Proceedings of the ASME 2010 3rd Joint US-European Fluids Engineering Summer Meeting and 8th International Conference on Nanochannels, Microchannels, and Minichannels
FEDSM-ICNMM2010
August 1-5, 2010, Montreal, Canada

FEDSM-ICNMM2010-31230

REYNOLDS STRESS ANISOTROPY IN ROTATING HOMOGENEOUS DECAY

Karuna Koppula
Department of Chemical Engineering and Materials Science
Michigan State University
East Lansing, MI 48824

André Bénard
Department of Mechanical Engineering
Michigan State University
East Lansing, MI 48824

Charles Petty
Department of Chemical Engineering and Materials Science
Michigan State University
East Lansing, MI 48824

ABSTRACT

In rotating homogeneous decay, the prolate quadratic form associated with the normalized Reynolds (NR-) stress is elongated by a coupling between velocity fluctuations and the Coriolis acceleration. This paper shows that this well-known turbulence phenomenon is consistent with an algebraic anisotropic prestress (APS-) closure for the NR-stress that unifies the study of turbulent flows in rotating and non-rotating frames-of-reference. The APS-closure is a non-negative mapping of the NR-stress into itself and is, thereby, *universally realizable* for all turbulent flows.

INTRODUCTION

Previous experimental, numerical, and theoretical results related to rotating and non-rotating homogeneous decay of turbulence support the idea that the turbulent kinetic energy and the turbulent dissipation satisfy the following autonomous, non-linear, ordinary differential equations[1,2,3,4,5,6,7]:

$$\frac{dk}{dt} = -\varepsilon \quad ,$$

$$k \equiv \mathrm{tr} < \underline{u}'\underline{u}' > /2 \quad , \tag{1}$$

$$\varepsilon \equiv \nu \, \mathrm{tr} < (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T >$$

$$\frac{d\varepsilon}{dt} = -C_D \frac{\varepsilon}{\tau_R} . \tag{2}$$

For $k^2/(\nu\varepsilon) \to \infty$, the dissipation turnover time τ_R depends on k/ε and $1/\|\underline{\Omega}\|$, where $\underline{\Omega}$ is the angular velocity of the rotating frame. Koppula et al.[6] formulated the following equation for τ_R by using the results of a spectral analysis of the decay process developed by Park and Chung[7]:

$$\tau_R = C_D \frac{dk}{d\varepsilon} = C_{R1} \frac{k}{\varepsilon} \tilde{\tau}_R = C_{R1} \frac{k}{\varepsilon} \frac{(1 + C_{R3} N_F^{3/2})}{(1 + C_{R2} N_F^{3/2})} = \quad ,$$

$$\frac{11}{16} = \frac{C_{R3}}{C_{R2}} = \tilde{\tau}_R(\infty) \le \tilde{\tau}_R(N_F) \le \tilde{\tau}_R(0) = 1 \quad . \tag{3}$$

The parameters in Eq.(3) are related to the Saffman energy spectrum $E_S(\kappa) = B\kappa^{+2}$, $0 \le \kappa \le \kappa_\ell = \varepsilon / k^{3/2}$; and, the

