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BEPU METHODOLOGIES AND PLANT TECHNICAL SPECIFICATIONS

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ABSTRACT

The Technical Specifications (TS) of a nuclear power plant define the conditions for a safe normal operation. With such an objective, the TS set limits on operational parameters of the plant and give surveillance requirements for the observation of such bounds. The values of TS limits are obtained from the safety analyses of the plant. In fact, the traditional conservative methodologies of deterministic safety analysis (DSA) have been profusely used in this task.

Nevertheless, in recent years realistic (also termed BEPU) methodologies have started to replace the conservative ones. This new methodologies use realistic models and assumptions and implement techniques for performing uncertainty analysis of their results. Many of them are statistical, with a probabilistic representation of uncertainty, and based on the random sampling of uncertain inputs and uncertainty propagation to the outputs.

In this paper the relation between BEPU safety analyses and TS is analyzed. The authors have a deep regulatory experience in the evaluation and licensing of DSA methodologies.

Safety analyses are aimed at showing that the *real* operation of the plant is safe, but they have a stronger goal: to prove that the *allowed* operation of the plant is safe. BEPU methodologies are not fitted for the estimation of TS bounds. They rather are used to prove the coherence of the safety analysis with the preestablished TS. Procedures for proving such coherence, with different degree of strictness, are discussed in the case of Monte Carlo- based methodologies.

1. INTRODUCTION

Realistic calculations are having a growing presence in nuclear safety analyses. Deterministic safety analyses (DSA) of nuclear power plants are based on the selection of transient and accident scenarios and the calculation of their consequences by means of predictive models. Traditionally, the models and assumptions used in the calculations have been pessimistic (i.e. conservative), in order to deal with the uncertainties about the real conditions of the plant and the performance of the predictive models.

In recent years, realistic state-of-the art models have been developed and implemented in computational codes, so that realistic consequence calculations for accident scenarios have been possible. Realistic calculations are useless unless they are supplemented with assessments of the uncertainty of their results, and this fact has boosted the development of methods for analyzing uncertainty in the realm of Nuclear Safety.

In this paper a discussion about the compatibility of BEPU methodologies and Plant Technical Specifications is presented. This topic has a crucial importance in the licensing of realistic safety analyses. However, it is generally overviewed in the discussion and study of this kind of methodologies.

The authors belong to Spain’s nuclear regulatory authority, and have a deep experience in the evaluation and licensing of BEPU methodologies.

2. BEPU METHODOLOGIES

A DSA methodology is defined as the set of predictive models, ancillary tools, guidelines, etc, needed to perform analyses of a specific set or category of accident scenarios. In the core of a methodology stand the predictive models, implemented in computational codes. Methodologies can be conservative or realistic, depending on the nature of their models and assumptions. Realistic methodologies are usually termed BEPU ("Best-estimate plus uncertainty"), because they implement best-estimate (i.e. realistic) models and assumptions as well as methods for performing uncertainty analysis of the results.

We may represent the predictive model of a methodology as a function

$$Y = R(X) \tag{1}$$

, transforming the input X in the output Y. X and Y are both, in general, multidimensional variables. R joins the input space (X-space) with the output space (Y-space). Points in the X-space represent possible "input decks" for the model. Y is the safety output, composed by the output variables that represent the consequences of the accident scenario.

Predictive models used in DSA are deterministic, in the sense that equal inputs produce equal outputs (i.e. they do not contain random number generators):

$$X_1 = X_2 \;\Rightarrow\; R(X_1) = R(X_2) \tag{2}$$

The final goal of a DSA methodology is to check the acceptability of the consequences of accident scenarios. Such consequences are considered as acceptable when they fulfil some acceptance criteria imposed on the safety outputs. In the traditional DSA methodologies, where pessimistic models and assumptions are used, the acceptance criteria are simple, and express that some safety outputs must remain in a prescribed region of the input space [1]:

$$Y \in SR \tag{3}$$

Here Y is the safety output, conservatively calculated. It is, in general, a multidimensional magnitude. SR is the "safe region" in the Y-space [1], established by the regulatory authority. Usually it is defined by setting limits on the values of the components of Y. We term "forbidden region" the complementary set of SR.

