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AN ANALYTICAL MODEL OF THE INSTANTANEOUS TRANSVALVULAR PRESSURE GRADIENT THROUGH AN AORTIC STENOSIS

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ABSTRACT

Diagnosis and treatment of aortic stenosis largely depends on accurate determination of the pressure difference before and after the valve, known as transvalvular pressure gradient (TPG). Clinically, TPG is obtained using Doppler echocardiography though sometimes invasive cardiac catheterization has to be used to confirm Doppler echocardiography findings. By solving analytically coupled fluid and solid domain equations, we suggest a formulation that with a good degree of accuracy can be used to calculate TPG. Analytical result is validated using experimental data from literature. The suggested methodology is an alternative to cardiac catheterization and helps to prevent its risks.

INTRODUCTION

Aortic valve stenosis is considered as the most frequent cardiac disease after arterial hypertension and coronary artery disease and it is the most common cause of valvular replacements in North America (100,000 per year) [1]. The severity of an aortic stenosis can be clinically determined by measuring the transvalvular pressure difference between the left ventricular pressure and the aortic pressure, clinically called: transvalvular pressure gradient (TPG). The determination of TPG and the effective orifice area of the stenotic valve are crucial for making decision about native valve replacement. Thus underestimation or overestimation of TPG can lead to improper treatment.

In a clinical setting, the determination of transvalvular pressure gradient (TPG) is usually performed using Doppler echocardiography which gives TPG_{Max} (the difference between the left ventricular pressure and the pressure at the vena contracta). However, sometimes in clinical practice it is required to confirm the transvalvular pressure gradient measured using Doppler echocardiographyby to perform invasive cardiac catheterization which gives TPG_{net} (the difference between the left ventricular pressure and the recovered aortic pressure). However, pressure recovery phenomenon causes a notable difference between these two $TPGs$ measured by Doppler echocardiography and cardiac catheterization.

In this study, in order to better investigate the TPG induced by a stenotic valve, coupled Navier-Stokes and elastic deformation equations with reasonable boundary conditions in the aortic valve region are solved analytically. This explicit model will improve the comprehension of the flow through an aortic stenosis. The analytical formulations obtained in this study could be used to calculate a reference pressure gradient (TPG_{Max} or TPG_{net}) for a given reference cross sectional area which in turn provides a more accurate estimate of the severity of the stenotic valve. Good agreement was found between the analytical model and in vivo data from the literature.

MATHEMATICAL ANALYSIS

Governing Equations

The following assumptions are used for the analytical formulation of the problem analyzed in this present study.

1. We consider the flow in the aorta as Newtonian fluid. The assumption of a Newtonian fluid behavior is realistic for blood flow in large arteries like the aorta as the shear rates in large arteries are sufficiently large [2].
2. The fluid is incompressible.
3. The valve opens and closes instantaneously thus its EOA remains constant throughout systole.
4. No secondary or swirling flows have been allowed so the flow is independent of θ. Therefore the total velocity is defined by the radial and axial components.

The continuity and unsteady Navier-Stokes equations which describe the motion of the fluid can be written in the form as follows.

$$\frac{1}{r}\frac{\partial}{\partial r}(rv)+\frac{\partial u}{\partial z}=0$$

$$\frac{\partial u}{\partial t}+(V.\nabla)u=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\upsilon\nabla^2 u$$

$$\frac{\partial v}{\partial t}+(V.\nabla)v=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\upsilon\left(\nabla^2 v-\frac{v}{r^2}\right) \qquad (1)$$

Where the axial velocity, axial coordinate, radial velocity and radial coordinate are denoted as $u=u(z,r,t)$, z, $v=v(z,r,t)$ and r respectively. Note that r varies along the axial axis.