Osmidov energy spectrum, $E_O(\kappa) = C_\Omega(\varepsilon|\Omega_x|)^{1/2}\kappa^{-2}$, $\kappa_\ell \le \kappa \le \kappa_\Omega = (|\Omega_x|^3/\varepsilon)^{1/2}$. The group $N_F(\equiv 2\sqrt{2}|\Omega_x|k/\varepsilon)$ is related to a length scale associated with the energy containing "eddies" and a length scale associated with the Osmidov "eddies": $\kappa_\Omega/\kappa_\ell \propto N_F^{3/2}$. In general, $\tilde{\tau}_R$ depends on $N_F(\equiv \| <\underline{\underline{F}}> \| k/\varepsilon)$, which compares the turbulent time scale k/ε with the mean field time scale $1/\| <\underline{\underline{F}}> \|$, where $\| <\underline{\underline{F}}> \| \equiv (<\underline{\underline{F}}>:<\underline{\underline{F}}>^T)^{1/2}$ and $<\underline{\underline{F}}> = \nabla <\underline{u}> + 2\underline{\underline{\Omega}}$. For rotating homogeneous flows, $<\underline{\underline{F}}> = 2\Omega_x(\underline{e}_y\underline{e}_z - \underline{e}_z\underline{e}_y)$ and $\| <\underline{\underline{F}}> \| = 2\sqrt{2}|\Omega_x|$. Experimental and DNS results for homogeneous decay and for non-rotating homogeneous simple shear were used to estimate the parameters in Eqs.(2) and (3) with the result that $C_D/C_{R1} = 11/6$, $C_{R3}/C_{R2} = 11/16$, $C_{R1} = 0.0036$, and $C_{R2} = 0.076$ (see Ref. 6). Eqs.(1) and (2)) were solved for a range of rotation numbers subject to the initial conditions $k(0) = k_0$ and $\varepsilon(0) = \varepsilon_0$. A 4th-order Runge-Kutta algorithm supported by MatLab® with a time increment limiter of $\Delta\tilde{t} = 10^{-6}$ was used to integrate Eqs.(1) and (2). The dimensionless time $\tilde{t}$ is defined as $t\varepsilon_0/k_0$.

Figures 1 and 2 summarize the influence of $\tilde{\Omega}_0(\equiv|\Omega_x|k_0/\varepsilon_0)$ on $\tilde{k} \equiv k/k_0$, $\tilde{\varepsilon} \equiv \varepsilon/\varepsilon_0$, and $\tilde{k}^2/\tilde{\varepsilon}$. The results for $\tilde{\Omega}_0 = 10$ and $\tilde{\Omega}_0 = \infty$ are about the same. The decay results for $0 < \tilde{\Omega}_0 < 10$ all lie between the two curves shown on Figure 1. During the initial stage (i.e., $0 < \tilde{t} < 0.1$), an enhanced rate-of-decay of $\tilde{\varepsilon}$ arises due to a decrease in the dimensionless turnover time $\tilde{\tau}_R$ as the rotation number $N_F(= 2\sqrt{2}\,\tilde{\Omega}_0\tilde{k}/\tilde{\varepsilon})$ increases. The influence of rotation on $\tilde{k}$ is relatively small during the initial stage. However, during the intermediate stage (i.e., $0.1 < \tilde{t} < 4$), the rate of decay of $\tilde{k}$ is mitigated by the smaller dissipation that developed during the initial stage. For the final (non-viscous) decay period (i.e., $\tilde{t} > 4$), the rate-of-decay of $\tilde{\varepsilon}$ decreases due to the persistence of a relatively large value of $\tilde{k}$ as $\tilde{k}/\tilde{\varepsilon} \to \infty$. Thus, for $\tilde{t} \gg 4$, $N_F \to \infty$ and $\tilde{\tau}_R \to C_{R3}/C_{R2} = 11/16$. It is noteworthy that the dissipation for $\tilde{\Omega}_0 = 10$ eventually exceeds the dissipation for $\tilde{\Omega}_0 = 0$. This feature, which was not noted by Park and Chung[7], shows that rotation actually retards the rate-of-decay of dissipation during the final stage of the decay process. For $0 < \tilde{\Omega}_0 < 1$, $\tilde{k}^2/\tilde{\varepsilon}$ decreases during the initial stage and increases during the final stage (see Figure 2). For $\tilde{\Omega}_0 \ge 1$, $\tilde{k}^2/\tilde{\varepsilon}$ increases monotonically. However, for $\tilde{\Omega}_0 = 0$, $\tilde{k}^2/\tilde{\varepsilon}$ decreases monotonically as $\tilde{t} \to \infty$.

