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### NEW CORRELATION FOR TWO-PHASE FLOW IN 90 DEGREE HORIZONTAL ELBOWS

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#### ABSTRACT

The comparison of experimental data and results obtained from four global models – homogeneous, Dukler, Martinelli and Chisholm, used to evaluate the two-phase flow pressure drop in circular 90° horizontal elbows – is presented in this paper. An experimental investigation was carried out using three galvanized steel 90° horizontal elbows ( $E1$ ,  $E2$ ,  $E3$ ) with internal diameters of 26.5, 41.2 and 52.5 mm, and curvature radii of 194.0, 264.0 and 326.6 mm, respectively. According to the experimental results, the model proposed by Chisholm best fitted them, presenting for each elbow an average error of  $E1 = 18.27\%$ ,  $E2 = 28.40\%$  and  $E3 = 42.10\%$ . Based on experimental results two correlations were developed. The first one is the classical Chisholm model modified to obtain better results in a wider range of conditions; it was adjusted by a dimensionless relationship which is a function of the homogeneous volumetric fraction and the Dean number. As a result, the predictions using modified Chisholm model were improved presenting an average error of 8.66%. The second developed correlation is based on the entire two-phase mass flow taken as liquid and adjusted by the homogeneous volumetric fraction ratio. The results show that this last correlation is easier and accurate than the adjusted Chisholm model, presenting an average error of 7.75%. Therefore, this correlation is recommended for two-phase pressure drop evaluation in horizontal elbows.

**Keywords:** Two-Phase Flow, Pressure Drop, 90° Horizontal Elbow, Chisholm Model, Dean Number.

#### INTRODUCTION

In most of the industrial processes fluids are used as raw materials, in power hydraulic systems, as a material transport medium, and many other applications. The complete

knowledge of the principles that rule the phenomena involved in fluids transportation leads not only to their better handling but also to more efficient and secure systems. However, in certain industries, such as chemical, geothermal, nuclear and oil, fluids mainly are present as two-phase flow [1].

Heat, mass and momentum transfer enhancement are some of the effects produced by the simultaneous presence of several phases in a mixture. Two-phase flow generally produces a higher-pressure drop in the piping components, which is not desirable in the system. Therefore, a reliable model for the pressure drop prediction in pipelines and fittings for two-phase flows is needed.

In industrial installations, among others, elbows are widely used fittings. In order to give flexibility to the system, they are used to direct the flow; moreover, they can be used as primary elements to measure the mass flow rate flowing through them [2, 3, 4]. Since these fittings are also used to install instruments to monitor the main parameters of industrial processes, and the right location is a crucial factor to have good measurements, so it is important to have a reliable method to evaluate pressure drop in elbows [4].

Below, a review of recent studies regarding pressure drop in elbows is presented. In paper [5], authors evaluated pressure drop on different types of horizontal bends for gas-liquid flow and developed correlations to predict the two-phase friction factor. After comparing the predicted values from Chisholm correlation and the measured values of the frictional pressure drop for 90° bends, they found an average relative error of 30.393%. In the work [6], authors carried out an experimental study of air-water flow pressure loss in a vertical bend and proposed a prediction model based on a two-phase flow multiplier. They found a logarithmic ratio scatter, of the experimental and the predicted values, of around 25%. This

result is lower than the obtained with some models recommended in the literature, such as Chisholm model and extended homogeneous flow model of which they found a logarithmic ratio of 40% and 33%, respectively. In paper [7], authors investigated the geometry effect of 45° and 90° elbows on the pressure drop in horizontal bubbly flow. They compared the experimental pressure loss results in elbows with the ones obtained from the Lockhart-Martinelli correlation. They found that the correlation failed to predict the pressure drop, so they developed a new correlation analogous to Lockhart and Martinelli's. Applying the new correlation, with  $C = 65$  and the minor loss factors of  $k = 0.58$  and  $0.35$  for the 90° and 45° elbows respectively, they got an average percent differences, between the predictions by the new correlation and the data, of  $\pm 2.1\%$  and  $\pm 1.3\%$  for the 90° and 45° elbows respectively. There have been some efforts to develop more accurate models, [8, 9] among others, but due to the two phase flow complexity in elbows, it is necessary to include more parameters to describe the phenomenon.