An artery is not a rigid tube but is an elastic one. Hence it is important to take into account the elastic nature of the vessel. The elastic tube motion equations in cylindrical coordinates can be written as follows.

$$\frac{\partial \tau_{rr}}{\partial r}+\frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta}+\frac{\tau_{rr}-\tau_{\theta\theta}}{r}+\frac{\partial \tau_{rz}}{\partial z}=\rho_w\frac{\partial^2\eta}{\partial t^2}$$

$$\frac{\partial \tau_{zr}}{\partial r}+\frac{1}{r}\frac{\partial \tau_{z\theta}}{\partial \theta}+\frac{\tau_{zr}}{r}+\frac{\partial \tau_{zz}}{\partial z}=\rho_w\frac{\partial^2\xi}{\partial t^2} \qquad (2)$$

Here (η,ξ) are the radial and axial displacements and τ and ρ_w are stress tensor and the density of the wall respectively.

Geometry

The schematic diagram for the model includes the left ventricle, aortic valve and the aorta is shown in figure 1. The flow pattern across an aortic stenosis is characterized by flow narrowing as far as the vena contracta, followed by a sudden expansion (Fig.1). As figure 1 shows, the vena contracta corresponds to the region where the cross sectional area of the jet is minimum. The area at the vena contracta is called clinically the effective orifice area (EOA) of the valve.

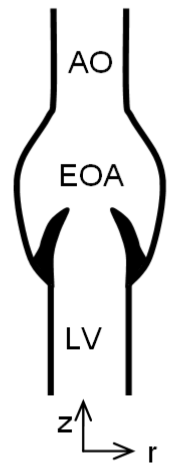

Figure 1. Schematic of the flow across an aortic valve. LV: Left Ventricle, AO: Aorta, EOA: Effective Orifice Area

Boundary conditions

We consider the radial displacement of the wall as η and $-\eta$. Then, the equation of the wall is given by $R(z,t)=\pm\eta=\pm a\left[1+\varepsilon\sin\frac{2\pi}{\lambda}(z-ct)\right]$, therefore, the velocity boundary condition on the arterial wall is taken as:

$$v(r,z,t)=\frac{\partial R}{\partial t},$$

$$R(z,t)=\pm\eta=\pm a\left[1+\varepsilon\sin\frac{2\pi}{\lambda}(z-ct)\right] \qquad (3)$$

Where ε,λ,c and a are the amplitude ratio, wave length, wave speed and characteristic radius of the tube.

It is further assumed that initially no flow takes place when the system is at rest.

$$v(r,z,0)=u(r,z,0)=0 \qquad (4)$$

Method of the solution

A sinusoidal wave was assumed to travel over the wall [3]. As the vessel wall is elastic and the flow is pulsating the solutions for axial and radial flow velocities, pressure, axial and radial wall deformations, are expanded as Fourier series. In the elastic model, due to the wall deformation, velocity and pressure have radial components.

$$u(z,r,t)=u_0(r)e^{i(\omega t-k_n z)}$$

$$v(z,r,t)=v_0(r)e^{i(\omega t-k_n z)}$$

$$P(z,r,t)=P_0(r)e^{i(\omega t-k_n z)}$$

$$\eta(z,r,t)=\eta_0 e^{i(\omega t-k_n z)}$$

$$\xi(z,r,t)=\xi_0 e^{i(\omega t-k_n z)} \tag{5}$$

Here $\omega=2\pi f$, $f={1}/{T}$ and λ are angular frequency, pulse frequency and wave length respectively.

Also pulse wave speed c is calculated by the following equation [5]. Where E and h are the wall's elastic modulus and thickness respectively.

$$c=\sqrt{\frac{Eh}{2a\rho}} \tag{6}$$

Using $c={\omega}/{k_n}$ and $\lambda={2\pi}/{k_n}$, k_n and λ are computed. With substituting the above expressions (eq.5) for u,v and p in Navier-stokes equations (eq.1), we can get the following equations,

$$u_0(r)=\frac{k}{\rho\omega}p_0+C_1J_0(\frac{i\omega\rho}{\mu}r)+C_2Y_0(\frac{i\omega\rho}{\mu}r) \tag{7}$$

$$v_0(r)=-C1\frac{k\mu}{\omega\rho}J_1(\frac{i\omega\rho}{\mu}r)-\frac{ik^2}{2\omega\rho}p_0r \tag{8}$$