The objective of this paper is to calculate the Reynolds stress anisotropy for rotating homogeneous decay based on a recently developed algebraic closure for the normalized Reynolds (NR-) stress, $\underline{\underline{R}} \equiv <\underline{u}'\underline{u}'>/\mathrm{tr}<\underline{u}'\underline{u}'>$. For a constant density constant viscosity fluid, the fluctuating velocity $\underline{u}'(\underline{x}, t)$ satisfies the continuity equation, $\nabla \cdot \underline{u}' = 0$ and the following dynamic equation in a rotating frame of reference [6]:

$$\frac{\partial \underline{u}'}{\partial t} + <\underline{u}> \cdot \nabla\underline{u}' - \nu\nabla^2\underline{u}' = -\underline{u}' \cdot <\underline{\underline{F}}> - \underline{f}', \qquad (4)$$
$$<\underline{\underline{F}}> \equiv \nabla <\underline{u}> + 2\underline{\underline{\Omega}}$$

In Eq.(4), $\underline{f}'(\equiv \nabla \cdot [\, p'\underline{\underline{I}}/\rho + \underline{u}'\underline{u}' - <\underline{u}'\underline{u}'>])$ is an intrinsic acceleration induced by pressure fluctuations and fluctuations in the instantaneous Reynolds momentum flux. The anti-symmetric rotation operator $\underline{\underline{\Omega}}$ and the angular velocity vector $\underline{\Omega}$ are related by $\underline{\underline{\Omega}} \equiv \underline{\underline{\underline{\varepsilon}}} \cdot \underline{\Omega}$. The Coriolis acceleration in Eq.(4) produces a fluctuating force (i.e., $-2\rho\underline{u}' \cdot \underline{\underline{\Omega}}$) orthogonal to the angular velocity $\underline{\Omega}$. Because $<\underline{u}'\underline{u}'>:\underline{\underline{\Omega}} \equiv 0$, the Coriolis force is unable to produce turbulent kinetic energy; however, it can induce anisotropy in the NR-stress. This phenomenon directly affects the primary and secondary normal stress differences and would, thereby, be an important factor in supporting secondary flows in inhomogeneous flows.

The quadratic form $Q(\underline{z})$ associated with the NR-stress is non-negative. Therefore,

$$Q(\underline{z}) \equiv \underline{\underline{R}} : \underline{z}\underline{z} \ge 0 \quad , \quad \forall \underline{z} \in E^3, \|\underline{z}\| = 1. \qquad (5)$$

This fundamental inequality stems directly from the physical condition that the underlying velocity distribution functional defined over an ensemble of turbulent flow fields is non-negative. Furthermore, Ineq.(5) implies that the eigenvalues of the NR-stress are non-negative.[5] A Reynolds stress model that produces an NR-stress that satisfies Ineq.(5) for all non-rotating and for all rotating turbulent flows is, by definition, *universally realizable*. For homogeneous decay, the quadratic form $Q(\underline{z})$ may be oblate, prolate, or isotropic [3, 8, 9]. During the initial stage of decay, the anisotropy is sensitive to the initial conditions. If the initial state is isotropic ($R_{xx} = R_{yy} = R_{zz} = 1/3$), then prolate anisotropic states can develop due to strong non-linear interactions among the velocity components (i.e., $\| \underline{f}'(\underline{x},t) \| \cong \| \underline{u}'(\underline{x},t)\cdot\underline{\underline{\Omega}} \|$). As demonstrated by Cambon et al.[8], turbulent kinetic energy associated with velocity fluctuations in a plane orthogonal to $\underline{\Omega}$ is transferred to the fluctuating velocity component aligned with $\underline{\Omega}$; therefore, $0 < R_{yy} = R_{zz} < 1/3 < R_{xx} < 1$.

The new closure for the NS-stress relates the hydrodynamic/kinematic operator $\tau_R < \underline{\underline{F}} >$ to a prestress operator $\underline{\underline{B}}\ (\equiv < \underline{f}'\underline{f}' > / \mathrm{tr} < \underline{f}'\underline{f}' >)$:

$$\underline{\underline{R}} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{B}} \cdot \underline{\underline{A}}}{\mathrm{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{B}} \cdot \underline{\underline{A}})} \quad , \quad \underline{\underline{A}} \equiv [\underline{\underline{I}} + \tau_R < \underline{\underline{F}} >]^{-1} \tag{6}$$