In BEPU methodologies, magnitudes are treated as uncertain, and the criterion (3) transforms to

$$Y \in SR \quad \text{"with a high level of certainty"} \tag{4}$$

At present, most BEPU methodologies make use of a probabilistic description of uncertainty, where uncertain magnitudes are represented by random variables, and uncertainty is broadly identified with probability distribution. The uncertainties are calculated with statistical methods, using random samples. In [1] it is described how, for these probabilistic methodologies, the acceptance criteria (4) adopt the following shape:

$$PR\{PR_Y\{Y \in SR\} \geq P_0\} \geq C_0 \tag{5}$$

This reads "Y must remain in SR with a probability of at least P_0 and with a statistical confidence no less than C_0". It is thus a doubly probabilistic criterion, with an inner probability related to the uncertainty of Y and an outer probability representing statistical uncertainty. In (5) Y is realistically calculated, in contrast with (3) where it is produced by a conservative calculation. P_0 and C_0 are the prescribed levels of coverage and confidence, respectively, and their values, close to 1, are established by the regulatory authority.

The uncertainty of a calculated magnitude Y derives from two main sources:

- The inputs to the calculation are uncertain themselves. Outputs inherit the uncertainty of inputs
- Models are imperfect, and thus they introduce errors in the outputs. Such errors are imperfectly known.

The uncertainty methods in the BEPU realm have three main stages:

1) Important inputs are identified, and assigned probability distributions. Here, "important" means "influential on the safety outputs involved in the acceptance criteria of the analyses"
2) Uncertainty is propagated from inputs to outputs
3) The fulfilment of the acceptance criteria by the safety outputs is checked

Most BEPU methodologies are based on Monte Carlo analysis, where input parameters are randomly sampled according to their assigned probability distributions. The predictive models transform the input random sample into an output random sample. Then, statistical inferences are performed on the random sample of safety outputs to estimate their uncertainty or, alternatively, to check the fulfilment of acceptance criteria like (5). A very popular procedure, nowadays, is based on simple random sampling and construction of nonparametric tolerance intervals by means of Wilks' theory [3]. A minimum value of the random sample size is derived from this theory, as a function of the prescribed coverage and confidence levels. This size is a measure of the computational effort needed in the uncertainty assessment.

3. THE ROLE OF INPUTS AND THE INVERSE PROBLEM

The assignment of probability distributions is essential for adequately performing an uncertainty analysis. If the input uncertainty is poorly estimated, the resulting output uncertainty will not be reliable.

Every safety analysis, be it conservative or realistic, must produce results that bound (in the conservative side) the real ones. The uncertain input parameters are sometimes fixed to conservative values in a BEPU analysis, instead of being assigned a probability distribution, due for instance to lack of information on them. Other parameters (for instance, those describing the spatila discretization) use to be chosen with a conservative point of view.

Safety analyses are aimed at proving that, with a specific choice of the input variables, the safety output satisfies the acceptance criteria, in the pertinent form ((3) or (5)). In this context we can define the *inverse problem* as the inference of values for the input parameters such that the resulting safety outputs fulfil the acceptance criteria.

For conservative methodologies, the inverse problem focuses on finding safe regions in the input space, i.e. regions in the X-space which are transformed by R in subsets of SR. It is a relatively easy task, because conservative models use to be fast-running and do not need uncertainty assessment.

On the other hand, BEPU methodologies are not very fitted for solving inverse problems. In this case, the problem may be formulated as finding out probability distributions for the inputs which produce outputs satisfying criteria like (5).

4. PLANT TECHNICAL SPECIFICATIONS AND SAFETY ANALYSIS

A group of input parameters especially important in safety analyses are those that describe the operational state of the plant when accidents occur. Some of them are so important that their value must be continuously surveyed to make sure that the operation is safe.

The Technical Specifications (TS) of the plant define the conditions for which the normal operation of the plant is considered as safe. In the TS, limitations are imposed on the value of some plant parameters (physical magnitudes) in order to ensure that the plant is always operating in an analyzed condition, namely that the hypotheses of the DSA are valid, so that the occurrence of a design basis event in such condition does not lead to the violation of safety limits. These constraints are sometimes termed limits and conditions for normal operation [2].

