# FEDSM-ICNMM2010-30534 

# Mercury Scaling of a Swirling Jet Micro-Bubble Generator 

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#### Abstract

Swirling jets with co-axial gas filament flow have been used for production of small bubbles in environmental and chemical processing industries for some time. The modeling of the physics for the gas filament break-up is not well established, and this impedes scaling of the device to use with fluids other than water and organics where data is available. High speed photographic studies of the gas filament break-up are used to examine the physical phenomena, and support model development for the bubble production that may be used to scale the device to alternate applications, such as bubble production in liquid metals. Bubble break-up models based on energy dissipation generate a power-law, with exponent of $\alpha=8 / 5$, relating Weber number to Reynolds number at the nozzle exit. Those models are compared to empirical models found in the literature providing a link between mechanistic models, scaling arguments, and legacy empirical models.


## INTRODUCTION

Energy deposition in the liquid metal target of the Spallation Neutron Source leads to a pressure pulse resulting in cavitation and pitting of the stainless steel target vessel (Riemer, 2010). The introduction of a $30 \mu \mathrm{~m}$ homogenous He bubble cloud with $0.5 \%$ volume fraction is a promising method of adding compressibility to the mercury target. The SNS High Power Target project plans tests at the Weapons Neutron Research facility at Los Alamos National lab in the fall of 2010. A commercial swirling jet bubbler will be one of several approaches for producing bubbles in mercury examined during these tests.

A swirling jet bubbler is a candidate device to accomplish a suitable bubble population in the SNS target. Coaxial swirl jet bubble production devices induce a high rate of rotation in a liquid flow, sometimes achieving over 1000 revolutions per second, and inject gas into the center of this
flow to form a gas filament. The swirling liquid flow and gas filament are then injected into a larger volume of liquid to form a swirling liquid jet. The gas filament diverges to the outer jet perimeter when the swirling jet enters the larger liquid volume due to the rapid liquid deceleration and associated adverse pressure gradient. The gas filament is broken into small bubbles in the high shear region where the swirling jet meets the more quiescent fluid in the larger liquid volume. An Olympus I-speed video imaging system is used to examine the bubble formation in this high shear region for a water-helium system. Models are developed from this data and used to develop scale parameters for the bubble generation physics. These are used to scale the device geometry, flow, pressure loss and bubble production diameter using water-helium data.

## SWIRLING-JET BREAKUP PHYSICS

The physics of the Nitta-Moore device is described in terms of the three distinct regions pictured in Figure 1. Region (A) introduces an axisymmetric swirl to the liquid. In region (B) the gas is injected along the low pressure centerline, and the swirling flow is accelerated through a nozzle. The flow enters a plenum in Region (C), creating a swirling jet. Flow instabilities at the gas/liquid interface within region (B) and interaction of the gas with a strong shear layer in region (C) have both been suggested as possible breakup mechanisms which fit this physical description. High-speed video of the flow at the nozzle exit indicates the radial pressure gradient confines the gas jet to the flow centerline in region (B). As the jet exits the nozzle into region (C) the pressure gradient is altered as the liquid jet decelerates and the gas jet follows the steepest decent of the pressure gradient into the high shear region where the rotating gas jet meets the relatively quiescent fluid in the plenum. The small bubbles are produced where the gas filament intersects this region of high shear.


Figure 1: Sketch of Nitta-Moore Cross-Section.

The gas filament rotates with the liquid jet as it diverts to the jet surface, and breaks into small bubbles in an annulus located a few exit orifice diameters downstream. The free shear layer formed where the jet meets the quiescent fluid in the plenum causes breakup of the gas filament.

To accomplish closed form scale model, an axisymmetric jet model is adopted which defines a turbulent viscosity as Eq. (1) (White, 2006).

$$
\begin{equation*}
v_{t}=0.016 \cdot U_{0} \cdot b_{0} \tag{1}
\end{equation*}
$$

Where $U_{0}$ defines the liquid velocity and $b_{0}$ is the jet width at a reference axial distance, $x_{0}$. The similarity variable assumes isotropic turbulence.