Eq.(6) follows from a formal analysis of Eq.(4) for the velocity fluctuating field [6,10,11,12]. The prestress operator $\underline{\underline{B}}$ is related to an intrinsic fluctuating force $\underline{f}'$. The preclosure operator $\underline{\underline{A}}$ depends on the kinematic operator $< \underline{\underline{F}} >$ and a turbulent transport time τ_R introduced by a statistical smoothing *anzatz* based on the idea that turbulent space-time correlations have finite memories compared with the relaxation of a convective/viscous Green's function. Dimensional reasoning implies that τ_R depends on three different time scales: a viscous time scale, ν / k; a turbulent time scale, k/ε; and, a mean field time scale, $1/ < \underline{\underline{F}} >$:

$$\begin{aligned} \tau_R &= C_{R1} \frac{k}{\varepsilon} \tilde{\tau}_R (\mathrm{Re}_t, N_F) \\ &\xrightarrow{\mathrm{Re}_t \equiv k^2/\varepsilon\nu \to \infty} C_{R1} \frac{k}{\varepsilon} \tilde{\tau}_R (\infty, N_F) \end{aligned} \tag{7}$$

The prestress operator $\underline{\underline{B}}$ is assumed to depend on the NR-stress, $\underline{\underline{R}}$. This self-consistent closure hypothesis can be represented in closed form by the application of the Cayley-Hamilton theorem[5]:

$$\begin{aligned} \underline{\underline{B}} &= \underline{\underline{R}} + C_1 (\underline{\underline{R}} - \underline{\underline{I}}/3) + C_2 (\underline{\underline{R}} \cdot \underline{\underline{R}} - \mathrm{II}_R\, \underline{\underline{R}}), \\ \mathrm{II}_R &\equiv \underline{\underline{R}} : \underline{\underline{R}} \end{aligned} \tag{8}$$

The CH-coefficients in Eq.(8) can be expressed as $C_1 = 27\beta \det(\underline{\underline{R}})$ and $C_2 = -\alpha (3\mathrm{II}_R - 1)/3$. If α and β satisfy the following two inequalities

$$-3/2 < \alpha < 9 \tag{9}$$

$$-1 < \beta < 0.0370\, \alpha + 4/5, \tag{10}$$

then Eq.(8) is a non-negative CH-mapping of $\underline{\underline{R}}$ into $\underline{\underline{B}}$ (see Ref. 6). Therefore, the anisotropic prestress (APS-) closure, defined by Eq.(6) and Eq.(8), is a non-negative mapping of $\underline{\underline{R}}$ into itself. The mapping is *realizable* for all rotating and non-rotating turbulent flows. This theoretical conclusion does not depend on a specific benchmark flow used to calibrate the mapping parameters α and β; however, α and β must satisfy Ineqs.(9) and (10). Koppula et al.[6] used experimental results related to asymptotic non-rotating homogeneous shear and determined that α = + 0.10 and β = – 0.010. Eq. (8) generalizes the isotropic prestress (IPS-) closure developed earlier by Parks et al.[10,11] and Weispennig et al.[12] wherein $\underline{\underline{B}} = \underline{\underline{I}}/3$.

For rotating homogeneous flows, $\tau_R < \underline{\underline{F}} > = 2\tau_R \underline{\underline{\Omega}}$. Therefore, the preclosure operator $\underline{\underline{A}}$ (see Eq.(6) above) and its inverse $\underline{\underline{A}}^{-1}$ can be expressed as

$$\begin{aligned} \underline{\underline{A}}^{-1} &= \underline{\underline{I}} + \tau_R < \underline{\underline{F}} > = \\ &\underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z + \tilde{N}_\Omega (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y) \end{aligned} \tag{11}$$

$$\begin{aligned} (1+\tilde{N}_\Omega^2)\underline{\underline{A}} &\equiv \\ &(1+\tilde{N}_\Omega^2)\, \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z - \tilde{N}_\Omega (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y) \end{aligned} \tag{12}$$