The motivation of this paper is to develop simply and accurate correlations for two-phase flow pressure drop evaluation in 90° horizontal elbows. To achieve this objective, three models developed for calculating the pressure drop on straight pipes – homogeneous [10], Dukler [11] and Lockhart-Martinelli [12] – and a model obtained for estimating the pressure drop on fittings – Chisholm [13] – will be used to evaluate the two-phase pressure drop in circular 90° horizontal elbows.

## NOMENCLATURE

$D$	Pipe diameter [m]
$De$	Dean number
$m$	Mass flow rate [kg/s]
$P$	Pressure [Pa]
$R_c$	Elbow curvature radius [m]
$Re$	Reynolds number
$X$	Martinelli's parameter
$x$	Mass quality
$U$	Velocity [m/s]
$\phi$	Martinelli's multiplier
$\lambda$	Homogeneous volumetric fraction

## Subscripts

$BLO$	Total two-phase flow as liquid
$BT$	Two-phase Chisholm approach
$BTP-adj$	Chisholm approach adjusted
$exp$	Experimental
$G$	Gas
$L$	Liquid
$M$	Mixture
$SG$	Superficial of gas
$SL$	Superficial of liquid
$T$	Total
$TP$	Two phase

## EXPERIMENTAL SET UP

In order to obtain the data used for comparison with the four global models of two-phase flow pressure drop evaluation, a research was carried out in an experimental facility designed to study and visualize low pressure air-water two-phase flows.

The experimental facility is integrated by an air supply, a water supply, a flow measurement section, an experimental zone, and a phase separation section. Figure 1 shows a diagram of the two-phase flow experimental facility.

The air supply section includes two alternative air compressors (C1 and C2) of 10 and 5 HP connected in parallel; each one has its own storage tank. At the exit of the tanks a pressure regulation valve (VP) allows to keep the sonic flow condition, and then a stable flow condition in the test section is maintained. On the other hand, water supply section consists of a 0.5 m<sup>3</sup> main water tank, a 5 HP centrifugal pump and a galvanized steel pipe. This pipe has a recirculation valve (VRW) in order to reduce the pressure at pump discharge when the experiments require small flows.

Both water and air mass flow rate measurement systems are composed of a couple of 52.8 mm internal diameter pipes connected in parallel. The measurement elements are two orifice plates for each flow, installed and calibrated according to the ISO-5167 [14], and the BS 1042 [15] standards for pressure differential devices.

Once the flow rates are measured, both are conducted to a 30° Y mixer. The resulting mixture continues through a 25.6 m long pipe in order to have a developed flow before the test section, which is interchangeable, in order to test other diameters.

After it, the two-phase flow passes through a pipe section of 4.9 m and discharges into a cyclone separator, which has a deflector that makes the water level to descend gradually. Finally, a 1 HP pump returns the water to the suction tank, and the separated air is vented to the atmosphere [2], and [16].

Three galvanized inconel alloy elbows were tested; their geometrical characteristics are presented on Table 1. Pressure taps were located 30 diameters upstream and downstream of the elbow where the fully developed flow condition was guaranteed; in figure 2 downstream details in the elbow are shown. Pressure taps were drilled each 15° on the internal and external elbow's walls, to measure static pressure variation as flow pass through the fitting. All pressure taps were connected to calibrated pressure transducers.

**Table 1. Tested elbows geometrical characteristics.**

$ELBOW$	$D, [mm]$	$R_c, [mm]$
$E1$	26.5	194.0
$E2$	41.2	264.0
$E3$	56.5	326.6

To locate the test zone the Mandhane chart and the values of  $U_{SL}$  and  $U_{SG}$  were used [10]. The experimental set up capacity and stability allow testing velocities  $U_{SG}$  in a range from 15 to 35 m/s and  $U_{SL}$  from 0.36 to 3.27 m/s, which were slug flow conditions for a 26.5 mm internal diameter pipe.

