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PORE SCALE SIMULATION OF TWO-PHASE FLUID FLOW IN BEREA SANDSTONE CORE USING LATTICE BOLTZMANN METHOD

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ABSTRACT

Study of flow through porous media has been an area of major interest due to its application in diverse areas like Enhanced Oil Recovery. In order to gain a better understanding of the physical processes taking place inside a porous structure, a large number of attempts have been made to computationally simulate multiphase fluid flow at pore-scale. Recently, application of Lattice Boltzmann Method has gained popularity for this very purpose, considering its relative superiority in dealing with complex boundaries and multiphase flow. However, in order that such a numerical analysis is successful, a proper understanding of the geometry of the pore structure at the microscale is required. This paper uses a Micro-CT scan image of a Berea Sandstone core, which displays a two dimensional representation of pore network inside the scanned sample. The processed image has been imported and simulation of an immiscible two-phase flow has been carried out by using a Lattice Boltzmann program. The resident fluid (oil) has been displaced by the invading fluid (water) due to application of a pressure gradient. The pore surfaces have been treated as solid boundaries and bounce back scheme has been implemented on them to account for the no-slip condition. The ability of the code to import an arbitrary porous geometry and perform numerical analysis of fluid flow has been demonstrated.

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INTRODUCTION

Present oil recovery from reservoirs include a number of secondary and tertiary methods involving injection of water, chemical solvents and gases into the pores of rock structures in order to drive out the resident oil. In order to study the various physical processes occuring inside a porous medium, pore-scale simulations have to be performed. Owing to the complex geometry of a porous medium and the complexity of physical processes taking place inside the micron sized pores, traditional computational tehniques prove to be inadequate for a comprehensive analysis. In this work, a meso-scale approach of Lattice Boltzmann method (LBM) has been used to simulate two dimensional multi-phase fluid flow in a deterministic pore structure of a Berea sandstone. In order that the geometry of the problem is an actual representative of the Berea pore structure, an image obtained from a micro-CT scan of the Berea core is imported into the LBM code. Displacement of resident oil by injected water due to applied pressure gradient is simulated by developing a multi-phase LBM model.

LITERATURE REVIEW

One of the more recent numerical techniques in the context of pore scale study is the Lattice Boltzmann method (LBM). Rothman (1988) initially used LBM for a simple porous medium and verified Darcy's law [1]. Ferreol et. al. (1995) made use of digitized image to regenerate a porous medium inside Fontainbleau sandstone and modeled flow by using LBM [2]. Kang et. al. (2002) proposed a LBM scheme to study fluid flow in

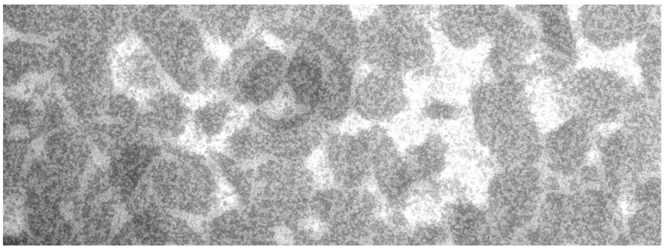

FIGURE 1. Original image depicting two dimensional pore structure inside a Berea sandstone core obtained by micro-CT scan.

porous media where multiple pore scales coexist [3]. Psihogios et. al. (2007) performed a study of non-newtonian fluid flow through digitally reconstructed porous media by using LBM [4]. Study of solvent diffusion into oil saturated porous media considering immiscibility with surface tension was performed by Can Ulas Hatiboglu and Tayfun Babadagli (2007) [5]. Two phase flow simulation in a digitally reconstructed porous media using LBM and including interfacial tension and wetting properties was peformed at two and three dimensional scales by Ramstad et. al. (2009) [6]. The present work attempts to use the two-color scheme in order to visualize displacement of resident oil by water, wherein the geometry of the porous medium is imported from a random micro-CT image of Berea sandstone core.

MICRO-CT OF BEREA SANDSTONE CORE

In order to generate a porous geometry which represents the actual porous structure inside a Berea sandstone, micro-CT of a $3mm \times 3mm$ core is performed on a Skyscan 1072 X ray microtomograph equipment. Resolution of the performed scan is $3\mu m$. A scan image representing the two dimensional pore structure of a random cross section is selected as a pore representative. This image is shown in Fig.1. A section of the image is considered for numererical analysis. This section is processed in the software ImageJ where it undergoes noise reduction and thresholding. The final image obtained in binary format is shown in Fig.2. This image is considered as the representative of the Berea porous structure and is imported into the LBM code by using its pixel values.

