, and C_s and C_c are currently unknown coefficients for the in-phase (sine) and out-of-phase (cosine) components of the plate motion.

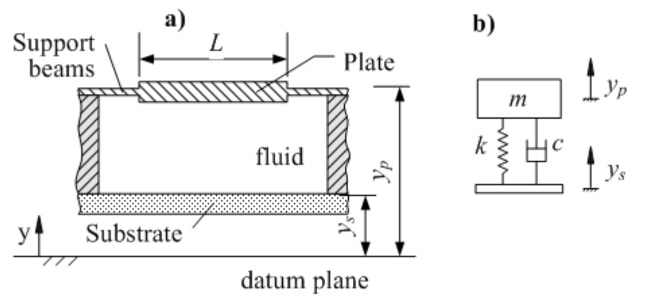

Fig. 1. a) Elastically supported, free-standing plate. Sinusoidal displacement is imposed on substrate. b) Schematic of system idealized as single degree-of-freedom.

From Newton's second law, the equation describing the motion of the plate is:

$$m \frac{d^2 y_p}{dt^2} - \left(F_{elas} + F_{hydr} - mg \right) = 0 \quad (3)$$

where F_{elas} is the elastic force, F_{hydr} is the force acted upon the plate by the fluid and g is the gravitational constant. The dynamic elastic force F_{elas} of the system is given by:

$$F_{elas} = -k \left[\left(y_p - y_s \right) - H_n \right] \quad (4)$$

The hydrodynamic force F_{hydr} is the sum of viscous force F_{visc} and inertia force F_{iner}. Determination of these expressions follows the methodology of Tichy and Modest [4] and Tichy and Winer [5]:

$$F_{hydr} = F_{visc} + F_{iner} \quad (5)$$

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$$F_{visc} = -\frac{L^3 W}{\left(Y_{p0} - Y_{s0}\right)^3} \eta c_v \left(\frac{dy_p}{dt} - \frac{dy_s}{dt} \right) \quad (6)$$

$$F_{iner} = -\frac{L^3 W}{\left(Y_{p0} - Y_{s0}\right)} \rho c_i \left(\frac{d^2 y_p}{dt^2} - \frac{d^2 y_s}{dt^2} \right) \quad (7)$$

where η is the viscosity and ρ is the density of fluid, respectively. The symbols c_v and c_i denote dimensionless viscous and inertia coefficients, which in the case of two-dimensional parallel plates are found to be $c_v = 1$, and $c_i = 1/10$, respectively [4,5]. Equations 6 and 7 are found by solving for pressure from the thin-film Navier-Stokes equations, and integrating over the plate length L and width W. Due to small amplitude vibrations, the nonlinear convective inertia terms are negligible relative to the unsteady terms, enabling a relatively straightforward closed form solution.

C_s and C_c are determined by numerically solving Eq. 3 after substituting the expression 6, 7 and 4 for fluid and elastic force. The resonance response of the system can then be determined from the magnification factor, denoted by C_{mag} [6]:

$$C_{mag} = \sqrt{C_s^{\ 2} + C_c^{\ 2}} \quad (8)$$

To illustrate the effect of inertia on the dynamic response of the system, the response of a system with geometrical parameters in a range relevant to microsystems is presented next. In this case study the fluid gap $H_n = 35$ µm, plate length $L = 500$ µm, and width $W = 500$ µm. The effective mass (plate mass and added mass) is taken as $m = 1.38$ mg. The spring constant k of the parallel cantilevers is taken as 2344 N/m, thus $\omega_0 = 4.12\times10^4$ rad/s, where ω_0 is the natural frequency of the system.

The magnification C_{mag} is plotted in Fig. 2 as a function of ω for different (ρ, η) pairs to shows the response of the system in different media (air and water) and to illustrate the difference in the dynamic response between inertia and viscous dominated regimes. The continuous curve in Fig. 2 shows the response in air ($\rho = 1.2$ kg/m^3, $\eta = 1.9\times10^{-5}$ Pa·s). The resonant frequency is almost identical with natural frequency of the system. The dashdot line is plotted keeping the same density as that of air, while increasing the viscosity by almost two orders of magnitude ($\rho = 1.2$ kg/m^3, $\eta = 0.001$ Pa·s). In this case the resonant frequency of the system does not change much. However, the amplitude decreases significantly, consistent with classical results for single degree of freedom damped vibration systems [6]. The dot line is plotted keeping viscosity same as air, while increasing density by two orders of magnitude ($\rho = 1000$ kg/m^3, $\eta = 1.9\times10^{-5}$ Pa·s). Comparing the continuous and dot lines, it can be seen that a larger density shifts the resonant frequency to a lower range, while it has little effect on the magnitude of the magnification factor. This shows that inertia produces an increase in the effective mass of the system. Finally, the dash line is obtained for both density and viscosity of water ($\rho = 1000$ kg/m^3, $\eta = 0.001$ Pa·s). In this case, both the magnitude and the position of the magnification factor are distinct from that of air.

These results clearly indicate that the inertia effects must be accounted for when designing microsystems operating in liquid media and are relevant to a wide range dynamic MEMS systems operating in a liquid environment.

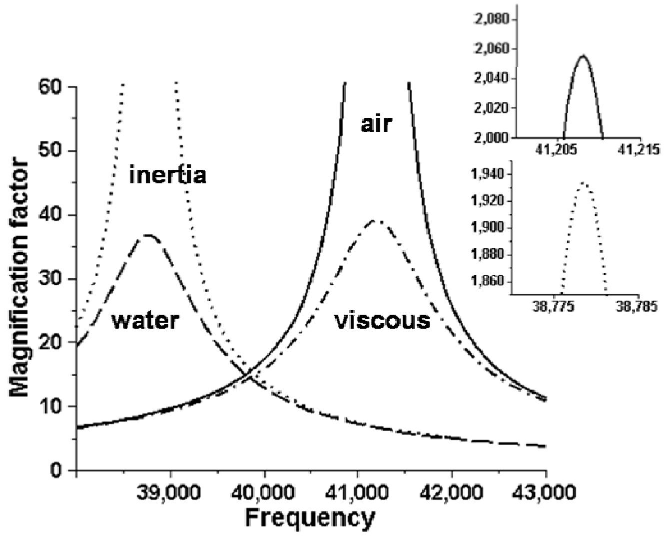

Fig. 2 Magnification factor versus frequency. Solid line shows the response in air (ρ=1.2 kg/m^3, η =1.9×10^{-5} Pa·s); dashdot line shows viscous effects (ρ = 1.2 kg/m^3, η = 0.001 Pa·s); dot line show inertia effect (ρ = 1000 kg/m^3, η =1.9 ×10^{-5} Pa·s) and dash line shows the dynamic response in water (ρ = 1000 kg/m^3, η =1.9 x10^{-5} Pa·s).

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