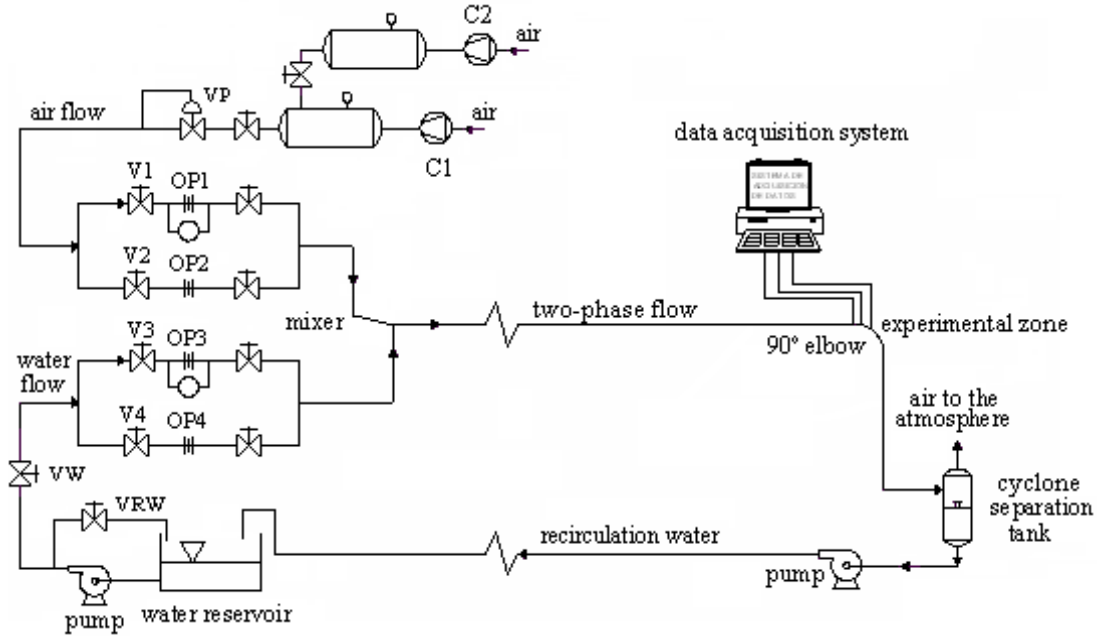


Figure 1. Low pressure air-water two-phase flow facility.

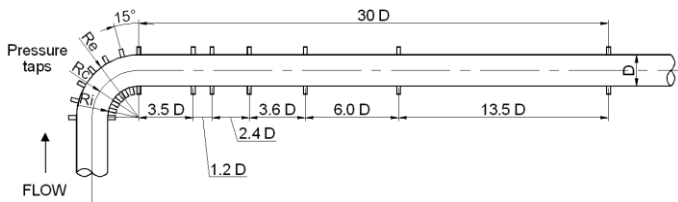


Figure 2. Pressure taps location in the elbow and branches.

## RESULTS ANALYSIS

The two-phase flow pressure profile upstream and downstream elbow was plotted and the pressure drop ( $\Delta P_{exp}$ ) was obtained for each experimental flow condition. Figure 3 shows the pressure profile and the pressure drop due to the elbow ( $\Delta P_{max} = \Delta P_{exp}$ ). In order to compare the two-phase flow models, all of them were programmed and computed under similar flow conditions as in experimental data.

In figures 4, 5, 6 the experimental results ( $\Delta P_{exp}$ ) for each tested elbow are compared versus the results obtained with the four models ( $\Delta P_{model}$ ) using the same experimental conditions; ideally, the points should be aligned on the 1/1 slope. Using this criterion, it is possible to observe that both the Chisholm and Lockhart-Martinelli models fit better all the experimental results at low mixture velocities; however, they present a bigger dispersion when the mixture velocity augments. Another important thing to remark is that the data dispersion for E3 is bigger of all three tested elbows; this means that there is clear influence of the diameter. The reason could be that the flow pattern characteristics as the hold-up, among others, change with the pipe diameter, so this factor must be considered in the correlation.