By considering boundary condition, $v=\pm a\varepsilon\frac{2\pi c}{\lambda}\cos\frac{2\pi}{\lambda}(z-ct)$ at $r=R(z,t)$, C_1 is Following and C_2 is zero since $u_0(r)$ becomes meaningless at $r=0$.

$$C_1=\frac{\omega\rho}{k\mu J_1(\frac{i\omega\rho}{\mu}R)}$$
$$\left[\frac{-ik^2}{2\omega\rho}p_0R+\frac{2\pi ca\varepsilon}{\lambda}\cos[\frac{2\pi}{\lambda}(z-ct)]e^{i(\omega t-k_n z)}\right] \tag{9}$$

Next by substituting waveform expressions for u,v and by considering equations 7 and 8 in the dynamic equations (eq.2), we obtain the following result for P_0 ,

$$P_0(r)=$$
$$\left(\begin{array}{l}\frac{C_1Ehk\mu}{i\omega^2\rho a^2(1-\upsilon^2)}J_1(\frac{i\omega\rho}{\mu}a)+\frac{C_1Ehk\mu}{a(1-\upsilon^2)}J_0(\frac{i\omega\rho}{\mu}a)\\ -\frac{C_1\rho_w hk\mu}{i\rho}J_1(\frac{i\omega\rho}{\mu}a)\end{array}\right)\Big/$$
$$\left(1-\frac{Ehk^2}{2(1-\upsilon^2)\omega^2\rho a}+\frac{Ehk^2\upsilon}{(1-\upsilon^2)\omega^2\rho a}+\frac{\rho_w hk^2a}{2\rho}\right) \tag{10}$$

In summary, we obtained the solution by coupling between the two sets of partial differential equations (Navier Stokes and elastic tube motion equations) as follow,

$$u(z.r,t)=\left[\frac{k}{\rho\omega}P_0+C_1J_0(\frac{i\omega\rho}{\mu}r)\right]e^{i(\omega t-k_n z)} \tag{11}$$

$$v(z,r,t)=\left[[-C1\frac{k\mu}{\omega\rho}J_1(\frac{i\omega\rho}{\mu}r)-\frac{ik^2}{2\omega\rho}P_0r\right]e^{i(\omega t-k_n z)} \tag{12}$$

$$P(z,r,t)=e^{i(\omega t-k_n z)}$$
$$\left(\begin{array}{l}\frac{C_1Ehk\mu}{i\omega^2\rho a^2(1-\upsilon^2)}J_1(\frac{i\omega\rho}{\mu}a)+\frac{C_1Ehk\mu}{a(1-\upsilon^2)}J_0(\frac{i\omega\rho}{\mu}a)\\ -\frac{C_1\rho_w hk\mu}{i\rho}J_1(\frac{i\omega\rho}{\mu}a)\end{array}\right)\Big/$$
$$\left(1-\frac{Ehk^2}{2(1-\upsilon^2)\omega^2\rho a}+\frac{Ehk^2\upsilon}{(1-\upsilon^2)\omega^2\rho a}+\frac{\rho_w hk^2a}{2\rho}\right) \tag{13}$$

Where ν is Poisson's ratio. Doppler echocardiography can measure the EOA which is used to calculate non deformed radius (a) in the above equations. Pressure difference between two points upstream and downstream of this specific point gives TPG . As mentioned earlier, each EOA allows the computation of the corresponding TPG , therefore, $TPG_{Max}=P_{LeftVentricle}-P_{VenaContracta}$ and $TPG_{net}=P_{LeftVentricle}-P_{Aorta}$.

VALIDATION AND DISCUSSIONS

Using present analytical formulation, just by knowing EOA, ascending aorta diameter and diameter of the opening before the aortic valve, TPG (pressure drop) can be determined and no further information from the invasive cardiac catheterization method is required.