LATTICE BOLTZMANN MODEL

In order to study the displacement of resident oil from the pores by injected water, a multi-component fluid flow analysis in the porous structure obtained from the micro-CT image is performed. The numerical Lattice Boltzmann model used for this purpose is presented in this section. The final image obtained for numerical analysis is imported into the Lattice Boltzmann code by processing its pixel values. Each pixel is considered to be a single lattice with a property of being a solid particle or pore

FIGURE 2. Final image of a section of the original image obtained after processing. Black region depicts pore space and white region depicts solid grains.

space. The flow equations can then be applied to each lattice accordingly. Consequently, the original image is converted into a two dimensional grid of interconnected lattices, over which the LBM equations are applied and the multi-phase fluid flow is studied.

D2Q9 Lattice Model

The **D2Q9** lattice model is used for the application of transport equations, in order to conserve isotropy of the system. In this model a typical lattice can be visualized as a square with a lattice site stationed at the centroid and the center of each edge in addition to ones at the vertices. A particle population at the centroid node can advect to any of the 8 different nodes surrounding it or reside stationary at the site itself, giving rise to 9 different velocity configurations which have been presented in Fig. .

The weight factors associated with each velocity configuration are listed in Eq. 1.

$$W_0 = \frac{4}{9} \tag{1}$$

$$W_{1,2,3,4} = \frac{1}{9}$$

$$W_{5,6,7,8} = \frac{1}{36} \tag{2}$$

Governing Equations

Governing equations used for the Lattice Boltzmann model are presented in this section. It consists of the Boltzmann transport equation which is stated in Eq. 3. [7]

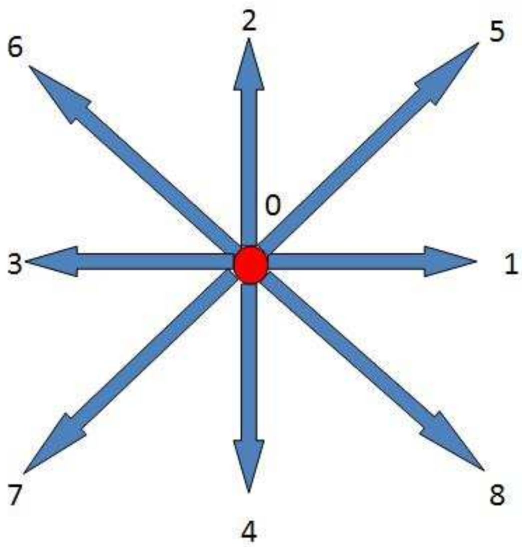

FIGURE 3. A typical D2Q9 lattice. Arrows denote 8 velocity directions and an additional stationary state is included.

$$\frac{\partial f}{\partial t} + c\nabla f = \Omega \tag{3}$$

In this equation c denotes the velocity of system and Ω is the collision operator.

In the presence of an external force such as pressure and surface tension, the equation can be modified as Eq. 4.

$$\frac{\partial f}{\partial t} + c\nabla f = \Omega + F \tag{4}$$

The main task in hand is the calculation of this collision operator and a number of schemes have been proposed for this very purpose. The present analysis uses the Bhatnagar-Gross-Krook (BGK) approximation developed in 1954. According to this scheme, the collision operator is replaced by a term containing the equilibrium distribution function f_{eq} snd a relaxation parameter.

$$\Omega = \omega(f - f_{eq}) = \frac{f - f_{eq}}{\tau} \tag{5}$$

Here f_{eq} is the equilibrium disbtribution function which represents the value of the distribution function when the system is in equilibrium. ω is the ralaxation and τ is the ralaxation time.

The relaxation parameter ω is related to the kinematic viscosity of the fluid by Eq.6.

$$\omega = \frac{1}{3\nu + 0.5} \tag{6}$$

Equilibrium distribution function in case of fluid flow is calculated by using the Chapman-Enslog expansion to obtain the distrubution function in terms of velocity.

$$f_k^{eq} = w_k \rho (1 + 3\frac{c_k.\mathbf{u}}{c_s^2} + \frac{9}{2}\frac{(c_k.\mathbf{u})^2}{c_s^4} - \frac{3}{2}\frac{\mathbf{u}.\mathbf{u}}{c_s^2}) \tag{7}$$

Here c_k denotes a typical lattice direction and c_s denotes the velocity of sound in the fluid which is given by

$$c_s = \frac{c_k}{\sqrt{3}} \tag{8}$$

With the application of BGK approximation, Boltzmann equation can be written as,

$$\frac{\partial f}{\partial t} + c\nabla f = \omega(f - f_{eq}) = \frac{f - f_{eq}}{\tau} + F \tag{9}$$

The Boltzmann equation is discretized into the Streaming and Collision equations stated in Eq. 10 and Eq. 11 respectively. These equations are the governing equations for the numerical model.