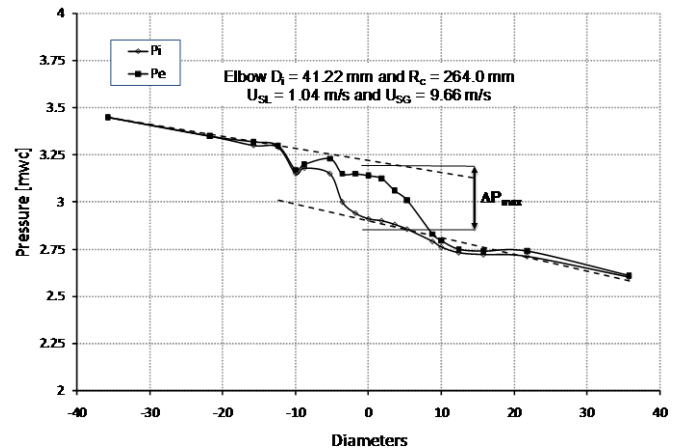
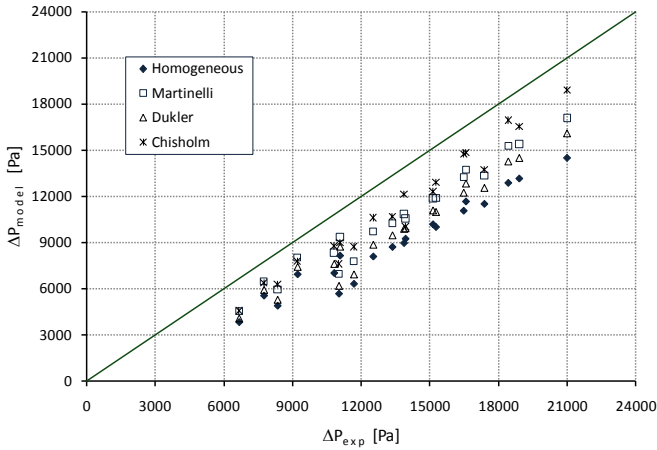


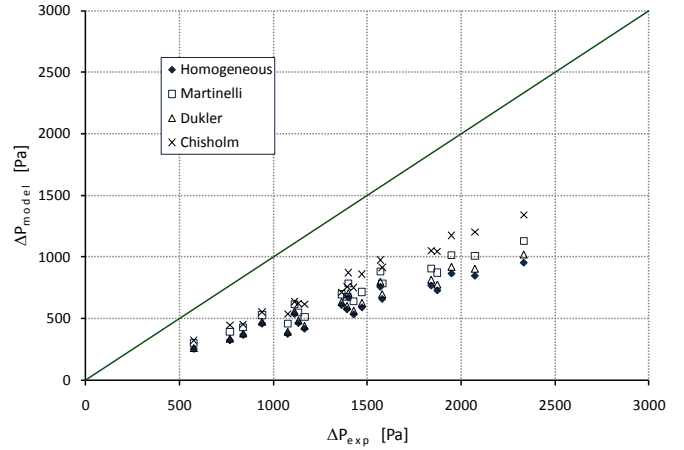
Figure 3. Pressure drop due to the elbow in two-phase flow.

The Chisholm model provides the best results presenting an average variation from 18.2 to 42.1 % respect to experimental data. Theoretically, gas content in mixture influences the pressure drop magnitude because a small increment in gas mass flow produces an important rise in both mixture velocity and void fraction. As a result, pressure drop augments because it is directly proportional to the square of the mixture velocity.

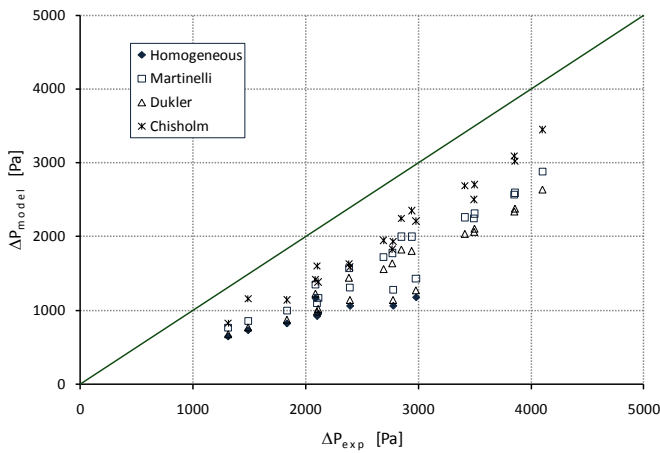
In figure 7 it is possible to observe a linear correlation between the mixture quality ( $x$ ) and the liquid Martinelli's multiplier, which was obtained keeping constant the mass liquid flow rate and varying the mass gas flow rate.



