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INVERSE METHOD USED FOR THE DETERMINATION OF THE WALL SHEAR STRESS IN A SLIDING RHEOMETER USING SANDWICH PROBES

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ABSTRACT

The inverse method, based on a numerical sequential estimation, has been applied for the determination of the wall shear stress of a liquid single phase flow in a sliding rheometer using multi-segment probe. This method requires the inversion of the convection diffusion equation in order to apply it to instantaneous mass transfer measurements. Polarography technique, known as the limiting diffusion current method, has been used. This requires the use of Electro-Diffusion ED probe which allows the determination of the local mass transfer rate for known flow kinematics. In addition, two-segment platinum probe was mounted flush to the inert surface of the upper disk of the sliding rheometer. Hydrodynamic oscillations have been imposed to the torsional flow (type sinusoidal), in order to study the frequency response of the sandwich probe for a fixed polarization voltage. Possible error sources which are likely to affect the interpretation of the results e.g. the directional angle effect, the inertial effect, the diffusion effect and the frequencies of oscillations effect have been studied in order to test the robustness of the inverse method within the presence of such impacts. Furthermore, to demonstrate the possible effect of non-negligible inertia and diffusion, we refer to ED results for both modified Reynolds number defined by [1] and Peclet number ranges as well as for different directional angles. An algorithm has been developed for the numerically filtering of the mass transfer signals, and therefore the wall shear stress signals. It permits to eliminate any possible noise effect due to the imposed vibrations to the torsional flow. The analysis shown that the inverse method is in a good agreement with the ED experimental results for the different cases of study, i.e. for different dimensionless Reynolds numbers, for high and low oscillation frequencies, as well as for different directional angles. The little difference is probably caused by the sensitivity of the double probe to such directional angles or to the neglecting of the insulating gap effect on the inverse method solution as a first step of the study of the inverse method for double probes signals.

1. INTRODUCTION

Torsional flows have been widely used on industrial application on the basis of controlling the properties of flows and its structures. This type of flow behaviors still receives attention. In particular, sliding rheometers are frequently used in chemical industries to simulate the flow structures between two parallel plates. They are also used for the study of the shear stress effect on the crystallization of polymers [2]. To study the flow behaviors at a wall, electrochemical probes are generally used. There are simple probes (circular and rectangular), double probes i.e. two-segment probes and three-segment probes. The circular probes allow the measurement of the local transfer coefficient, while the rectangular probes allow the measurement of the total transfer coefficient. The two-segment probe has been frequently recommended for the determination of the instantaneous wall shear stress and the flow direction near the wall [3-5]. For instance, it has been used for the investigation of an adiabatic single phase flow inside two-dimensional and three dimensional channels to the description of flows at a wall [6]. This kind of multi-segment probe can be used successfully for small superposed flow oscillations with known direction [7]. Otherwise, they are unsuitable for strongly fluctuating flows of low velocity and varying direction e.g. recirculating two-phase flows in bubble columns. In addition, we have restricted our study to low oscillation amplitude (β<1). We should note also that our setup allows a maximum amplitude of 100%. In the literature, the approaches proposed to calculate the wall shear rate from the limiting diffusion current have some restrictions. In fact, most of them are valid for specific ranges of Peclet numbers and for moderate frequency noises on the ED measurements. A numerical study of the frequency response of sandwich electrochemical probes was carried out by [8]. These probes allow giving more coherent results in the case of reverse flow [9]. Thanks to the use of sandwich Electro-Diffusion probes, the measurement of wall shear stress components and the flow angle can be carried out. The principle of measurement is based on the comparison of the two currents resulting from the segments of the probe [10-11]. However, the sensitivity of this type of probe is important when the flow becomes perpendicular to the insulating gap. For this reason, we have focused our study to different cases, i.e. different directional angles, different modified Reynolds numbers and different oscillations frequencies.The study was done on a parallel plate (Sliding) rheometer, which permits to impose a known experimental shear stress. This later was used to check the validity of the wall shear gradient given from mass transfer measurements by using the numerical inverse method. It deals with torsional flows of Newtonian fluids in sliding rheometers for the study of the frequency response of a sandwich probe (two segment probe) to hydrodynamic fluctuations of a single phase flow under the control by the convection diffusion phenomena. The inverse method has been applied and compared to the experimental ED data base constructed using a sandwich probe mounted flush on the fixed upper disk of the sliding rheometer. The result analysis is based on sources of errors generally re-encountered while using a sliding rheometer and/or a sandwich probe. In sliding rheometer, the most sources of errors are related to the end and edge effects, the inertia effect (apparition of a secondary flow). To minimize the "End effect", authors proved that the gap *h* has to be thin. In addition, in order to neglect the end effects on our study, the thickness of the gap between the rotating disk and the bottom of the fixed plate is equal to $h = 0.43 \pm 0.02$ *mm* and we used a Newtonian fluid (Electrochemical solution ferri-ferrocyanide of potassium).