A critical bubble diameter for bubble break-up in a turbulent jet flow is defined such that turbulent shear stresses equal surface tension forces (Martinez-Bazan, 1999),
$D_{c}=1.26 \cdot\left(\frac{\sigma}{\rho}\right)^{3 / 5} \cdot \varepsilon^{-2 / 5}$

The energy dissipation per unit mass is found from the definition of a turbulent viscosity and the assumption that the strain-rate, $s_{i j}$, only varies with swirl,
$\tau_{i j}=2 v_{t} S_{i j} \Rightarrow \varepsilon=\frac{2 \cdot S_{i j}}{l^{2}} v_{t}=f(\varpi) \cdot v_{t}$

Combination of Eqs. (2) and (3) defines a critical bubble diameter for a swirling jet,

$$
\begin{equation*}
D_{c}=6.59 \cdot\left(\frac{\sigma}{\rho}\right)^{3 / 5} \cdot\left(f(\varpi) \cdot U_{0} \cdot b_{0}\right)^{-2 / 5} \tag{4}
\end{equation*}
$$

## FUNCTIONAL ANALYSIS

Functional analysis of the flow variables assists in scaling from water to mercury systems. The variables pertinent to the jet characteristics are listed in Table 1. The literature indicates that the gas flow rate has a minimal effect on average bubble size (Tabei, 2007). This leads to the functional form for the average bubble diameter given as

$$
\begin{equation*}
d_{\text {bub }}=d_{\text {bub }}\left(D_{\text {nozzle }}, \dot{V}_{l i q}, \omega_{\text {exit }}, \rho_{\text {liq }}, v_{\text {liq }}, \sigma_{\text {liq }}\right) \tag{5}
\end{equation*}
$$

Define the nozzle diameter, liquid flow rate, and liquid density as primary variables according to

$$
\begin{align*}
& {[\text { Length }] \Leftrightarrow D_{\text {nozzle }}} \\
& {[\text { Time }] \Leftrightarrow D_{\text {nozzle }}^{3} / \dot{V}_{\text {liq }}}  \tag{6}\\
& {[\text { Mass }] \Leftrightarrow D_{\text {nozzle }}^{3} \cdot \rho_{\text {liq }}}
\end{align*}
$$

| Table 1: Range of Pertinent Variables |  |
| :---: | :---: |
| Diameter of Exit orifice, $D_{\text {nozzle }}$, (mm) | 4 |
| Liquid Flow Rate, $\stackrel{\bullet}{l i q}^{\text {, (L/min) }}$ | 2-4 |
| Angular Frequency, $\boldsymbol{\omega}_{\text {exit }},(\mathrm{Rev} / \mathrm{s})$ | 600-1300 |
| Liquid Density, $\boldsymbol{\rho}_{\text {liq }},\left(\mathrm{Kg} / \mathrm{m}^{3}\right)$ | 1000 |
| Liquid Viscosity, $\boldsymbol{V}_{\text {liq }}$, $\left(\mathrm{m}^{2} / \mathrm{s}\right)$ | $1 \times 10^{-6}$ |
| Liquid Surface Tension, $\boldsymbol{\sigma}_{\text {liq }}$, (N/m) | 0.075 |
| Average <br> Bubble <br> Diameter, $d_{b u b},(\mu m)$ | 50-200 |