The main important feature of current analytical solution is as following. Indeed, as EOA and mentioned diameters can easily be obtained by Doppler echocardiography, instantaneous TPG_{net} might therefore be estimated non-invasively in patients. This would allow analyzing the instantaneous TPG-flow relationship, which could provide new insights into the unsteady fluid dynamics of the aortic valve and its implication for the clinical assessment of aortic stenosis severity without performing an invasive cardiac catheterization.

Fig.2 shows instantaneous systolic value of the transvalvular pressure gradient (TPG_{Max}) for $EOA = 0.48 cm^2$ by using current analytical model. Analysis has been carried out 1 cm upstream from the aortic stenosis ($P_1 = P_{LeftVentricle}$) and at the level of the vena contracta ($P_2 = P_{VenaContracta}$), therefore, TPG_{Max} is equal to $P_1 - P_2$. The error between mean TPG_{Max} calculated using the current analytical model (Fig.2) and the experiment study in the literature [5] for $EOA = 0.48 cm^2$ is 6.9%. Considering variability and statistical parameters used in the experimental measurement as described by Garcia et al. [5], this error is in an acceptable range. Analysis for Fig.2 has been carried out by equations 13 and 9 using the following parameters values: $\mu = 4.144 \times 10^6 \, dyn/cm^3$, $\rho_w = 1062 \, kg/m^3$, $\rho = 1050 \, kg/m^3$, $a_{Aorta} = 1.25 cm$, $a_{LeftVentricle} = 1.25 cm$, E=4.8e^{5}N/m^2, $\nu = 0.45$.

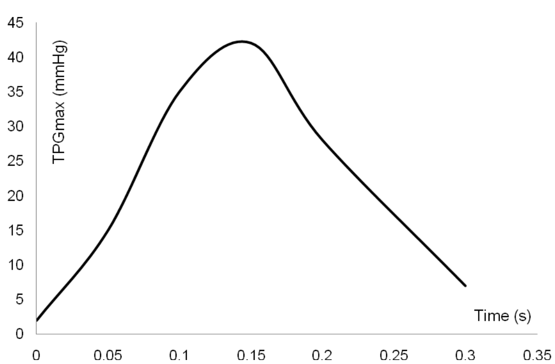

Figure 2. Predicted TPG_{Max} for EOA of 0.48 cm^2

CONCLUSIONS

Aortic stenosis is the most frequent valvular heart disease. The mean systolic value of the transvalvular pressure gradient (TPG) is commonly utilized during clinical examination to evaluate its severity and it can be determined either by cardiac catheterization or by Doppler echocardiography. In this study, we thus derived an analytical formulation, allowing determination of the transvalvular pressure gradient none-invasively which is the major criteria used to evaluate aortic stenosis severity. This analytical model was validated with in vivo experiments from the literature. There is a very good concordance between experimental and analytical results. The analytical model proposed and validated in this study provides a comprehensive description of the aortic valve hemodynamics that can be used to accurately predict the instantaneous transvalvular pressure gradient in aortic valves. Since the derived equations 9 and 11-13 validated for a given EOA and TPG_{Max} it can be used to any other EOA to calculate other $TPGs$ including TPG_{net}.

REFRENCES

[1] Pibarot, P., Dumensil, J.G., Mathieu, P., 2007. Nouveautes sur le retrecissement valvulaire aortique. Medecine/Sciences 23:81-87.

[2] Shahcheraghi N., et al., 2002. Unsteady and Three-Dimensional Simulation of Blood Flow in the Human Aortic Arch. ASME 124:378-387.

[3] Mazumdar J., 1999. An introduction to mathematical physiology & biology. Cambridge University Press, New York.

[4] Fung Y.C., 1981. Biomechanics: Mechanical properties of living tissues. Springer.

[5] Garcia D., Kadem L., Savery D., Pibarot P., Durand L.G., 2006. Analytical modeling of the instantaneous maximal transvalvular pressure gradient in aortic stenosis. Journal of Biomechanics 39:3036-3044.