The inertia effect was studied by [1], using a dimensionless criterion $Re^* = \Omega^* h^2/\upsilon$ derived for the study of flow regime of Newtonian liquids in a rheometer during measurements. In fact, the authors confirmed that the secondary flow has no significant effect on the relative torque if $Re^*<1$. The apparition of the secondary flow (inertia effect) corresponds in our case to an angular velocity equal to 50 rpm. With the aim of validating the inverse method, ED results are presented for high and low oscillation frequencies, for different dimensionless Reynolds numbers, as well as for different directional angles. It is shown that it is possible to determine the wall shear stress by using the ED data obtained from torsional oscillatory flows. For convenience of presentation, we develop firstly the results of the inverse method with experimental validation for the case of high and low frequency oscillations while the inertia effect remains non significant and for a directional angle equal to zero. Then, we focus on the case of significant inertia effect i.e. presence of secondary flow on the sliding rheometer and for the case of directional angle effect. In such part, we present the time evolution of the Sherwood numbers calculated via the inverse method and compared with the experimental one resulting from the mass transfer measurements, delivered by the ED probe. We later illustrate how the inverse method allows the estimation of the wall shear gradient by comparison of the experimental mass transfer and the numerical one for the different cases studied.

NOMENCLATURE

A	Surface of the probe (m^2)
C	Concentration (mol/m^3)
D	Coefficient of diffusion ($m^2.s^{-1}$)
F	Frequency of oscillation (Hz)
g	Gap thickness between the front and the back segments (m)
h	Length of the gap between the rotating disk and the fixed one (m)
i	Current (A)
l	width of the probe (m)
P	Perimeter of the probe (m)
R	Radial distance (mm)
S	Wall shear stress (s^{-1})
t	Time (s)
v	Velocity ($m.s^{-1}$)
X ,Y	Coordinates (m)

Greek symbols

β

ε	Amplitude of the oscillation
θ	Constant used in the inverse method
	Directional angle relative to a pointer, fixed on the
Ω	sandwich probe mantle (°)
υ	Angular velocity of the rotating plate ($rad.s^{-1}$)
	Kinetic viscosity ($m^2.s^{-1}$)
Superscripts	
*	
–	Dimensionless
	Temporal average
Subscripts	
exp	
num	Experimental
grad	Numerical
Pe	Gradient
Re	Peclet number
Sh	Reynolds number
	Sherwood number

2. EXPERIMENTAL INSTALLATION AND ED SOFTWARE

The experimental installation (Fig. 1) is composed of a sliding rheometer, a servo-motor and its controller, an ED interface EDIK2 and a PC. The lower disk of the sliding rheometer is a rotating Plexiglas disk, while the upper disk is fixed. It serves as a base for fixing the multi-segment ED probes.
The thickness of the gap between the rotating disk and the bottom of the fixed plate is equal to h = *0.43±0.02* mm, and the radial distance is r = *30* mm. The sliding rheometer achieves a torsional viscometric flow between the two coaxial disks.

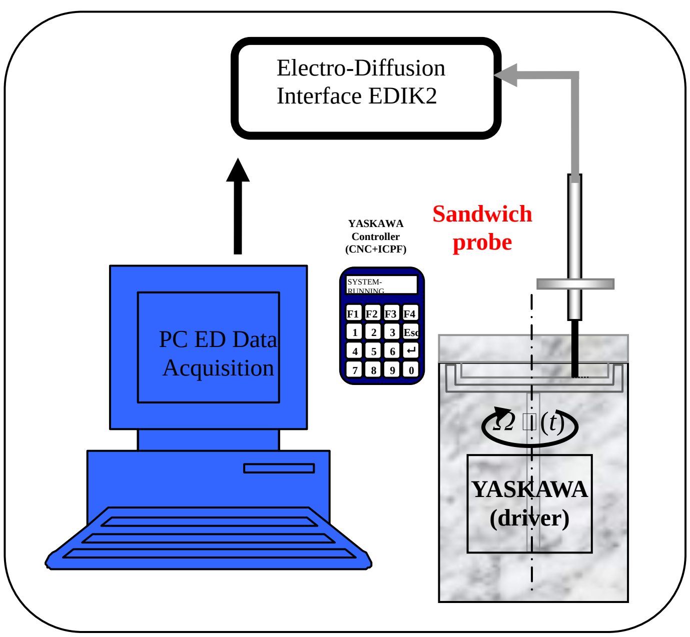

Figure 1: Experimental installation.

