Proceedings of the ASME 2010 3rd Joint US-European Fluids Engineering Summer Meeting and 8th International Conference on Nanochannels, Microchannels, and Minichannels
FEDSM-ICNMM2010
August 1-5, 2010, Montreal, Canada

FEDSM-ICNMM2010-30382

EFFECT OF SURFACE ROUGHNESS ON THE PRESSURE DROP OF LIQUID FLOW IN MICRO-CHANNELS

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ABSTRACT

Experimental results in the literature of the surface roughness effect on pressure drop of liquid flow in mini and micro channels were examined. Also, three prominent theories, which are the constricted flow model, the constricted model considering roughness distribution, and the roughness viscosity model are reviewed based upon this broad data base. It is found that all the normalized data of *fRe* can be predicted, within an error of about ±15%, by the classical pressure drop theory with the channel dimension using the constricted flow model. On the other hand, it appears that the viscosity model is difficult to use generally and the roughness distribution information is frequently hard to obtain for natural rough-surfaces. However, the data, which supports these two theories, can also be predicted well by the constricted flow method at a similar accuracy. It is possible that the viscosity model and the constricted model with roughness distribution can be further developed to become easier to use, for the time being the simple constricted flow model could be a general and reliable method for mini and micro channel pressure drop predictions.

1. INTRODUCTION

Mini- and Micro-channels are increasingly used in many applications due to their large surface area per unit volume, which results in the compact size and efficient heat transfer. Extensive research has been conducted on the fluid flow in micro-channels, and revealed that the flow characteristics are usually different from the classic theory in macro-channels. For instance, the friction factor for laminar liquid flow in micro-channels was found as much higher than that in macro-channels, and the transition to turbulence occurs earlier than *Re*=2300 (Reynolds number). Surface roughness of the channel walls is suspected as one of the most important factors for these deviations because when the channel dimension reduces the relative importance of roughness increases.

Mala and Li [1] experimentally examined the water flow in micro-tubes of stainless steel and fused silica with diameters ranging from 50 to 254 μm and *R*e up to 2500. For all the tests, the mean roughness is ±1.75 *u*m. Their results indicated that as *Re* increases there is a significant increase in pressure gradient compared to that predicted by the Poiseuille flow theory. They proposed a roughness-viscosity model to interpret the experimental data. Qu et al. [2] investigated the de-ionized water flow through trapezoidal silicon micro-channels with D_h = 51.3-168.9 μm and ε/D_h = 1.2-1.75% for *R*e <1500. The friction factors are 8-38% higher than the classical theory predictions and dependent upon the micro-channels' hydraulic diameters and Reynolds number.

Pfund et al. [3] measured the pressure drop in de-ionized water flow in high aspect-ratio rectangular micro-channels made in a shallow sandwich structure with depths ranging from 128 to 521 μm and Reynolds number between 60 and 3450. For the 257 μm deep roughened channel (maximum roughness of 14.67 μm), Po (the Poiseuille number, as shown in equation (1)) at low Reynolds number reached about 29±2.4, which is significantly above the classical value of 23.2.

$$Po = f \times \mathrm{Re} \quad (1)$$

Wu and Cheng [4] performed experiments with 13 different trapezoidal silicon micro-channels. They also reported that at the same Reynolds number, the Po number of channels with large surface roughness is much higher than those with smaller roughness values. However, their results show an unexpected increase in Po even for very small roughness and low Reynolds numbers. Kim et al. [5] experimentally investigated the effects of surface roughness (ε/D_h = 1-3%) on the flow characteristics in rectangular PDMS micro-channels with a

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hydraulic diameter ranging from 66.67 to 200 μm and the Reynolds number up to 830. The results showed that the slopes of the measured pressure gradient versus Re relationships are approximately 2 to 46% higher than those of the theoretical curves.

Celata et al. [6] carried out experiments on smooth glass/fused silica capillary tubes from 259 μm down to 31 μm. The results showed that friction factor of smooth channels agrees well with the classical Hagen–Poiseuille law ($f = 64/Re$ for circular channels); For roughened glass channels, an increase in Darcy friction factor above 64/*R*e was observed only at the smallest diameter of 126 μm. They explained that the deviations may be caused by actual deformation of channel circularity.

Hao et al. [7, 8] fabricated discrete rectangular rough elements on the side wall of shallow rectangular channels and examined the water flow with micro-PIV. They found that for smooth channels, the friction factor for laminar flow agreed with that predicted by classic theory; for channels with rough side-elements the friction factor was higher.