Therefore the TS define a "safe operation region" in the input space of the safety analyses, where the DSA hypotheses are fulfilled and thus the plant is operated in accordance with the design as documented in the safety analysis report (SAR).

The safe operation region is thus derived from the safety analyses, and, conversely, every safety analysis must keep coherence with the established TS.

Setting up the limits for the operating parameters is clearly related to the solution of the inverse problem. As stated in section 3, conservative DSA methodologies are fitted for this task. Traditionally, the limits of TS have been obtained in a straightforward fashion from these conservative safety analyses. The procedure is based on trying different values of the parameters and checking when Y reaches the boundaries of the safe region SR. The safe operation region is transformed by R in a subset of SR.

On the other hand, the estimation of TS limits by means of BEPU methodologies may be a really cumbersome task. The uncertainty assessment requires, for a Monte Carlo-based BEPU methodology, at least tens of code runs. A conservative methodology may require only one calculation to check the acceptance criteria.

Therefore BEPU methodologies should be applied to prove that established TS limits and conditions are coherent with a specific safety analysis. In other words, they check the acceptance criteria assuming that the operation of the plant is fulfilling the TS.

The most obvious way of accomplishing this task is by fixing the TS variables to their limit values and then proving that (5) is satisfied. Such procedure will take into account all the others sources of uncertainty in the calculations.

A less stringent procedure should be based on the assignation to the operational parameters of probability distributions compatible with the limitations stated in the TS. In this respect, it is important to point out that the plant TS include surveillance requirements [2], in order to ensure that the limits and conditions for normal operation are fulfilled. The surveillance activities must be planned and may include monitoring, inspection, checking, calibration and testing. The probability distribution assigned to the operational parameters must take into account the uncertainty introduced by this surveillance activities.

However, the probability distribution assigned to TS parameters must not reflect the real operation, but the allowed operation of the plant. In other words, such distributions must assign a significant probability to values of the parameters beyond their operating limits. In this way, the BEPU analysis will prove the safety of the allowed operation, and not only of the real operation.

Different procedures may be envisaged for assigning these modified probability distributions to the TS parameters. A simple possibility is to introduce a conservative bias in the parameter, shifting the real distribution in the conservative direction. Care must be exercised in order to prevent the parameter from taking values excessively conservative or unphysical.

To fix ideas let us suppose a very simple case, where Y is a continuous scalar output with an acceptance criterion

$$Y < L \tag{6}$$

, L being an upper safety limit. (6) is a particular form of (5). Let us also suppose that two of the input parameters, say X_1 and X_2, are scalar operating parameters limited by TS for normal operation as:

$$\begin{aligned} X_1 &< L_1 \\ X_2 &< L_2 \end{aligned} \tag{7}$$

, L_1 and L_2 being operating limits. It is easy to suppose that R is an increasing function of both X_1 and X_2 in the region of allowed operation of the plant.

Let us further suppose that a BEPU methodology, based on Monte Carlo is used to analyze a specific design basis event with the criterion (6). As previously mentioned, the best way to prove the compatibility between safety analysis and TS is by fixing $X_1 = L_1$ and $X_2 = L_2$ and then performing the Monte Carlo analysis, by sampling the remaining uncertain inputs, running the code and finally checking (5).

But the licensee may try a less strict procedure, assigning probability distributions to X_1 and X_2. In order to judge the adequacy of such procedure, the following probabilities must be considered:

$$\begin{aligned} P_{EX,1} &\equiv PR\{X_1 \geq L_1\} \\ P_{EX,2} &\equiv PR\{X_2 \geq L_2\} \\ P_{EX,12} &\equiv PR\{X_1 \geq L_1 \quad and \quad X_2 \geq L_2\} \end{aligned} \tag{8}$$

In (8) $P_{EX,1}$ (resp. $P_{EX,2}$) is the probability that X_1 (resp. X_2) exceeds the operating limit L_1 (resp. L_2). $P_{EX,12}$ is the probability of the simultaneous violation of both limits.