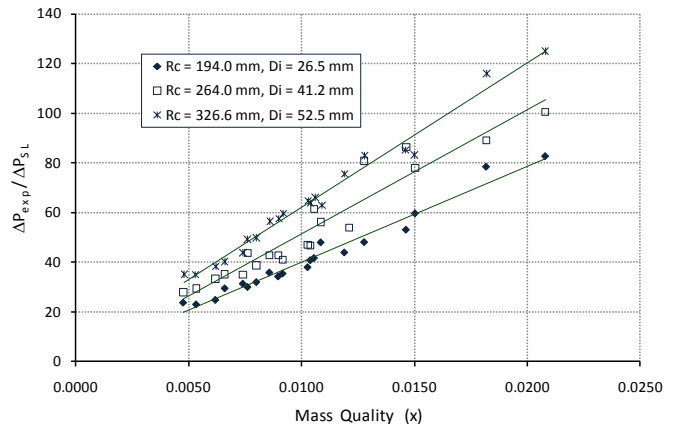
**Figure 4. Comparison between experimental and theoretical pressure drop for E1.**



**Figure 6. Comparison between experimental and theoretical pressure drop for E3.**



**Figure 5. Comparison between experimental and theoretical pressure drop for E2.**

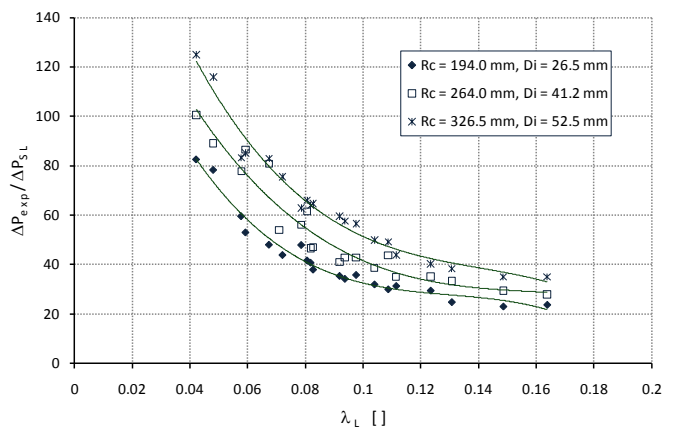


**Figure 7. Comparison of the liquid Martinelli's multiplier respect to mass quality using entire experimental results.**

The mixture quality is defined as the gas mass flow rate and the total mass flow rate ratio,  $x = (\dot{m}_G / \dot{m}_T)$ . The pressure drop increases as the quality augments.

Data presented in figure 7 corresponds to all evaluated elbows, so it is possible to remark that by increasing  $\dot{m}_T$  and  $x$ , the pressure drop in the elbow also augments. On the other hand, for the same flow rate conditions in all three elbows the pressure drop is bigger for the smaller diameter pipe, so the diameter pipe influence must be taken into account.

In figure 8 the experimental results obtained of the three elbows as a function of  $\lambda_L$  are presented. The liquid Martinelli's multiplier  $\phi_L^2 = (\Delta P_{TP} / \Delta P_{SL})$  is plotted as a function of  $\lambda_L = (U_{SL} / U_M)$ . It can be seen that as  $\lambda_L$  reduces, the  $\phi_L^2$  augments; moreover,  $\phi_L^2$  increases, for the same two-phase flow conditions, as the elbow's diameter rises, and decreases as  $\lambda_L$  augments.



**Figure 8. Comparison of the liquid Martinelli's multiplier respect to homogeneous liquid volumetric fraction for all experimental results.**

A comparison between the Chisholm approach and the entire experimental data is shown in figure 9. As was described above the Chisholm approach fitted better the experimental data with less dispersion than the other three models compared in this study. In this figure the pressure drop increases as the pipe diameter reduces and the mass flow rate augments, which is in agreement with the single-phase flow theory.

Figures 8, 9 and 10 yield the conclusion that there is a correlation between the Martinelli's multiplier, the mass quality of the mixture, the homogeneous volumetric fraction and the elbow curvature radius; therefore, the Dean number must be included in order to consider the diameter effect and it is defined as follows [17].

$$De = Re \left[ \frac{D}{2Rc} \right]^{0.5} \quad (1)$$

Consequently, to improve the Chisholm model for a wider range of application is needed to find a dimensionless correlation as a function of  $\lambda_G$  and the Dean number. The proposed dimensionless group consists of  $\lambda_G$  times the Dean number to the  $n$  power,  $\lambda_G De^n$ . This new parameter was plotted against the ratio between experimental pressure drop and the pressure drop given by the Chisholm approach ( $\Delta P_{exp} / \Delta P_{BTP}$ ). The curve that best fitted the entire experimental data was determined, so the correction factor for the Chisholm theoretical model was obtained.

$$\frac{\Delta P_{exp}}{\Delta P_{BTP}} \approx 15.33(\lambda_G De^n)^{0.520} \quad (2)$$