Hrnjak and Tu [9] investigated R134a liquid and vapor flow through rectangular micro-channels with hydraulic diameters varying from 69.5 to 304.7 μm and aspect ratios from 0.09 to 0.24. The Reynolds numbers were varied between 112 and 9180. It was reported that when the channel surface roughness was low, the laminar friction factor approached the conventional values, even for the smallest channel tested. And surface roughness was suggested to be responsible for higher (9%) laminar flow friction in one of the channels tested.

Tang et al.'s [10] experimental data of the de-ionized water flow in glass micro-tubes with diameters ranging from 50 to 530 μm showed that the friction factors are in good agreement with the conventional theoretical predictions. However, the friction factors in stainless steel micro-tubes with diameters of 119 and 172 *u*m are much higher than the conventional theoretical predictions. The discrepancy was attributed to the large surface relative roughness or dense roughness distribution in the stainless steel tubes.

Gamrat et al. [11] investigated de-mineralized water flow in rectangular channels with periodically distributed block roughness and randomly distributed particle roughness by experimental measurements, 3D numerical simulation and 1D rough layer model, respectively. It was shown that the Poiseuille number, Po, increases with the relative roughness and is independent of Re in the laminar regime ($Re<2000$). Experiments on water flow in copper micro-tubes by Hakamada et al. [12] also showed that the rough surface caused a higher pressure gradient.

From the above review of former experimental studies on liquid flow in circular, rectangular and trapezoidal micro-channels, it is seen that surface roughness plays an important role in friction factors. But the exact mechanism is still not fully understood. Nevertheless, several prominent models have been proposed.

Kandlikar et al. [13] presented a constricted flow model to interpret the roughness effect on the flow in micro-channels. Considering high roughness, the flow after the rough elements does not re-attach to the channel wall, thus decreasing the available channel area of fluid flow. The new flow boundary is suspended above the base surface wall at some distance. The effective diameter of the flow is, then, not the base diameter of the channel, but is reduced by the size of the roughness elements, i.e. the constricted flow diameter, D_{cf}, defined as

$$D_{cf} = D - 2\varepsilon \tag{2}$$

where D is the base or root diameter of the channel. It was reported that the use of constricted flow hydraulic diameter in the classical laminar flow theory results in a good agreement (within 5%) with their own experimental data for $0 \le \varepsilon/D_{cf} \le 0.14$. They used the peak values of roughness (ε_p) instead of average roughness (ε_a) described in Figure 1(a).

Croce et al. [14] numerically investigated the results of different choices of hydraulic diameter for the plane channel flow. They found that, with roughness pitch included, the hydraulic diameter defined by

$$D_{h,cf} = 2b(1 - 2\varepsilon r / s) \tag{3}$$

with parameters shown in Figure 1(b), gives values of Nu and Po in very good agreement with those predicted by the classical theory for conical roughness in their data set. As aforementioned, the roughness-viscosity model presented by Mala and Li [1] also agrees well with their own experimental data.

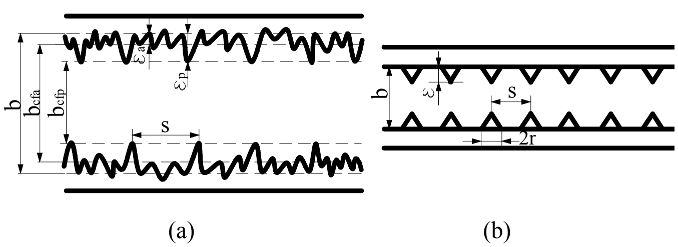


Fig. 1. Schematic sketch of roughness in micro-channels. (a) natural roughness; (b)conical roughness

Although all these three models have been validated by the authors' own data sets, it is important to make a broad-based comparison of the experimental data of liquid flow in micro-channels reported in literature with the predicted values of the three models. Since there are increased use of micro-channels in the future, a broadly validated guideline will benefit its further developments applications.