where $\tilde{N}_\Omega \equiv 2\tau_R \Omega_x = 2 C_{R1} \tilde{\tau}_R\, k \tilde{\Omega}_0 / \tilde{\varepsilon}$ and $\tilde{\Omega}_0 \equiv k_0 \Omega_x / \varepsilon_0$. For $\nabla < \underline{u} > = \underline{0}$, an analysis of Eqs.(6) and (8) shows that $B_{xy} = 0$, $B_{xz} = 0$, $R_{xy} = 0$, and $R_{xz} = 0$. Furthermore, with $B_{yz} = 0$ and $B_{yy} = B_{zz}$, the shear component of the NR-stress is also zero (i.e., $R_{yz} = 0$); therefore, for rotating homogeneous decay, Eq.(6) implies that

$$\begin{aligned} R_{xx} &= [(1+\tilde{N}_\Omega^2)^2 B_{xx}] / \\ &\left([(1+\tilde{N}_\Omega^2)^2 B_{xx}] + [B_{yy} + \tilde{N}_\Omega^2 B_{zz}] + [B_{zz} + \tilde{N}_\Omega^2 B_{yy}] \right) \end{aligned} \tag{13}$$

$$\begin{aligned} R_{yy} &= [B_{yy} + \tilde{N}_\Omega^2 B_{zz}] / \\ &\left([(1+\tilde{N}_\Omega^2)^2 B_{xx}] + [B_{yy} + \tilde{N}_\Omega^2 B_{zz}] + [B_{zz} + \tilde{N}_\Omega^2 B_{yy}] \right) \end{aligned} \tag{14}$$

$$\begin{aligned} R_{xx} &= [B_{zz} + \tilde{N}_\Omega^2 B_{yy}] / \\ &\left([(1+\tilde{N}_\Omega^2)^2 B_{xx}] + [B_{yy} + \tilde{N}_\Omega^2 B_{zz}] + [B_{zz} + \tilde{N}_\Omega^2 B_{yy}] \right) \end{aligned} \tag{15}$$

It is noteworthy that the preclosure operator $\underline{\underline{A}}$ couples with the prestress operator $\underline{\underline{B}}$ to shift turbulent kinetic energy from the plane orthogonal to the rotation axis into the fluctuating velocity aligned with the angular velocity. Eqs.(13)-(15) shows how the anisotropy depends on the rotation group $\tilde{N}_\Omega$. The components of $\underline{\underline{R}}$ are not affected by the sign of Ω_x.

Eq.(8) implies that the prestress components are related to the components of the NR-stress by the following scalar equations:

$$B_{xx} = R_{xx} + C_1(R_{xx} - 1/3) + C_2(R_{xx}^2 - II_R R_{xx}) \quad (16)$$

$$B_{yy} = R_{yy} + C_1(R_{yy} - 1/3) + C_2(R_{yy}^2 - II_R R_{yy}) \quad (17)$$

$$B_{zz} = R_{zz} + C_1(R_{zz} - 1/3) + C_2(R_{zz}^2 - II_R R_{zz}). \quad (18)$$

In (16)-(18), $II_R \equiv \underline{\underline{R}} : \underline{\underline{R}} = R_{xx}^2 + R_{yy}^2 + R_{zz}^2$ inasmuch as the shear components of $\underline{\underline{R}}$ are zero. For $\tilde{N}_\Omega > 0$, solutions to Eqs.(13)-(18) are prolate anisotropic and satisfy the following inequalities: $0 < R_{yy} = R_{zz} < 1/3 < R_{xx} < 1$. Eqs.(14) and (15) together with Eqs.(17) and (18) imply that $B_{yy} = B_{zz}$ and $R_{yy} = R_{zz}$. Therefore, with $B_{yy} = B_{zz} = (1 - B_{xx})/2$ and $R_{yy} = R_{zz} = (1 - R_{xx})/2$, Eq.(13) simplifies to

$$R_{xx} = \frac{(1 + \tilde{N}_\Omega^2) B_{xx}}{(1 + \tilde{N}_\Omega^2 B_{xx})}. \quad (19)$$