Figure 10 shows the pressure drop ratio versus the proposed correction factor, given by the equation 2; it can be noticed that the correlation is quite good. Furthermore, the Dean number exponent was found to be  $n = -0.5$ , so from the equation (2) the adjusted Chisholm model is,

$$\Delta P_{BTP-adj} = 15.33 \Delta P_{BTP} (\lambda_G De^{-0.5})^{0.52} \quad (3)$$

Equation 3 fits well the entire experimental data; moreover, it includes the Dean number in order to generalize the model application. The results obtained with the adjusted Chisholm approach are plotted in figure 11. The predictions using adjusted Chisholm model were improved presenting an average error of 8.66 % with a standard deviation of 6.04 and an average dispersion of 4.79 %.

In order to obtain an easier and accurate correlation, the pressure drop produced by the entire mixture flow, taken as a liquid  $\Delta P_{BLO}$ , was used, i.e., the entire two-phase flow is considered as liquid flowing in the pipe and filling it up. Therefore, a dimensionless group which includes the Dean number and the homogeneous volumetric fraction was developed, and the correlation found is,

$$\Delta P_{TP} = 245.66 \Delta P_{BLO} \left[ \frac{\lambda_G}{\lambda_L} De^{-0.5} \right]^{-0.7208} \quad (4)$$

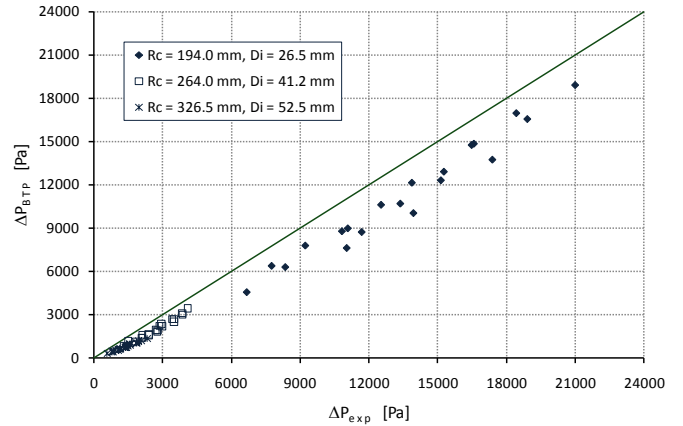


Figure 9. Comparison of the experimental pressure drop versus Chisholm model using entire experimental results.

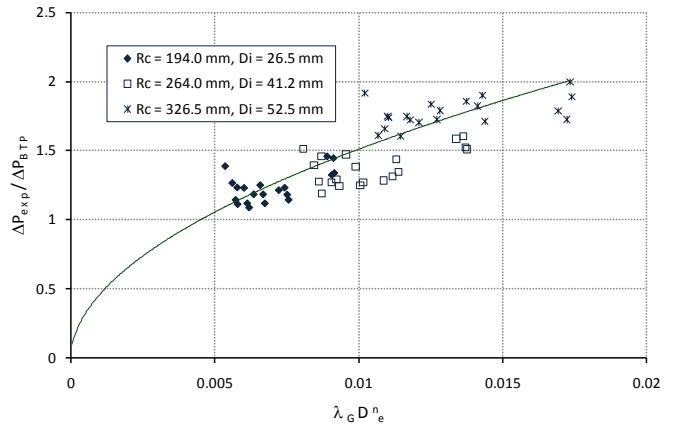


Figure 10. Determination of the factor to adjust Chisholm's approach.

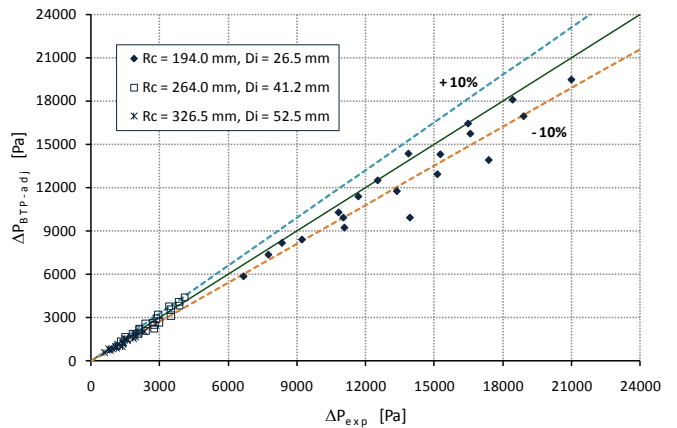


Figure 11. Comparison of the adjusted Chisholm model versus experimental data.