2. DATA ANALYSIS

2.1 classical theory

For laminar flow in circular tubes, we have

$$Po_{th} = f \times \mathrm{Re} = 64 \tag{4}$$

where f is Darcy friction factor and *R*e is Reynolds number defined as

$$\mathrm{Re} = \frac{\rho u D}{\mu} \tag{5}$$

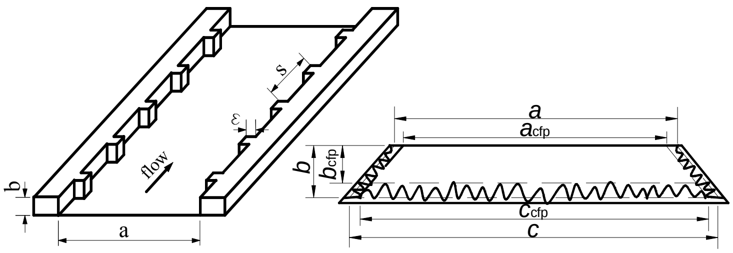
In the above equation, D is the root diameter of the tube.

For rectangular channels, Po_{th} is predicted by Kakac et al. [15] as

$$Po_{th} = 96(1 - 1.3553\alpha + 1.9467\alpha^2 - 1.7012\alpha^3 + 0.9564\alpha^4 - 0.2537\alpha^5) \quad (6)$$

where α is the aspect ratio defined as $\alpha = \frac{b}{a}$ and a, b are defined in Figure 2.

For trapezoidal channels, Po_{th} can be calculated by Shah and London's tabulated data [16] corresponding to various aspect ratios and bottom angles.



(a) (b)

Fig. 2. Schematic sketch of roughness in non-circular channels. (a) rough side walls in rectangular channel; (b) cross section of trapezoidal channel (glass top-lid)

2.2 constricted flow model

According to Kandlikar et al.'s idea [13], the constricted flow diameter for circular channels is defined as $D_{cf} = D\text{-}2\varepsilon$, where ε is the roughness height. For non-circular channels in literature, the trapezoidal can be considered as a complex geometry with rectangular shape as a special case. The constricted hydraulic diameter is used as

$$D_{h,cf} = \frac{4A_{cf}}{P_{cf}} = \begin{cases} \dfrac{4 \times a_{cf} \times b_{cf}}{2 \times (a_{cf} + b_{cf})}, & rectangular \\ \dfrac{4 \times (a_{cf} + c_{cf}) \times b_{cf} / 2}{2 \times \sqrt{b_{cf}^2 + (\frac{c_{cf} - a_{cf}}{2})^2} + a_{cf} + c_{cf}}, & trapezoidal \end{cases} \quad (7)$$

where A_{cf}, P_{cf}, a_{cf}, c_{cf} and b_{cf} are constricted values of cross sectional area, perimeter, up width, bottom width and height of the trapezoidal channel, respectively (see Figure 2 (b)). For rectangular channel, $a_{cf}=c_{cf}$. Surface roughness can be considered in this equation. For example, if there are rough elements on all the sides along the perimeter, we have $a_{cf}=a\text{-}2\varepsilon$, $c_{cf}=c\text{-}2\varepsilon$ and $b_{cf}=b\text{-}2\varepsilon$. If there are rough elements only on the bottom of the channel, we have $a_{cf}=a$, $c_{cf}=c$ and $b_{cf}=b\text{-}\varepsilon$. For roughness only on the side walls of the channel, $a_{cf}=a\text{-}2\varepsilon$, $c_{cf}=c\text{-}2\varepsilon$ and $b_{cf}=b$. Other cases can be deduced correspondingly.

Accordingly, the Poiseuille number based on the constricted flow diameter, Po_{cf}, in the laminar flow can be expressed as:

$$Po_{cf} = f_{Darcy,cf}\,\mathrm{Re}_{cf} \quad (8)$$

where,

$$\mathrm{Re}_{cf} = \frac{\rho u D_{h,cf}}{\mu} = \frac{\rho Q D_{h,cf}}{\mu A_{cf}}. \quad (9)$$

and $f_{Darcy,cf}$ is the Darcy friction factor based on the constricted channel diameter.