Dimensional analysis of the remaining variables determines the equivalent form
$\frac{d_{\text {bub }}}{D_{\text {nozzle }}}=f\left(\frac{\pi \cdot \omega_{\text {exit }} \cdot D_{\text {nozzle }}^{3}}{8 \cdot \dot{V}_{\text {liq }}}, \frac{\cdot \dot{V}_{\text {liq }}}{D_{\text {nozzle }} \cdot v_{\text {liq }}}, \frac{\rho_{\text {liq }} \cdot \cdot^{2}}{D_{\text {liq }}^{3}}\right) \Leftrightarrow$
$W e=f\left(\bar{\omega}, \mathrm{Re}_{\text {exit }}\right)$
with the average Weber number, We, non-dimensional swirl rate, $\bar{\omega}$, and Reynolds number at the nozzle exit, $\mathrm{Re}_{\text {exit }}$, defined as
$W e \Leftrightarrow \frac{\rho_{\text {liq }} \cdot \dot{V}_{\text {liq }}^{2} \cdot d_{\text {bub }}}{D_{\text {nozzle }}^{4} \cdot \sigma_{\text {liq }}}$
$\bar{\omega} \Leftrightarrow \frac{\pi \cdot \omega_{\text {exit }} \cdot D_{\text {nozzle }}^{3}}{8 \cdot \dot{V}_{\text {liq }}}$
$\operatorname{Re}_{\text {exit }} \Leftrightarrow \frac{\dot{V_{\text {liq }}}}{D_{\text {nozzle }} \cdot v_{\text {liq }}}$

In a similar fashion, dimensional analysis of the pressure drop across the device and the radial pressure gradient
within the nozzle generates relationships useful to engineering design.

$$
\begin{align*}
& \Delta P_{\text {loss }}=f\left(\operatorname{Re}_{\text {exit }}\right)  \tag{9}\\
& \omega_{\text {exit }}=f\left(\mathrm{Re}_{\text {exit }}\right) \tag{10}
\end{align*}
$$

## SCALING CRITICAL BUBBLE DIAMETER

A power law, Eq. (11), is stated to represent the relationship in Eq. (7).
$W e=\beta \cdot \operatorname{Re}^{\alpha}$

Expanding terms from Eq. (8) while using Eq. (11) generates a functional relationship for the average bubble size
$d_{b u b}=\beta \cdot\left(\frac{\sigma}{\rho}\right) \cdot\left(\frac{\pi}{4}\right)^{\alpha} \cdot\left(\frac{D_{\text {noz }}^{\alpha}}{v_{\text {liq }}^{\alpha}}\right) \cdot U_{0}{ }^{\alpha-2} \Rightarrow W e=\beta \cdot \mathrm{Re}^{8 / 5}$

Comparing Eqs. (4) and (12) implies $\alpha=8 / 5$ and the form of $\beta$ is offered by Eq. (13), where $h(\sigma)$ relates strain-rate, fluid properties and turbulence closure coefficients.

$$
\begin{equation*}
\beta=\left(\frac{D_{n o z}}{v_{\text {liq }}}\right)^{-8 / 5} \cdot h(\Phi)^{-2 / 5} \tag{13}
\end{equation*}
$$

A power law relationship is also presented for characterizing the measured performance of swirling jet bubblers by Tabei, 2007,
$W e=\beta \cdot \mathrm{Re}^{3 / 2}$
with $\beta=\left(1.72 \times 10^{-6}\right) \cdot \bar{\sigma}^{-3 / 2}$. The power law coefficients for these two models are dimensionless quantities relating swirl and fluid properties. Therefore, the coefficient in Eq. (11) will take a value of $\beta=\left(1.72 \times 10^{-6}\right) \cdot \sigma^{-3 / 2}$ in both models. Interesting to note, the physics which supports Eq. (12) explicitly generates the coefficient of Eq. (13) for water as $\left(D_{\text {noz }} / v_{\text {liq }}\right)^{-8 / 5}=1.72 \times 10^{-6}$. However, this physical interpretation is not provided by Tabei, 2007.

## EXPERIMENT

The functional relationships presented in Eqs. (7), (9) and (10) are compared against data found in water using a commercial swirling jet device. Table 1 shows the range of values taken in Eq. (5) for this study. A loss coefficient for the device of $K_{\text {loss }} \cong 20$ is estimated from pressure loss and liquid flow rate measurements at the nozzle exit. Also, the direct correlation between swirl rate and device geometry is measured which establishes the relationship in Eq. (10). Finally, the power law exponent of $\alpha=3 / 2$ from Tabei, 2007, and the power law exponent of $\alpha=8 / 5$ from this study are compared to the data.