The servo-motor and its controller allow the imposition of a known wall shear stress (type sinusoidal). The ED interface EDIK2 is composed of two 3-channel connections to two 3-segment ED cells (stages A and B). It serves for the ED processes control. Using the ED interface EDIK2 and the AD/DA NI card PCI-6221 (68pin) under PC environment of NI LabView, we have achieved the control of the ED data and the preliminary analysis. More details are available on [12].

3. QUALIFICATION OF THE ED MEASUREMENTS IN THE PARALLEL PLATE RHEOMETER USING SANDWICH PROBE

The polarography technique is widely used for the determination of flow behaviors at a wall using electrochemical probe. It is mostly recommended for the local wall shear rate measurements [13], and it is a non-instructive method based on oxydo-reduction reaction at the ED probe surface. The double probe (the cathode) is fixed on the wall of the upper fixed disk. The counter electrode (the anode) is bigger than the working electrode in order to have diffusion control on the probe. The electrolyte used is ferri-ferrocyanide of potassium of concentration 25 mol/m^3, and the chemical reaction equation is:

$$Fe(CN)_6^{3-} + \bar{e} \longleftrightarrow Fe(CN)_6^{4-} \qquad (1)$$

Generally, the elimination of the migration current caused by the electric field is guaranteed using a supporting electrolyte. In our tests, we have used an access of the sulfate of potassium K_2SO_4 (230 mol/m^3) as supporting electrolyte.
An ideal sandwich probe is schematically presented on Figure 2. It is geometrically characterized by two segments on platinum separated by an insulated gap. The microphotography of the probe is presented on Figure 3. It illustrates the existence of roughness on the surface of the probe comparing to the ideal one. Indeed, at the operations of joining and polishing of the multi-segment probe on the surface of the experimental device, a modification of the geometry of the probes is detected. The active surface of the probe is larger than that deduced starting from the sectional surface of the platinum wire used.

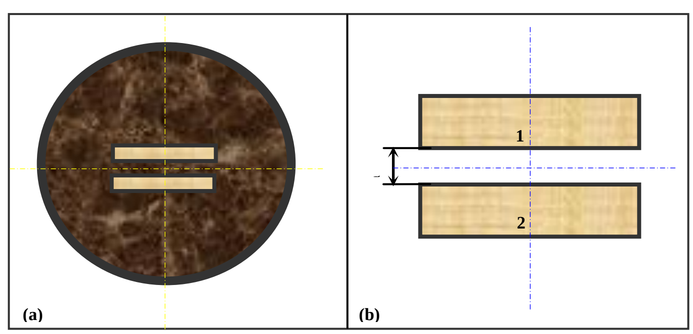

**Figure 2: Sandwich probe composed of two rectangular segments separated by an isolated gap
(a): Glabal view of an ideal sandwich probe; (b): View of the two segments separated by the isolated gap**

The geometrical characteristics of the probe are resumed in table 1. In this same table, we compare the characteristics of the real probe to those of an ideal probe. We note that the length of the isolated gap between the two segments of the double probe is equal to 0.0271mm.

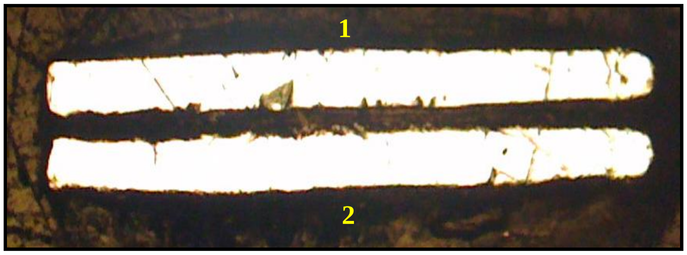

Figure 3: Microphotography of the sandwich probe used

Table 1: Geometrical characteristics of the sandwich probe used

N° of the segment of the sandwich probe	Segment n° 1	Segment n° 2
Area of the segment (mm^2)	0.0421	0.0422
width of the segment (mm)	0.0507	0.0496