From the classical pressure drop equation

$$\Delta p = f_{Darcy}\frac{\Delta x}{D_h}\frac{\rho u^2}{2} \quad (10)$$

it gives

$$f_{Darcy} = D_h\frac{\Delta p}{\Delta x}\frac{2}{\rho u^2} = D_h A^2\frac{\Delta p}{\Delta x}\frac{2}{\rho Q^2} \quad (11)$$

Correspondingly, for constricted flow

$$f_{Darcy,cf} = D_{h,cf}A_{cf}^2\frac{\Delta p}{\Delta x}\frac{2}{\rho Q^2} \quad (12)$$

then

$$\frac{f_{Darcy,cf}}{f_{Darcy}} = \frac{D_{h,cf}A_{cf}^2}{D_h A^2} \quad (13)$$

For Reynolds number, we have

$$\frac{\mathrm{Re}_{cf}}{\mathrm{Re}} = \frac{D_{h,cf}A}{D_h A_{cf}} \quad (14)$$

From equations (8), (13) and (14), the following correlation is obtained:

$$\frac{Po_{cf}}{Po} = \frac{D_{h,cf}^2 A_{cf}^3}{D_h^2 A^3} \quad (15)$$

Based on the above equations, we can evaluate the constricted parameters such as Re_{cf}, $f_{darcy,cf}$ and Po_{cf} if we know the experimental results of Re, f and Po in literature.

For circular tubes, the theoretical Poiseuille number, $Po_{th,cf}$ is also 64; for rectangular channels, it is again predicted by the equation (6) but α is substituted with α_{cf} which is the constricted aspect ration defined as $\alpha_{cf} = \frac{b_{cf}}{a_{cf}}$ and a_{cf}, b_{cf} are defined in Figure 2.

Also, for trapezoidal channels, $Po_{th,cf}$ is calculated by Shah and London's tabulated data [16] corresponding to various constricted aspect ratios and bottom angles.

2.3 constricted model considering roughness distribution

Croce et al. [14] examined different choices of hydraulic diameter for plane channel flow with the pitch of roughness included. Using numerical modelling, they found that the hydraulic diameter defined by equation (3) results in very good agreement of Nu and Po with the predictions of the classical theory for conical roughness. However, it is generally not easy to determine the pitches of the natural roughness on a surface. In the present study, experimental data of artificial and discrete roughness with known distribution information were examined with this model and compared with Kandlikar et al.'s constricted flow model. When include the roughness on bottom or sidewalls, equation (7) was used to evaluate the hydraulic diameter with

$$a_{cf} = a - 2\varepsilon r / s \quad (16)$$

and $b_{cf} = b - 2\varepsilon r / s$ (17)

2.4 roughness-viscosity model

Mala and Li [1] proposed a roughness-viscosity model to illustrate the effect of roughness in terms of a roughness-viscosity function, which is

$$\frac{\mu_R}{\mu} = B \,\mathrm{Re}_R \frac{y}{\varepsilon}\left(1 - \exp\left(\frac{\mathrm{Re}_R}{\mathrm{Re}} \frac{y}{\varepsilon}\right)\right) \quad (18)$$

The numerical results agree well with their own experimental data. However, the value of coefficient *B* needed in the model has to be determined using experimental data, which may depend on channel shape and roughness distribution and thus may vary from channel to channel. For circular and trapezoidal micro-channels, correlations were given by authors [1, 2] to evaluate the values of B. However, for other different channels such as triangular and rectangular micro-channels, the B value is still needed but not provided to users. This restricts the easy use of the model. In the present study, the experimental results from Mala and Li [1] and Qu et al. [2], which were analyzed with the roughness-viscosity model, were also compared with the constricted flow model for cross-check.

Although there are many data sets appear in literature, only those containing complete information are selected as the data base for comparison with these models. The selected experimental data sets on laminar flow in circular, rectangular and trapezoidal channels from literature are listed in Table 1. It should be mentioned that some literatures only give the maximum value of roughness ε_p, some others only give the average value of roughness ε_a, while only a few give both of them. Also, frequently it is not clear whether the diameter of the circular channels reported in the literature were the root diameter or not. If so, they were all assumed to be the root diameter *D*. To compare these three models of roughness effect on laminar flow, we use a normalized parameter Po_{exp}/Po_{th}, where Po_{th} is predicted by each of these three models.