$$f_i(\mathbf{r}, t) = f_i(\mathbf{r} + \mathbf{c}, t + dt) \tag{10}$$

$$f_i(\mathbf{r}, t + dt) = f_i(\mathbf{r}, t) + \Omega + F \tag{11}$$

Conservation of mass

Lattice Boltzmann method applies the principle of conservation of mass at each lattice site in the entire grid which can be summed up to be realized as the conservation of mass for the entire fluid volume. The density at each lattice site is calculated as,

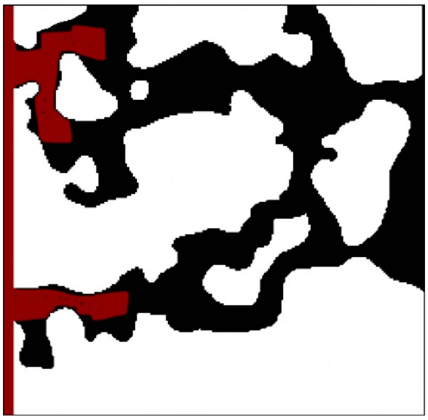

FIGURE 4. Displacement of resident oil (black) by injected water (red) after 37500 time steps.

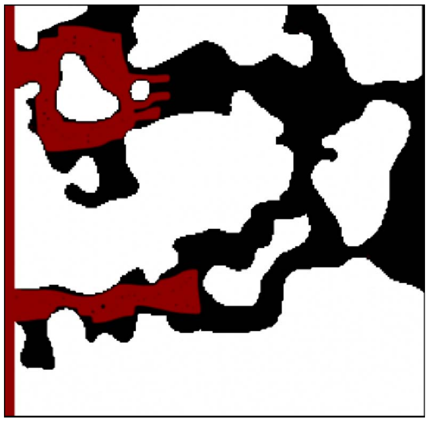

FIGURE 5. Displacement of resident oil (black) by injected water (red) after 75000 time steps.

$$\rho = \sum f_i \tag{12}$$

Here the value of n ranges over all the velocity states at the site.

Calculation of momentum

Momentum is calculated after every time step at each node in the fluid volume. Momentum is considered to be in the form of momentum per unit mass per unit volume or velocity.

$$\mathbf{v} = \frac{1}{\rho} \sum f_k.c_k \tag{13}$$

Here c_k denotes the unit vector in the direction k. In case of two dimensional fluid flow discretized by D2Q9 model, the velocity components can be calculated as listed in Eq. 14

$$\begin{aligned} v_x &= f_1 + f_5 + f_8 - f_3 - f_6 - f_7 \\ v_y &= f_2 + f_5 + f_6 - f_3 - f_7 - f_8 \end{aligned} \tag{14}$$

Multi-Fluid Approach

In order to incorporate two-fluid analysis in the LBM model, the two-color scheme developed by Gunstensen and Rothman [8] is used. It involves the solution of two distinct governing equations for two fluids present in the flow volume. The population distribution of these fluids can be denoted by R and B. The streaming equation can then be written as in Eq. 15.

$$\begin{aligned} R_i(\mathbf{r},t) &= R_i(\mathbf{r}+\mathbf{c},t+dt) \\ B_i(\mathbf{r},t) &= B_i(\mathbf{r}+\mathbf{c},t+dt) \end{aligned} \tag{15}$$

Similarly, the collision equation can be written as in Eq. 16.

$$\begin{aligned} R_i(\mathbf{r},t+dt) &= R_i(\mathbf{r},t) + \Omega_R + F_R \\ B_i(\mathbf{r},t+dt) &= B_i(\mathbf{r},t) + \Omega_B + F_B \end{aligned} \tag{16}$$

Here Ω_R and Ω_B are the collision functions calculated from the distinct relaxation parameters relating to the two fluids which are in turn calculated from the viscosities of the two fluids according to Eq. 6.

Mass and momentum of both fluids is calculated individually according to Eq. 12 and 14. The total mass and momentum at lattice sites is determined by summing up the individual masses and momenta of the two fluids.

$$\rho = \rho_R + \rho_B \tag{17}$$

$$\begin{aligned} v_x &= v_{xR} + v_{xB} \\ v_y &= v_{yR} + v_{yB} \end{aligned} \tag{18}$$

Presence of one fluid at a lattice site is determined by calculating mass fraction of one of the fluids (say R) at the lattice site.