Figure 3 summarizes the solutions to Eqs.(16) and (19) for different rotation numbers, $\tilde{\Omega}_0$. Changes in $\tilde{N}_\Omega$ are calculated based on the decay process defined by Eqs.(1) and (2). The APS-closure predicts that the Coriolis force causes a redistribution of energy from an initial isotropic state to a prolate state as the turbulent kinetic energy decays. The algebraic nature of the APS-closure causes a rapid (instantaneous) redistribution of energy at $\tilde{t} = 0$ followed by a slower transfer of energy for $\tilde{t} > 0$. During the decay process, the quadratic form associated with the NR-stress changes from a spherical isotropic form to a prolate ellipsoidal form with an aspect ratio of $2R_{xx}/(1 - R_{xx})$. The anisotropic states identified as ‘a’, ‘c’, ‘e’, and ‘g’ in Figure 3 for which $R_{xx} = 1/2$ all have quadratic forms with an aspect ratio of 2:1. The anisotropic states identified as ‘b’, ‘d’, ‘f ’, and ‘h’ in Figure 3 for which $R_{xx} = 3/4$ all have quadratic forms with an aspect ratio of 6:1. As the turbulent transport time increases (i.e., $\tau_R >> \Omega_x^{-1}$), the energy redistribution process continues and is effectively completed (i.e., $R_{xx} \cong 1$) in a finite amount of time $\tilde{t}_f = t_f \varepsilon_0 / k_0 < \infty$. Figure 3 shows that the time needed to shift the energy to the R_{xx} component depends on the rotation number $\tilde{\Omega}_0$. For $\tilde{\Omega}_0 = 1$, the transfer time is significant inasmuch as $\tilde{t}_f \cong 33$. However, for $\tilde{\Omega}_0 = 15$, $\tilde{t}_f \cong 1.8$.

The normalized prestress $\underline{\underline{B}}$ and the NR-stress $\underline{\underline{R}}$ have significant anisotropic components. The “extra” anisotropy $\underline{\underline{\Delta}}$ in the CH-representation (see Eq.(8) above) is defined by

$$\underline{\underline{\Delta}} \equiv \underline{\underline{B}} - \underline{\underline{R}} = \underline{\underline{\Delta}}_1 + \underline{\underline{\Delta}}_2,$$
$$\underline{\underline{\Delta}}_1 \equiv C_1(\underline{\underline{R}} - \underline{\underline{I}}/3), \quad (20)$$
$$\underline{\underline{\Delta}}_2 \equiv C_2(\underline{\underline{R}} \cdot \underline{\underline{R}} - II_R \underline{\underline{R}})$$

For rotating homogenous decay, the APS-closure predicts that Δ_{xx} and R_{xx} are related by:

$$\Delta_{xx} = -\frac{\tilde{N}_\Omega^2 R_{xx}(1 - R_{xx})}{1 + \tilde{N}_\Omega^2 (1 - R_{xx})}. \quad (21)$$

The anisotropic states ‘a’, ‘c’, ‘e’, and ‘g’ in Figure 3 have the following characteristics: $\tilde{N}_\Omega = 0.082$; $R_{xx} = 0.500$; $B_{xx} = 0.498$; and $\Delta_{xx} = -0.002$. For the anisotropic states ‘b’, ‘d’, ‘f’, and ‘h’ (see Figure 3), $\tilde{N}_\Omega = 0.150$; $R_{xx} = 0.750$; $B_{xx} = 0.746$; and $\Delta_{xx} = -0.004$. The second invariant associated with $\underline{\underline{\Delta}}$ is $II_\Delta \equiv tr(\underline{\underline{\Delta}} \cdot \underline{\underline{\Delta}})$. For prolate states, the third and the second invariants are related by $II_\Delta = 6(III_\Delta / 6)^{2/3}$ where $0 \le III_\Delta \le 8/36$. The invariants of the first-order and the second-order “extra” anisotropic operators $\underline{\underline{\Delta}}_1$ and $\underline{\underline{\Delta}}_2$ are defined similarly.