If $P_{EX,1}$ is high enough it is expected that a significant fraction of the Monte Carlo runs will be performed with values of X_1 equal or beyond the TS limit. In such case we can say that the Monte Carlo sampling is exploring the forbidden region of X_1. The same can be said about $P_{EX,2}$ and X_2. Furthermore, if $P_{EX,12}$ is high enough, it is expected that a significant fraction of the Monte Carlo runs will be exploring simultaneously the forbidden region of both variables.

A condition for such an exploration may be

$$E[n_i] \geq r_i \qquad i = 1, 2 \tag{9}$$

, where n_i is the number of Monte Carlo runs with $X_i \geq L_i$. r_1 and r_2 are integers, $r_i \geq 1$. (9) is the condition that the expected number of runs where the operating limit is surpassed must be higher than a prescribed number.

A stronger condition than (9) is

$$PR\{n_i \geq r_i\} \geq C_i \qquad i = 1, 2 \tag{10}$$

, where it is required that, with probability no less than C_i, the number of runs with exceedance of L_i be at least a prescribed value r_i. C_i is a number in the interval (0,1) and close to 1.

Thus (9) and (10) are a type of conditions that should be imposed in the analysis in order to assure an adequate exploration of the forbidden regions of X_1 and X_2. The conditions are characterized by the values of r and C.

n_1 is a binomial variable with parameters N and $P_{EX,1}$, and n_2 is also binomial with parameters N and $P_{EX,2}$. By using the binomial distribution, (9) is simply

$$P_{EX,i} \geq \frac{r_i}{N} \qquad i = 1, 2 \tag{11}$$

, and (10) transforms into

$$\sum_{k=r_i}^{N} \binom{N}{k} P_{EX,i}^{k} \left(1 - P_{EX,i}\right)^{N-k} \ \geq \ C_i \qquad i = 1, 2 \tag{12}$$

It is a well known result (see e.g. [4]) that (12) can be recast in terms of the beta probability distribution, as follows

$$r_i \binom{N}{r_i} \int_0^{P_{EX,i}} x^{r_i - 1} (1 - x)^{N - r_i} \, dx \ \geq \ C_i \qquad i = 1, 2 \tag{13}$$

The left hand side of (13) is the cumulative distribution function (cdf) of the beta random variable with parameters r_i and $N-r_i+1$. Then (13) can be rewritten in a very compact form

$$P_{EX,i} \geq {}_{C_i}beta(r_i, N - r_i + 1) \qquad i = 1, 2 \tag{14}$$

The right hand side of (14) is the quantile C_i of the beta variable. The expressions (12) to (14) coincide with the condition derived from Wilks' theory on nonparametric tolerance intervals [3] for the coverage of a prescribed quantile of a random variable X by an order statistic, with a certain level of statistical confidence. A difference is that here we are using the probability of exceeding the operating limit instead of the level of coverage.

The coincidence is not surprising, because both our reasoning and Wilks' theory are based on the binomial distribution. In fact, requiring $n_i \geq r_i$ is equivalent to requiring that the order statistics of X_i with order $N-r_i+1$ exceeds L_i.

Stronger conditions are obtained when *simultaneous* exceedances of the operating limits are required. For instance

$$E[n_{12}] \geq r_{12} \tag{15}$$

, n_{12} being the number of runs with $X_1 \geq L_1$ and $X_2 \geq L_2$ simultaneously, and r_{12} the prescribed minimum value. n_{12} is binomial with parameters N and $P_{EX,12}$, and thus (15) is

$$P_{EX,12} \geq \frac{r_{12}}{N} \tag{16}$$

An even stronger condition is

$$PR\{n_{12} \geq r_{12}\} \geq C_{12} \tag{17}$$

, or, equivalently

$$P_{EX,12} \geq {}_{C_{12}}beta(r_{12}, N - r_{12} + 1) \tag{18}$$

, provided C_{12} is high enough.