## Bubbler Description

The commercial swirling jet bubbler is pictured in Figure 2a. Liquid is injected at Part (b) and gas is injected at Part (c), the liquid gas mixture exits Part (a). A cross-section of the device is presented in Figure 2(b) for clarification. The swirl and gas filament injection described as physics in Regions (A) and (B) in Figure 1 occur in Part (a) of Figure 2(a). The jet physics of Region (C) in Figure 1 occurs in a liquid tank and is pictured in Figure 2(c).

## SCALING RESULTS

The viscous losses through the bubbler can be described using the one-dimensional steady incompressible energy equation in terms of the pressure losses, dynamic pressure, and loss coefficient, $\mathrm{K}_{\text {loss, }}$, following the form,

$$
\begin{equation*}
K_{\text {loss }}=\frac{\Delta P_{\text {loss }}}{8 \cdot \pi^{2} \cdot \rho_{\text {liq }} \cdot \dot{V}_{l i q} \cdot D_{\text {noz }}^{4}} \tag{15}
\end{equation*}
$$

The liquid flow rate is measured using an Omega turbine meter with $0.3 \mathrm{~L} / \mathrm{min}$ accuracy in the $1-5 \mathrm{~L} / \mathrm{min}$ operating range. The pressure drop across the device is measured using an upstream pressure gauge with 2 psi resolution in the 10-50 psi range. Gas flow rates were measured using an inverted tube assembly with an accuracy of 1 sccm .

Figure 3 shows the upstream pressure versus exit Reynolds number, with the downstream pressure near one atmosphere. The pressure losses scale according to the kinetic energy of the flow and the functional form given in Eq. (9) is confirmed quadratic. This suggests that the pressure losses scale according to the kinetic energy with a loss coefficient, $\mathrm{K}_{\text {loss }} \sim 20$, which is dependent only on the device geometry and body material; this data is compared in Figure 4.

(a)

(b)

(c)

Figure 2. (a) Image of commercial swirling jet micro-bubbler, (b) cross-section of commercial bubbler, and (c) image of commercial bubbler mounted in experiment tank.


Figure 3. Comparisons of pressure losses versus exit Reynolds number, Eq. (9) is confirmed quadratic.


Figure 4. Pressure drop versus dynamic pressure; $\mathrm{K}_{\text {loss }} \sim 20$.

## Swirl Scaling Results

The flow swirl is introduced via a fluted insert that forces the flow into a spiral within the nozzle; located inside Part (a) of Figure 2a. The flute pitch, $p_{\text {flute }}$, nozzle diameter and liquid viscosity determine the ratio of the exit angular frequency to the exit Reynolds number.
$\omega_{\text {exit }}=\kappa_{\omega} \cdot v_{\text {liq }} \cdot \operatorname{Re}_{\text {exit }}$
$\kappa_{\omega}=\frac{p_{\text {flute }}}{D_{\text {nozzle }}}$

Measurement of the flute pitch, $p_{\text {flute }}=4.1 \mathrm{~mm} / \mathrm{rev}$, generates the proportionality constant, $\kappa_{\omega}=6.1 \times 10^{4}\left[\mathrm{rev} / \mathrm{m}^{2}\right]$, which depends only on the bubbler geometry. The swirl rate in the flute is accelerated as the nozzle reduces the flow diameter while the angular momentum of the flow is conserved. Jet exit swirl rates are determined using an Olympus High-Speed camera to range from 600-1350 RPS; this data is presented in Figure 5. A proportionality constant of $\kappa_{\omega}=5.9 \times 10^{4}\left[\mathrm{rev} / \mathrm{m}^{2}\right]$ is calculated using a linear least squares fit to data from the high speed videos.