Figure 4 shows how the invariants of the “extra” anisotropy develop for $\tilde{\Omega}_0 = 1$. Although II_Δ is very small compared with the invariants of either $\underline{\underline{R}} - \underline{\underline{I}}/3$ or $\underline{\underline{B}} - \underline{\underline{I}}/3$, the presence of the “extra” anisotropic operator $\underline{\underline{\Delta}}$ in Eq.(6) is nevertheless important. For example, if $\Delta_{xx} = 0$, then Eq.(21) implies that either $R_{xx} = 1$ or $R_{xx} = 0$ for $|\tilde{N}_\Omega| > 0$. Figure 4 also shows the relative importance of the first-order and the second-order “extra” anisotropic operators $\underline{\underline{\Delta}}_1$ and $\underline{\underline{\Delta}}_2$ for $\tilde{\Omega}_0 = 1$. The “extra” anisotropic operators play complementary roles in the energy transfer process. Both are fundamentally important inasmuch as $\underline{\underline{\Delta}}_1$ triggers the reorganization of energy during the initial stage of decay ($\tilde{t} << 4$), whereas $\underline{\underline{\Delta}}_2$ sustains the anisotropy during for the final stage of the decay ($\tilde{t} >> 4$).

In summary, this brief paper shows that the APS-closure (see Eqs. (6) and (8) above), unlike other commonly encountered closures (see Ref. 5), predicts the formation of Reynolds stress anisotropy during rotating homogeneous decay. With an appropriate generalization of Eqs.(1) and (2), the APS-closure together with the Reynolds averaged Navier-Stokes (RANS-) equation may provide a means to unify the study of non-rotating and rotating turbulent flows. Koppula et al.[6] have recently used the APS-closure to predict the self-similar states in rotating homogeneous simple shear.

ACKNOWLEDGMENTS

This research was partly funded by the National Science Foundation Industry/University Cooperative Research Center for Multiphase Transport Phenomena (NSF/ECC 0331977). The authors thank the members of the Center for their support of this research.

REFERENCES

1. A. Ibbetson and D. J. Tritton, “Experiments on turbulence in a rotating fluid”, J. Fluids Mech., **68** (4), 639-672 (1975).
2. R. A. Wiegland and H. M. Nagib, “Grid generated turbulence with and without rotation about the streamwise direction”, IIT Fluid and Heat Transfer Report R78-1, Appendix B and C, Thesis.ST3B660, Illinois Institute of Technology, Chicago, Ill. (1978).
3. L. Jacquin, O. Leuchter, C. Cambon, and J. Mathieu, “Homogeneous turbulence in the presence of rotation”, J. Fluid Mech., 220, 1-52 (1990).
4. C. Morize, F. Moisy, M. Rabaud, and J. Sommeria, “Dynamics of the anisotropy in decaying rotating turbulence with confinement effects”, 18th Congrès Francais de Mécanique, Grenoble, 27-31 August 2007.
5. S. B. Pope, S. B., *Turbulent Flows,* Cambridge University Press, Cambridge (2000).
6. S. K. Koppula, A. Benard, C. A. Petty, “Realizable Algebraic Reynolds Stress Closure”, Chemical Engineering Science, M. M. Denn Festschrift (submitted, September 2008).
7. J. Y. Park, and M. K. Chung, “A Model for the Decay of Rotating Homogeneous Turbulence”, Phys. Fluids, **11** (6), 1544-1549 (1999).
8. C. Cambon, N.N. Mansour, F. S. Godefrerd, “Energy transfer in rotating turbulence”, J. Fluid Mech., **337**, 303-332 (1997).
9. Y. Morinishi, K. Nakabayashi, and S. Q. Ren, “Dynamics of anisotropy on decaying homogeneous turbulence subjected to system rotation”, Phys. Fluids, **13** (10), 2912-2922 (2001).
10. S. M. Parks, "Relaxation Model for Homogeneous Turbulent Flows", Ph.D. Dissertation, Michigan State University (1997).
11. S. M. Parks, K. Weispfennig, and C.A. Petty, "An Algebraic Preclosure Theory for the Reynolds Stress", Phys. Fluids, **10** (3), 645-653 (1998).
12. Weispfennig, K., S. M. Parks, and C. A. Petty, "Isotropic Prestress for Fully Developed Channel Flows", Phys. Fluids, **11**(5), 1262-1271 (1999).