For the same values of r and C, the conditions (15) and (17) are clearly stronger (i.e. more strict) than (9) and (10), respectively. In fact, when X_1 and X_2 are independent variables, or rather when *they are treated* as independent variables, which the usual practice in BEPU analyses, the exceedance probability factorises:

$$P_{EX,12} = P_{EX,1} \cdot P_{EX,2} \tag{19}$$

, and, in such case, a sufficient condition for (16) is

$$P_{EX,i} \geq \sqrt{\frac{r_{12}}{N}} \qquad i = 1, 2 \tag{20}$$

, and a sufficient condition for (18) is

$$P_{EX,i} \geq \sqrt{{}_{C_{12}}beta(r_{12}, N - r_{12} + 1)} \qquad i = 1, 2 \qquad (21)$$

Another possible condition, intermediate between (10) and (17), is

$$PR\{n_1 \geq r_1 \text{ and } n_2 \geq r_2\} \geq C \qquad (22)$$

When X_1 and X_2 are independent, a sufficient condition for (22) is

$$PR\{n_i \geq r_i\} \geq \sqrt{C} \qquad i = 1, 2 \qquad (23)$$

, or, equivalently

$$P_{EX,i} \geq {}_{\sqrt{C_i}}beta(r_i, N - r_i + 1) \qquad i = 1, 2 \qquad (24)$$

Let us compare the different choices in our simple example, when the Monte Carlo size is N=100 and X_1 and X_2 are independent random variables. Putting r=5 and C=0.95, the conditions for i=1, 2 , with growing strictness, are

- Condition (11)

$$P_{EX,i} \geq \frac{5}{100} = 0.05$$

- Condition (14)

$$P_{EX,i} \geq {}_{0.95}beta(5, 96) = 0.09$$

- Condition (24)

$$P_{EX,i} \geq {}_{0.975}beta(5, 96) = 0.10$$

- Condition (20)

$$P_{EX,i} \geq \sqrt{0.05} = 0.22$$

- Condition (21)

$$P_{EX,i} \geq \sqrt{0.09} = 0.30$$

It is evident that higher exceedance probabilities correspond to stronger conditions. An analyst using the condition (11) will assign, for the BEPU calculations, probability distributions to X_1 and X_2 producing exceedance probabilities no less than 0.05. In the case that the analyst prefers a much more strict condition like (21), the probability distributions will be chosen so as to give exceedance probabilities no less than 0.30 for each parameter.

Our example is very simple and only involves two parameters. For a general case with p parameters limited by TS, conditions requiring the simultaneous exceedances of any number of operational limits, between 1 and p, may be established in an analogous manner, and with different degrees of strictness. Of course, the most strict procedure upon performing a BEPU analyses is to fix the TS parameters on the value of their operating limits.

5. CONCLUSIONS

An adequate assignation of probability distributions to the uncertain inputs is essential in order to conduct reliable BEPU analyses. An important category of inputs are operational parameters controlled by the plant Technical Specifications. The TS define a region of allowed operation of the plant, and the complementary forbidden region. The limits in the TS are derived from safety analyses and, conversely, safety analyses must be compatible with the established TS. In fact, safety analysis must prove not only that the real operation of the plant is safe, but also that the allowed operation of the plant is safe

BEPU methodologies are not fitted for obtaining TS limits; they rather must prove their compatibility with them. A strict procedure to do that should fix the TS parameters on their limit values and perform the BEPU analysis. A looser procedure should be based on the assignation to the TS parameters of probability distributions which are not based on the real operation, but on the allowed operation of the plant, so that they assign a significant (non negligible) probability to the violation of the operating limits. For BEPU methodologies based on pure Monte Carlo, the goal is that the random sampling of the input space provides a significant exploration of the forbidden regions for the TS parameters (i.e. the regions beyond the limits).

In this paper we have proposed some simple criteria, proceeding from an easy example, to prove a feasible exploration around the TS limits. They are based on requirements about the number of Monte Carlo runs having values of the TS parameters beyond their limits. They have different degrees of strictness, depending on the number of required limit exceedances, the character (individual or collective) of such exceedances, etc.

The criteria stated in this paper may be useful for both the analyst performing a BEPU analysis or the regulator evaluating it. The strictness of the chosen criteria should be judged in conjunction with the conservatisms contained in the BEPU methodology.

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