To guarantee that the polarographic process is controlled exclusively by the convective diffusion of the working electrode, preliminary tests have been realized in order to well choose a polarization voltage. For the ED measurements, the polarization voltage of the electrochemical solution is fixed to U_p=*-800 mV*. It has been chosen in the diffusional plateau. A sampling of polarograms of current intensities of each segment of the double probe for polarization tension U_p is illustrated on Figure 4. It highlights that, for a completely polarized electrode, the local current densities of the disk edge have finite values [14] corresponding to the limiting diffusion current.

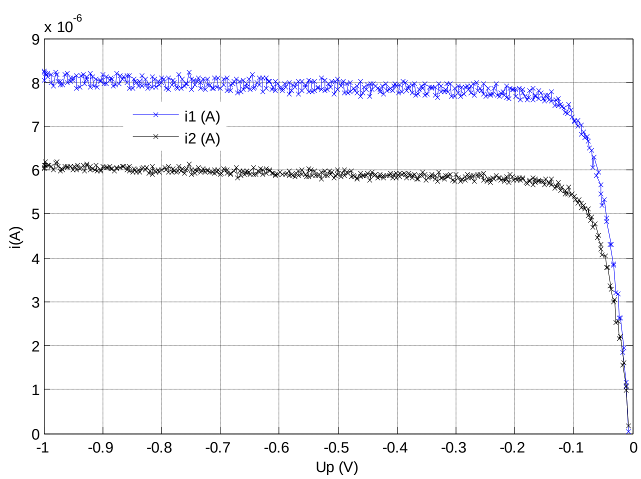

Figure 4: Sampling of diagrams of the current intensities of each segment of the double probe for polarization tension U_p

The sandwich probe is constituted by two segments separated by an insulated gap. Under the diffusion layer (DL) approximation, the current (mass transfer flux) delivered from the front segment is not affected by the downstream process. While, the back segment one is lower in comparison with a single probe of same dimensions [15]. For an ideal sandwich probe and under a steady state convective diffusion, [15] studied in particular the overall transient process in the case of a "*zero insulating gap*" (G=0) under a DL approximation. The currents to back segment can be calculated as the differences between two-segment electrode i_{f+b} and the front segment one i_f. [8] studied numerically the frequency response of double probe (two-segment probe) to fluctuations of the wall shear stress (type sinusoidal: $S^* = S0\left(1+\sqrt{2}\tau\sin\left(2\pi Ft\right)\right)$). He used the finite element method for resolving the equation of convection diffusion. He assimilated the double probe to a two segments of a fictive probe i.e. *non isolating gap* (G=0) obtained by cutting the probe into two segments. He proposed then a formula related to the mass transfer coefficients of every segment of the double probe. The formula has some restrictions. In particular, it is limited to low amplitude of oscillations i.e. $\tau < 40\%$. It is no longer available for large amplitude of fluctuations.

In this paper, we propose a typical application of the inverse method under the same conditions adopted by these researchers, i.e. the isolated gap effect was neglected as a first step of the study of double probe frequency response for the validation of the inverse method.

4. INVERSE METHOD FOR THE DETERMINATION OF THE WALL SHEAR STRESS

The inverse method has been introduced for the first time by [16] in order to solve heat transfer problems, then by [9, 17] to solve mass transfer problems. It is based on a sequential estimation in order to simulate the response of ED probe to a wall shear rate excitation *S(t)* by resolving the convection diffusion equation using the finite volume method [18]. This method is well developed by [19] with known simulated solutions of the diffusion-convection equation for the determination of the wall shear rate in simple phase flows with sinusoidal fluctuations using simple probe. The concentration profile, determined numerically, allows the calculation of the instantaneous Sherwood number by integration, using the Simpson method, of the equation:

$$Sh(t) = \overline{Pe}^{\frac{1}{3}} \int_0^1 \left(\frac{\partial c}{\partial y}\right)_{y=0} dx \tag{2}$$

where the Peclet number is written as:

$$\overline{Pe} = \overline{S}\,\frac{l^2}{D} \tag{3}$$

and the wall shear stress is defined under the condition of a linear velocity profile of the electrolyte flow along the wall surface:

$$S(x,t)=\left(\frac{\partial U_x}{\partial y}\right)_{y=0} \quad (4)$$

The numerical instantaneous Sherwood number $Sh_{num}(t_i^*)$ is calculated via the convection diffusion equation by injection of an estimated wall shear stress $\overleftrightarrow{S}^*_{num}(t_i^*)$. The minimization of the difference between the numerical instantaneous Sherwood number $Sh_{num}(t_i^*)$ and the experimental instantaneous Sherwood number $Sh_{\exp}(t^*)$ calculated via the limiting diffusion current $I_{\exp}(t)$ allows the determination of the wall shear stress:

$$S^*_{\exp}(t_i^*)=\overleftrightarrow{S}^*_{num}(t_i^*)+\frac{\left(Sh^*_{\exp}(t_i^*)-\overleftrightarrow{Sh}^*_{num}(t_i^*)\right)}{\left(\dfrac{\partial\overleftrightarrow{Sh}^*_{num}(t_i^*)}{\partial S^*}\right)_{\vec{S}^*_{num}(t_i^*)}} \quad (5)$$

Where, the quantity $\left(\dfrac{\partial\overrightarrow{Sh}^*_{num}(t_i^*)}{\partial S^*}\right)_{\vec{S}^*_{num}(t_i^*)}$ is estimated numerically via the equation [19]:

$$\left(\frac{\partial\overrightarrow{Sh}^*_{num}(t_i^*)}{\partial S^*}\right)_{\vec{S}^*_{num}(t_i^*)}=\frac{\left(\overrightarrow{Sh}^*_{num}(t_i^*,(1+\varepsilon)\vec{S}^*_{num})-\overrightarrow{Sh}^*_{num}(t_i^*,(1-\varepsilon)\vec{S}^*_{num})\right)}{2\varepsilon\vec{S}^*_{num}} \quad (6)$$

where $10^{-6}\le\varepsilon\le10^{-3}$ [16].

In our study, we have chosen $\varepsilon=10^{-4}$. This value permits to obtain a linear evolution between $Sh(t_i^*)$ and $S^*(t_i^*)$ [18]. It corresponds to an optimal value of the numerical derivate quantity $\left(\dfrac{\partial Sh^*_{num}(t_i^*)}{\partial S^*}\right)_{S^*_{num}(t_i^*)}$.

The new value of the wall shear stress obtained numerically $S^*_{num}(t_i^*)$ is then injected again on the convection diffusion equation in order to recalculate the new value of $Sh^*_{num}(t_i^*)$.

It is noted that a bad initialization of $S^*_{num}(t^*=t_0^*)$ deduce a long calculation time for the resolution of the inverse problem on quasi-steady regime which introduce a difference between the numerical solution and the experimental one on the beginning of the resolution. This problem is generally reencountered when the axial diffusion has a significant effect. More details are available on [19-20].

5. RESULTS AND DISCUSSIONS

The polarography measurements were conducted on the parallel plate rheometer using the multi-segment ED probe in order to validate the inverse method. The angular rotating disk velocity is adjusted to a modified Reynolds numbers Re*=0.103<<1. Therefore, we can directly test the robustness of the inverse method to high and low oscillations frequencies under a non-significant inertia effect. The directional angle is fixed to zero in this case of study. Then, we conducted a series of tests for different directional angles to highlight the directional angle effect. We referred to a modified Reynolds numbers Re*= 2.07>1 in order to consider the inertia effect in our study, i.e. there is presence of secondary flow in the sliding rheometer [1].

The results obtained for different critical positions, characterized by a directional angle relative to a pointer, fixed on the sandwich probe mantle, are studied. The first position corresponds to a directional angle equal to zero. The second position corresponds to a directional angle equal to 90° (vertical position comparing to the first one as reference). Finally, we have studied the effect of an angle equal to 180° (inverse direction taking the first position as reference). The results are presented and discussed below.

5.1. *Frequency oscillation effect on wall shear rate of single phase oscillatory torsional flow*

It is generally impossible to separate different factors effects during a study. However, from a theoretical point of view, the significant complications involved by the real case of coupled factors can be simplified by considering such assumptions or hypothesis. Indeed, factors impact are frequently separated or neglected in order to fulfil the study just on one effect. In addition, in this part of our study, our aims are to foresee the oscillation frequency effect on the instantaneous wall shear stress and to validate the inverse method applied to the double probe frequencies responses. For this reason, a series of tests has been realized for the cases where the inertia effect is neglected; i.e. absence of secondary flow in order to test the robustness of the inverse method to well determine the instantaneous wall shear stress. In addition, the modified Reynolds number Re* chosen corresponds to angular rotation of *5* rpm. It is equal to *Re*=0.103<1*, according to [1]. In this case, the secondary flow in the sliding rheometer does not appear yet. It has to be noted that the Peclet number in this case is equal to $7.63\ 10^3$. Consequently, the diffusion effect can be neglected [21]. This allows us to check exactly the robustness of the inverse method for high and low frequencies of oscillations in any absence of perturbing factors, except the insulating gap effect.