Table 1 Selected experimental data sets on liquid flow in micro-channels

Literature	Year	Channel	$D_h(\mu m)$	ε_p (μm)	ε_a (μm)
Mala and Li [1]	1999	circular	50-152	/	1.75
Pfund et al. [3]	2000	rectangular	501	14.67	1.9
Qu et al. [2]	2000	trapezoidal	114.2-168.9	/	2.0
Kandlikar et al.[13]	2005	rectangular	953	72.9	17.9
Celata et al. [6]	2006	circular	126,299	/	0.16,0.7
Hao et al. [7]	2006	rectangular	191	50	50
Hao et al. [8]	2007	rectangular	193.8-196.4	10,20	10,20
Hrnjak and Tu [9]	2007	rctangular	69.5-305	0.21-0.48	0.77-2.4
Tang et al. [10]	2007	circular	172	/	7
Gamrat et al. [11]	2008	rectangular	191-585	5.8,10.6	/

3. RESULTS AND DISCUSSION

Figure 3 shows the normalized Po number (Po_{exp}/Po_{th}) at various relative roughness based on the experimental data sets listed in Table 1, where Po_{th} is predicted by classical theory (equations (4) and (6)) using the root hydraulic diameter. It shows that most of the points are markedly higher than unity which indicates that the measured pressure drop is higher than that predicted by classical theory.

Figure 4 shows the recalculated normalized Po number using the constricted flow model. It should be noticed that the relative roughness in the figure are differentiated as ε_p/D_h or ε_a/D_h when including the various data of literature in this figure. This is a little different from the original constricted flow method of Kandlikar et al. who used $\varepsilon_p/D_{h,cfp}$ [13]. It indicates that the *P*o numbers based on the constricted flow diameter agrees reasonably well with the predictions by the classical theory. Particularly, when the maximum values of roughness ($\varepsilon_p/D_{h,cfp}$) is used, the error of this constricted flow method is within ±15%, which is better than using the average roughness.

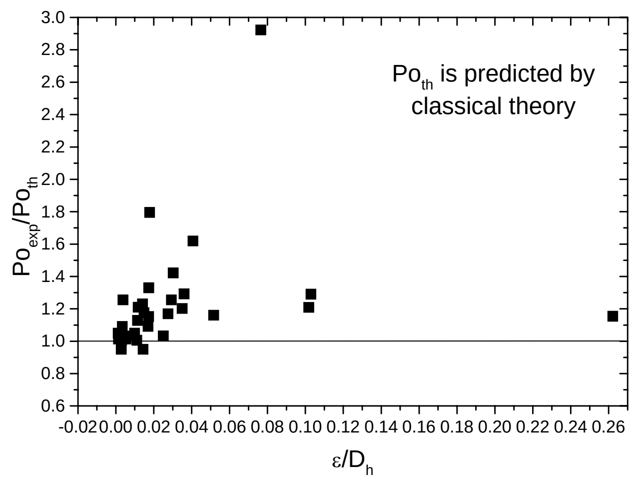


Fig. 3. Variation of normalized Po number versus relative roughness reported in literature.

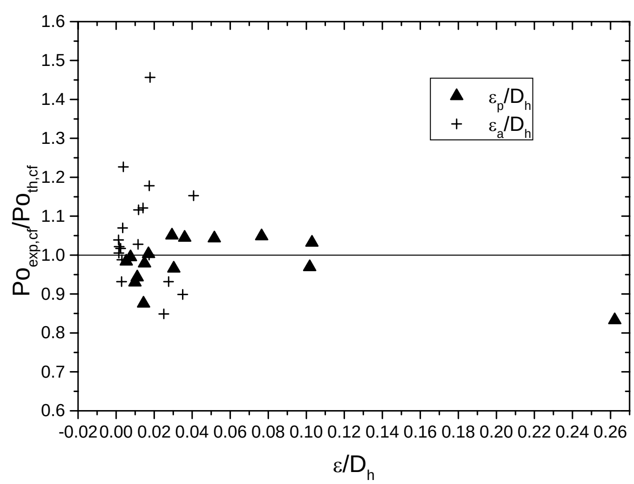


Fig. 4. Variation of normalized Po number versus relative roughness using the constricted flow model.

In examining the constricted flow model with roughness distribution, the spacing between roughness elements is not easily obtainable for natural roughness due to their random distribution. To investigate the roughness distribution effects, the data of artificial discrete roughness fabricated on channel bottom (Figure 1(a)) or on side walls (shown in Figure 2) are used. Figure 5 shows the normalized *P*o number predicted by the original constricted flow model [13] and that considering the roughness distribution [14]. It suggests that the roughness distribution model which modified the definition of hydraulic diameter using equations (7), (16) and (17) results in poorer predictions of normalized *P*o number than the original constricted model. Nevertheless, it is possible that future improvements of this model may have rooms for better results because the distribution of discrete roughness is intuitively important on some artificial roughness surfaces.