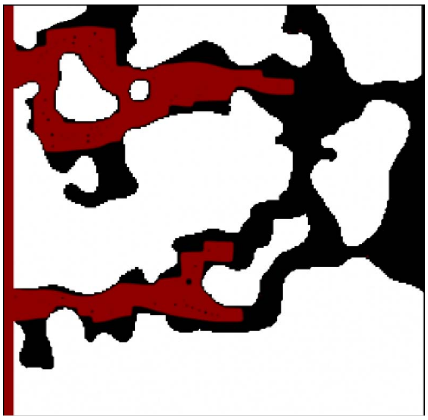

FIGURE 6. Displacement of resident oil (black) by injected water (red) after 112500 time steps.

$$M = \frac{\rho_R}{\rho} \tag{19}$$

This quantity is calculated at all the lattice sites throughout the grid.

$$M = 0 \quad (Bulk\ of\ fluid\ B) \tag{20}$$
$$M = 1 \quad (Bulk\ of\ fluid\ R) \tag{21}$$
$$0 < M < 1 \quad (Interface)$$

Interfacial tension force between the two fluids is calculated by using the quantity color gradient which is non-zero at the interface of the two fluids and calculated according to Eq.22. [8]

$$G(\mathbf{x}) = \Sigma(R_i(\mathbf{x}+\mathbf{c}_i) - B_i(\mathbf{x}+\mathbf{c}_i)) \tag{22}$$

The interfacial tension force F can then be calculated.

$$F \;=\; \sigma|G|cos(\theta_i - \theta_f) \tag{23}$$

Here, σ is the interfacial tension between the two fluids, θ_i denotes the direction of lattice direction and θ_f denotes the direction of the color gradient i.e. normal to the interface.In addition to this force, a minimization scheme is used [9] which ensures the immiscibility of the two fluids.

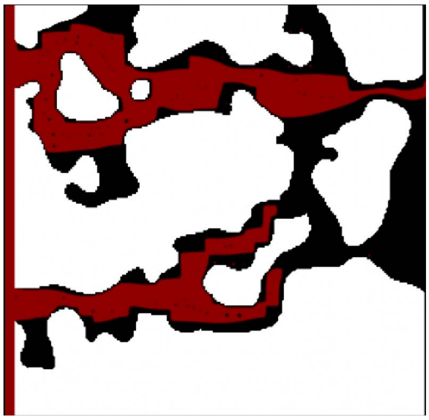

FIGURE 7. Displacement of resident oil (black) by injected water (red) after 150000 time steps.

Boundary Conditions

The Lattice Boltzmann model uses perioic boundary conditions at the domain boundaries in order to assure mass conservation in the flow volume. The no-slip boundary condition at the solid walls is applied by using the bounce-back scheme, wherein the population densities going inside the wall surface are simply reversed into the flow volume, so that the net velocity of the fluids at the wall is zero.

RESULTS AND DISCUSSION

Results obtained upon the application of the developed LBM model to geometry acquired from the image of the pore space for a particular pressure gradient are presented in this section. Initial fluid present in the pore space is oil (black). Water (red) enters the pore volume due to an applied pressure gradient of 10^{-6} lattice units and gradually displaces the resident oil out of the porous region. Density of both the fluids is considered to be 1 lattice unit. Applying a viscosity ratio of 2.0, viscosities of the red and the black fluid are considered to be 0.08 and 1.66 lattice units respectively. The interfacial tension between the two fluids is considered to be 2×10^{-5} lattice units which is in the range of the value observed in practical situaions. The simulation is run for 150000 which allows the simulation of complete traversal of a water front through the porous domain.

Images depicting the gradual displacement of resident oil by injected water are shown in Figures 4,5,6, and 7. It is observed that water enters the pore space through two inlets and displaces the resident oil through the outlets. The interaction between the two fluids is primarily at the interface, where the fluids remain immiscible and experience interfacial tension forces. The invasion of water front is predominantly independent from the walls of the pores, since wettability effects of the two fluids with respect to the pore walls are not considered in the developed model.

Hence, the displacement is incomplete and an amount of undisplaced oil is observed in the pores. Complete displacement of oil from the pores can be studied by inclusion of water-wet conditions for the pore walls.

CONCLUSION AND FUTURE WORK

In the present study, a Lattice Boltzmann model is used to study the displacement of resident oil by injected water from the pores of a geometry representing the porous structure inside a Berea sandstone core. The geometry is obtained from a random micro-CT image of a Berea core upon further processing in software ImageJ. The two-color approach is used in order to incorporate interfacial tension and viscosity ratio between the two fluids in the model and ensure immiscibility. It is observed that displacement of the resident is gradual and incomplete from the pore space, resulting in undisplaced oil in the pore region. In order to study the effect of preferential wettability on the displacement pattern, a surface tension force depending on the contact angle of the interface with the pore walls will have to be included in the LBM model. Then by varying the value of the equilibrium contact angle of the two-fluid and wall system, wettability effects can be incorporated in the model and more realistic results can be obtained at the pore scale.

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