To demonstrate the possible effect of oscillation frequencies, we referred to ED results with two different frequencies of oscillations (*F = 4 Hz*) and (*F = 8 Hz*). The time evolution of the mass transfer and the wall shear rate signals issued from the experimental ED results and the numerical one are presented for low frequency of oscillation in the first section. While, in the next section, we highlight the time evolution of the mass transfer and the wall shear rate signals for high frequency of the oscillatory torsional single phase flow. This allows us in particular to test the robustness of the inverse method to well predict the wall shear stress in both case of high and low oscillations frequencies.

Figures 5.a and 6.a present the time evolution of the mass transfer signals calculated from the experimental ED results and the inverse method signals respectively for low oscillation frequency (*F=4Hz*), and high oscillation frequency (*F=8Hz*).

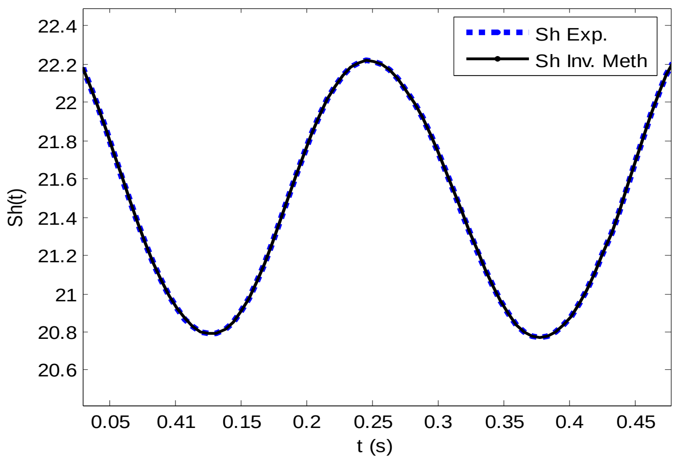

Figure 5 a: Time-evolution of the Sherwood number obtained by the inverse method and compared to the experimental one for F = 4 Hz, Re* = 0.103, θ =0°, A = 40%, Pe = 7.63 10^3

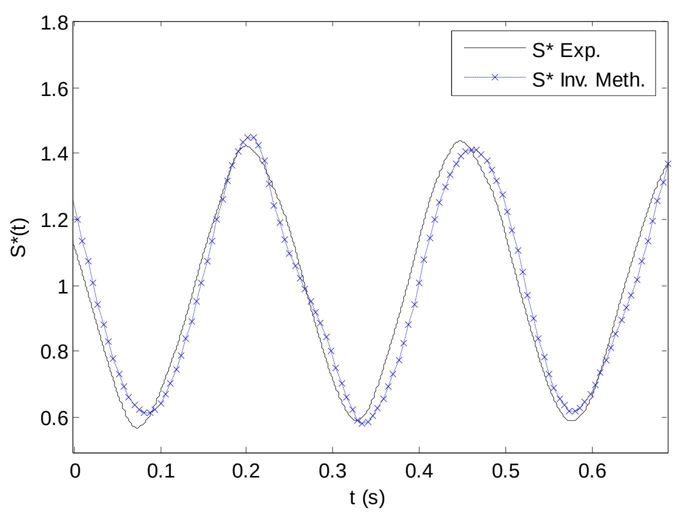

Figure 5 b: Time-evolution of the wall shear stress obtained by the inverse method compared to the experimental one for

F = 4 Hz, Re* = 0.103, θ =0°, A = 40%, Pe = 7.63 10^3

They illustrate the excellent agreement between the experimental results issued from the probe response and the numerical results issued from the inverse method in respect of both amplitude ratio and phase lag for both cases of high and low frequencies of oscillations under the condition of no secondary flow in the sliding rheometer.