An audible pitch change in the operating noise of the micro-bubbler is observed with increasing liquid flow rates; this suggests a method to verify bubbler operation in liquid metals. Therefore acoustic data was obtained using 1s time histories from a PCB dynamic pressure sensor mounted on the outside of the bubble discharge/observation tank and analyzed for peak amplitude in the frequency domain. Frequency peaks in the acoustic data correspond to rotation rate measurements taken using high-speed video at $\mathrm{Re}_{\text {exit }}>15,000$; this data is compared to high speed video measurements in Figure 5. The acoustic content associated with the bubbler swirl is indistinguishable from the background noise for $\mathrm{Re}_{\text {exit }}<15,000$. The frequency content of the acoustic data at $1.5 \times 10^{4} \leq \operatorname{Re}_{\text {exit }} \leq 2.1 \times 10^{4}$ is compared in Figure 6.


Figure 5. Angular frequencies as a function of exit Reynolds number; high-speed video data is compared to acoustic microphone data.


Figure 6. Frequency Spectrum of microphone data computed for various Reynolds numbers.

The data in Figure 5 confirms a linear relationship between exit flow rates and angular velocity. The angular velocity is found to be a function of $\mathrm{Re}_{\text {exit }}$ and $v_{\text {liq }}$ as well as a geometric proportionality constant, $\kappa_{\omega}$, which is independent of fluid flow properties. Finally, acoustic data correlates well with high-speed video provided $\mathrm{Re}_{\text {exit }}>15,000$.

## Critical Bubble Diameter Scaling Results

The average bubble diameter has been related to the Weber number associated with the kinetic energy of the liquid in Eq. (7). Also, Eq. (9) has been experimentally verified, suggesting that the dimensionality of the solution space is reduced from Eq. (7) to a power law relationship. A proposed power law exponent of $\alpha=8 / 5$ is compared to a value reported in the literature of $\alpha=3 / 2$ (Tabei, 2007).
$W e=\beta \cdot \operatorname{Re}^{\alpha}$
where,

$$
\begin{equation*}
\beta=\left(1.72 \times 10^{-6}\right) \cdot \varpi^{-3 / 2} \tag{17}
\end{equation*}
$$

A high resolution still camera is used to determine that the bubble size distribution ranges from 50-200 microns with an
average bubble diameter of $\sim 100$ micron; $\sim 10$ micron resolution provided bubble areas of about 50 pixels. An edge detection algorithm prepares raw images for a thresholding algorithm which creates binary images. These images are filtered for noise using a despeckle algorithm. The processed images are prepared for particle analysis using several morphological functions. Image processing is implemented using WCIF Image-J freeware.

The previous range of Reynolds numbers and gas flow rates were examined. Figure 7 compares the bubble size distribution at various $\mathrm{Re}_{\text {exit }}$ and gas flow rates. The 10 micron imaging resolution limits our ability to determine structure within the size distributions however examination of the average bubble diameter at various Reynolds numbers demonstrates the functional relationship between Weber number and Reynolds number at the nozzle exit.

In Figure 8, a power law exponent of $\alpha=8 / 5$ is compared to $\alpha=3 / 2$ using Eq. (11). This result demonstrates that mixing-length type turbulence models correctly predict the gas break-up in a co-axial swirl jet bubbler while the empirical model under predicts the average bubble size. The model coefficient, given in Eq. (13), is also verified.


Figure 7. Bubble size distributions from still image data analyzed using WCIF ImageJ.


Figure 8. Experimental relationship between Weber number and Reynolds number.

## CONCLUSION

Swirling jet bubblers have been used for production of small bubbles for some time. The modeling of the physics for the gas filament break-up is not well established. Dimensional analysis reveals the functional relationship between flow variables which assists in the development of closed form models for bubble breakup useful to engineering design.

Pressure loss and swirl rate relationships are also presented in order to assist in engineering design. These combined models will be used to extend the bubbler use to alternate fluids, such as liquid metals. A method for measuring jet swirl rates acoustically is verified using high speed video data. This will allow some validation of the bubbler performance in opaque fluids. Bubble break-up models based on energy dissipation generate a power-law relationship, with an exponent of $\alpha=8 / 5$, relating Weber number to Reynolds number at the jet exit. Those models are compared to empirical models found in the literature providing a link between mechanistic models, scaling arguments, and legacy empirical models.

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