Using equation 4 a better correlation is obtained; figures 12 and 13 show the results of the correlation factor and the comparison with the experimental data, respectively. As can be seen, the proposed model (equation 4) gives better results with an average error of 7.75 %, with a standard deviation of 5.48 Pa and an average dispersion of 4.13 %. Figures 11 and 13 show the comparison of the adjusted Chisholm model (equation 2) and the proposed model (equation 4) against experimental data, respectively. Because more data are within  $\pm 10\%$  error the proposed model predicted better the pressure drop for two-phase flow in a 90° horizontal elbow.

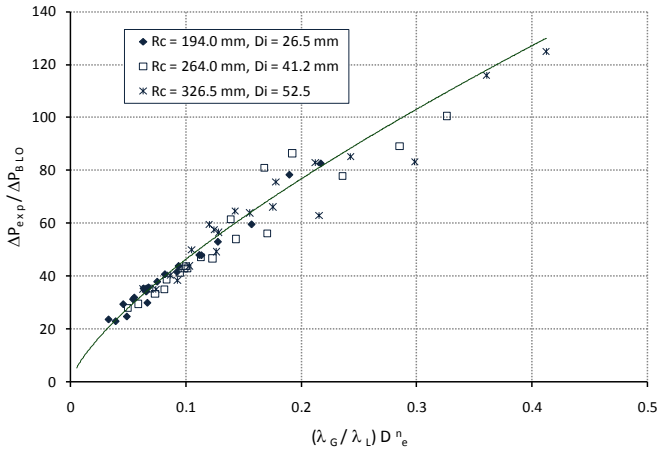


Figure 12. Determination of the proposed model.

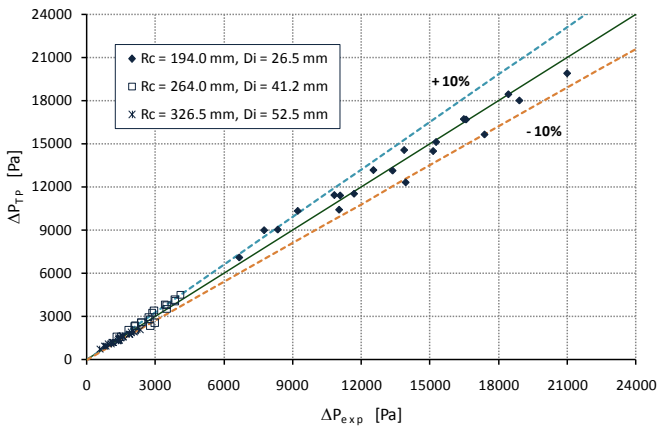


Figure 13. Comparison of the proposed model against all the experimental data.

## CONCLUSIONS

The results calculated by four models used to evaluate the two-phase flow pressure drop in elbows (Homogeneous, Lockhart-Martinelli, Dukler and Chisholm) were compared with the experimental data obtained in a two-phase flow horizontal rig. As was expected, the Chisholm approach was the model that best fit the experimental data, presenting a maximum average error of 42 % and a minimum of 18.3 %.

After analyzing the experimental results it was found that as  $\lambda_G$  increases, the liquid Martinelli's multiplier augments, and it decreases when  $\lambda_L$  rises. In addition, there is a clear influence of the pipe diameter which is in agreement with the single-phase flow theory. Another important fact that has to be considered is the correlation between the mass quality of the mixture and the liquid Martinelli's multiplier – as  $x$  increases the mixture velocity augments, consequently,  $\Delta p$  rises too.

As a final result of this work, two new correlations were developed. The first one is the Chisholm approach modified to reduce the dispersion, equation (3); the second one takes as a base the  $\Delta p$  produced by  $m_t$  when it is considered as a liquid, and is adjusted by a factor which considers the  $\lambda_L$  and the Dean number (equation 4). Experimental data obtained of three different diameter elbows was compared with the results of these two final correlations. It was found that equation (4) gives better results with an average error of 7.75 %, a standard deviation of 5.48 Pa and an average dispersion of 4.13 %. Therefore, due to its simplicity and accuracy equation 4 is recommended for two-phase pressure drop evaluation in 90° horizontal elbows.

## ACKNOWLEDGMENTS

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