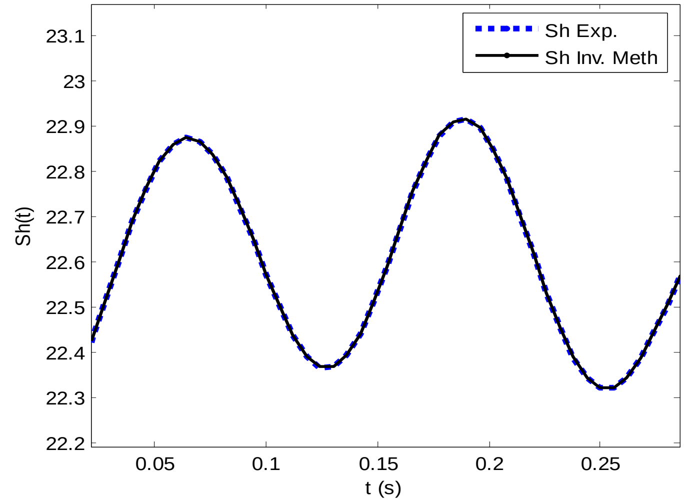

Figure 6 a: Time-evolution of the mass transfer obtained by the inverse method and compared to the experimental one for F = 8Hz, Re* = 0.103, θ =0°, A = 20%, Pe = 7.63 10^3

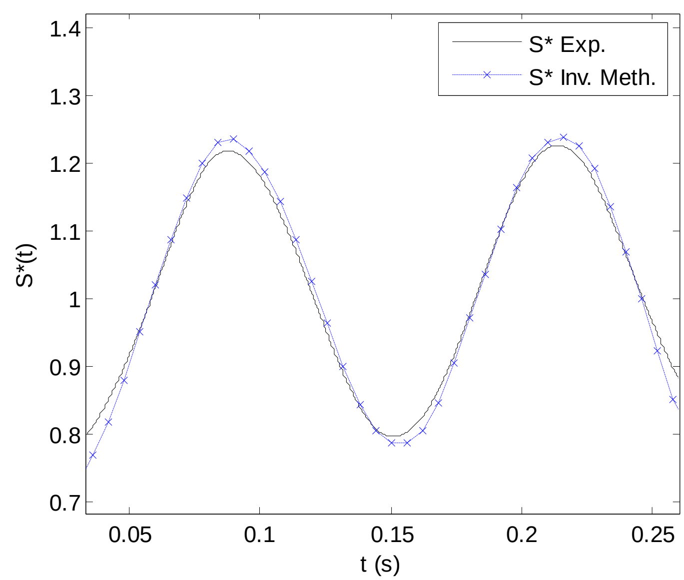

Figure 6 b: Time-evolution of the wall shear stress obtained by the inverse method and compared to the experimental one for F = 8 Hz, Re* = 0.103, θ =0°, A = 20%, Pe = 7.63 10^3

Figures 5.b, and 6.b indicate the time evolution of the dimensionless wall shear stress on the upper disk of the sliding rheometer for a negligible inertia effect ($Re^* = 0.103<1$), and for a Peclet number $Pe = 7.63\ 10^3$ respectively for $F = 4\ Hz$ and $F = 8\ Hz$. From these figures, we can deduce the following remarks. Firstly, they demonstrate that the dimensionless wall shear stress determined numerically leads to well follow the experimental one for both cases of high and low frequencies while neglecting the inertia effect. Secondly, they prove that, for a directional angle $\theta =0°$, the inverse method gives a good estimation of the wall shear rate in the presence of diffusion effect ($Pe = 7.63\ 10^3$), which is not the case for different linear approaches.

The previous illustrations show that the inverse method is experimentally validated for high and low frequencies of oscillations while neglecting the inertia effect.

5.2. Inertia effect and directional angle effect on wall shear rate of single phase oscillatory torsional flow

The protocol of ED measurement and its analysis using the especially designed ED interface EDIK2 and LabView are very important in the qualification of the ED results. The mass transfer principles are strongly depending on the flow gradient velocity and the current issue from the ED probe. To qualify our ED results especially when the inertia effect becomes significant, we studied the different factors of possible error sources.

The criterion for purely tangential laminar flow in a parallel plate rheometer studied experimentally by [1] has been tested. This criterion permits the characterization of flow regime for Newtonian liquid in a rheometer during measurements. The authors introduced the dimensionless number $Re^*=\Omega^* h^2/\upsilon$ and shown experimentally that when $Re^*> 1$, the secondary flow effects becomes significant. Basing on their analysis, in our study, the inertia effect is detected for 50 rpm revolution per minute of the rotating disk of the sliding rheometer (Table 2).