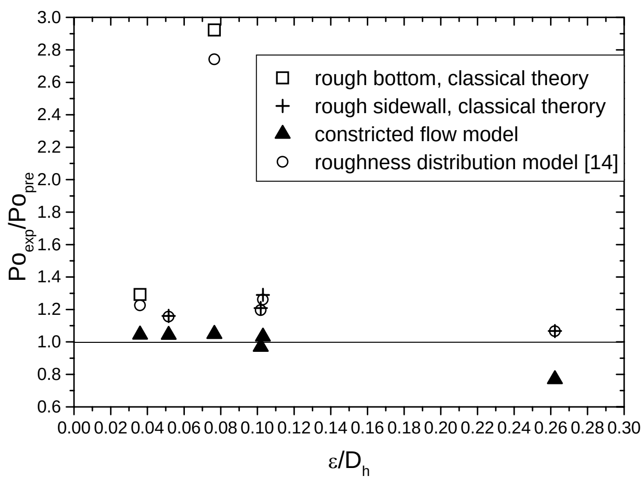


Fig. 5. Comparison of constricted flow model and roughness distribution model

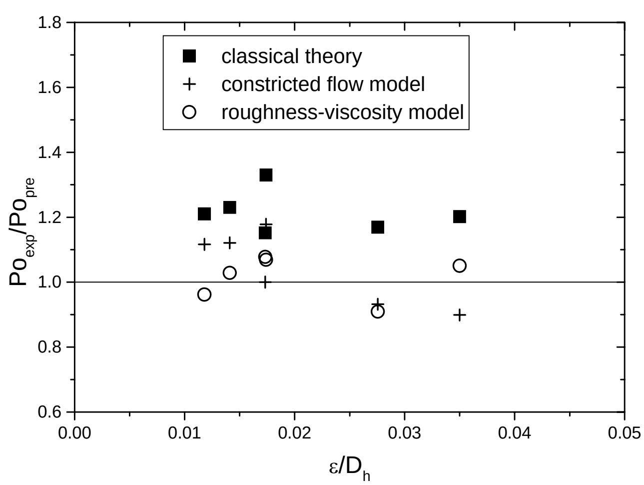


Fig. 6. Comparison of constricted flow model and roughness-viscosity model

Figure 6 illustrates the comparison of the constricted flow model and the roughness-viscosity model. It appears that within this limited data set the roughness-viscosity model predicts *P*o number slightly better than the constricted flow model. But it is also noticed that the roughness-viscosity is a far more complex method, which needs numerical computation and the empirical parameter *B* is required from experimental data.

4. CONCLUSIONS

Experimental results in the literature on the effect of surface roughness to the liquid flow pressure drop in mini and micro channels were examined. Three prominent theories, which are the constricted flow model, the constricted model considering roughness distribution, and the roughness-viscosity model are reviewed against this broad data base.

It is found that all the normalized data of *fRe* can be predicted, within an error of about ±15%, by the classical pressure drop theory with the channel dimension using the constricted flow model. On the other hand, it appears that the viscosity model is generally difficult to use, and the pitches of roughness distribution are usually hard to determine for natural rough-surfaces. However, the data, which supports these two theories, can also be predicted reasonably well by the constricted flow method at a similar accuracy. It is possible that the viscosity model and the constricted model with roughness distribution can be further developed in the future to become easier to use and covers more extensive physical nature of the phenomena. But, for the time being the simple constricted flow model could be considered as a general and reliable method for mini and micro channel pressure drop predictions at laminar flow.

NOMENCLATURE

A	cross sectional area of the channel (m^2)
a	up width of trapezoidal channel (m)
B	coefficient in roughness-viscosity function
b	height of the micro-channel (m)
c	bottom width of trapezoidal channel (m)
D	diameter (m)
f	Darcy friction factor
P	perimeter (m)
p	pressure (Pa)
*P*o	Poiseuille number, *fRe*
Q	volume flow rate (m^3/s)
r	roughness cone base radius (m)
Re	Reynolds number
s	roughness pitch (m)
u	velocity of flow (m/s)
x	length of the channel (m)
y	radial coordinate (m)

Greek Letters

α	aspect ratio, *b/a*
ε	roughness height (m)

μ	dynamic viscosity (Pa/s)
ρ	density (kg/m^3)

Subscripts

a	average roughness
cf	constricted flow
exp	experimental
h	hydraulic diameter
p	maximum roughness
pre	prediction
R	roughness
th	theoretical

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