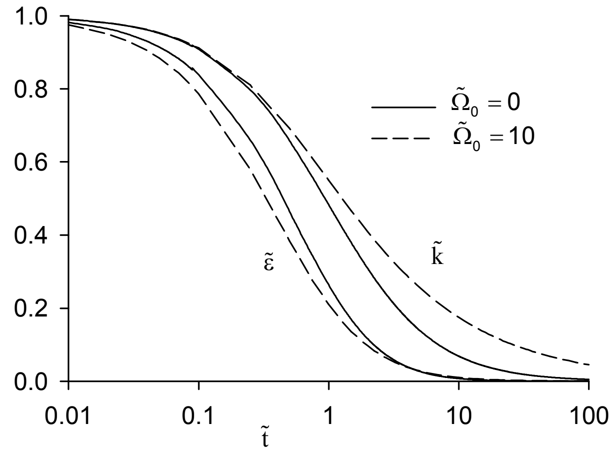

Figure 1. The influence of rotation on the turbulent kinetic energy and the turbulent dissipation for homogeneous decay ($0 \le \tilde{\Omega}_0 \le 10$).

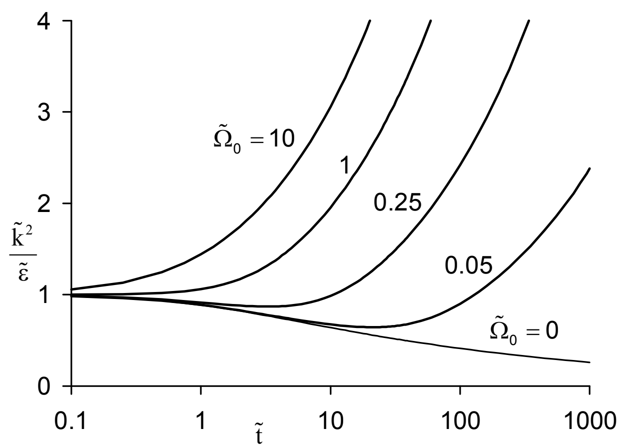

Figure 2. The influence of rotation on the turbulent dispersion coefficient for homogeneous decay ($0 \le \tilde{\Omega}_0 \le 10$).

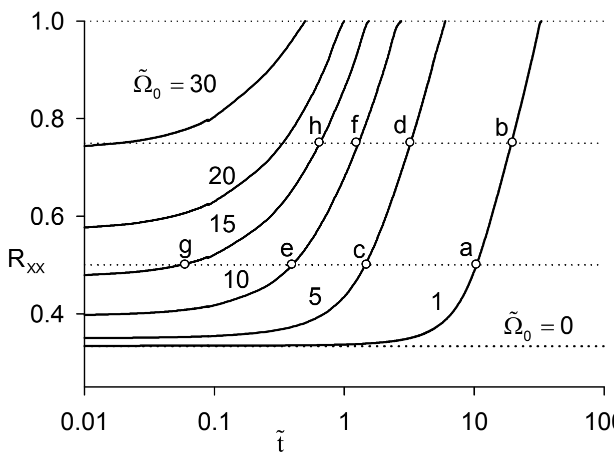

Figure 3. The influence of rotation on Reynolds stress anisotropy for homogeneous decay ($0 \le \tilde{\Omega}_0 \le 10$).

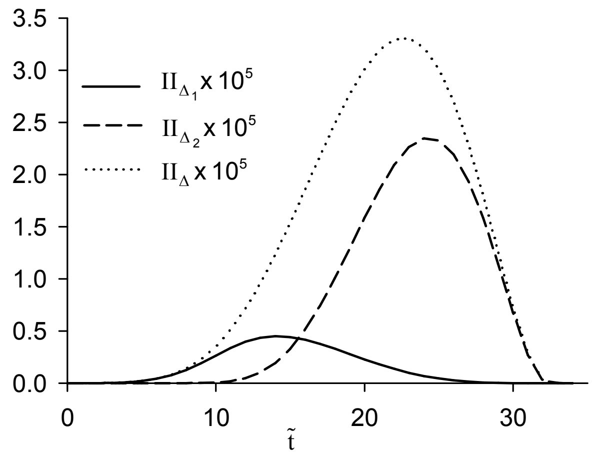

Figure 4. The development of "extra" prestress anisotropy for homogeneous decay ($\tilde{\Omega}_0 = 1$).