Table 2: Inertia effect analysis on the sliding rheometer

Inertia effect	W (rpm)	Ω (rad/s)	Re*
Neglected	5	0.523	0.103
Neglected	30	3.14	0.62161242
Apparition	50	5.23333333	1.0360207
moderated	70	7.32666667	1.45042898
Significant	100	10.4666667	2.0720414

To demonstrate the possible impact of non-neglecting inertia effect, we referred to a series of ED measurements for a dimensionless Reynolds number characterized by the significant presence of the inertia effect i.e. $Re^*=2.07>1$. The measurements have been performed for different directional angles. The results are presented according to an increasing directional angle chronometry, i.e. successively 90° and 180°.

Theoretical considerations for suitable ED measurements have been considerate during the ED measurements and the acquisition analysis. The diffusion effect presents an important factor in the interpretation of the ED results. The tangential diffusion has no significant effect for $Pe >5\ 10^3$ [21]. The boundary layer approximation [22-23], based on the assumption that the longitudinal and transversal diffusion of ions has no significant effect, leads to large errors for $Pe <10^4$ [14].In addition, the longitudinal and lateral diffusion effects should be taken into account.

Table 3: Diffusion effect analysis according to the evolution of the Peclet number

w (rpm)	Ω (rad/s)	Pe^*10^{-4}
5	0.523	0.76330466
30	3.14	4.579828
50	5.233333333	7.63304667
70	7.326666667	10.6862653
100	10.46666667	15.2660933

For high frequency of fluctuations where the regime is strongly unsteady, the different linear approaches are wrong because the average and fluctuating equations are coupled. The average value depends on the fluctuations. In our case of study, the Peclet number is equal to $1.5\ 10^5$. Consequently, the diffusion effect can be neglected (Table 3).

Some assumptions well defined by [6], has to be taking into account. In our case of study, the ED method was applied under the restriction of a large thickness of the viscous sublayer comparing to the concentration boundary layer one [6]. The thicknesses of the dimensionless concentration boundary layer:

$$\Delta^+ = \Delta\sqrt{\frac{S}{\upsilon}} = 3.68\sqrt{\frac{D}{\upsilon}}S^{1/6} \qquad (7)$$

of the different tests are calculated and illustrated on Table 4, where Δ is the thickness of the concentration boundary layer, assuring the condition:

$$y = \Delta \quad for\ \frac{C}{C_0} = 0.99 \qquad (8)$$

Table 4: Evolution of the dimensionless concentration boundary

W (rpm)	Δ (µm)	Δ^+	δ (µm)
5	31.70980906	0.191606053	339.17421
30	17.45058043	0.258286139	186.654761
50	14.71838956	0.281239225	157.4307226
70	13.15682046	0.297461364	140.7278794
100	11.68199354	0.315680356	124.9528473

The table illustrates that the concentration boundary layer is very thin compared to the viscous boundary layer. Under these hypotheses, the mass transfer is related to the instantaneous wall shear stress via the equation:

$$\frac{\partial C}{\partial t} + S.y.\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \qquad (9)$$

Where, the wall shear stress can be defined under the condition of a linear velocity profile of the electrolyte flow along the wall surface:

$$S(x,t) = \left(\frac{\partial U_x}{\partial y}\right)_{y=0} = \frac{\tau}{\mu_L} \qquad (10)$$

where τ is the torque (*Pa*) and μ_L is the dynamic viscosity of the liquid (*Pa .s*).

The primary analyses have been achieved using the especially designed ED interface EDIK2 which permits also a simultaneous control of the ED measurements. An algorithm, using Matlab®, has been developed for the post-processing, the filtering, the analysis and the interpretation of the mass transfer signals, and the wall shear stress signals. It permits in particular to numerically eliminate any possible noise effect due to the vibrations imposed to the torsional oscillatory single phase flow.

The time evolution of the Sherwood numbers results from the mass transfer delivered by the ED probe are illustrated for a directional angle equal respectively to 90° and 180° on Figures 7a and 8a. Also included in the figures is the numerical time evolution of the Sherwood numbers obtained from the inverse method. As we can see, under the conditions of high frequencies of oscillations, high modified Reynolds numbers (significant inertia effect), high Peclet number (non significant diffusion effect), the different figures illustrate the excellent agreement between the numerical results and the experimental one in respect of both amplitude ratio and phase lag.

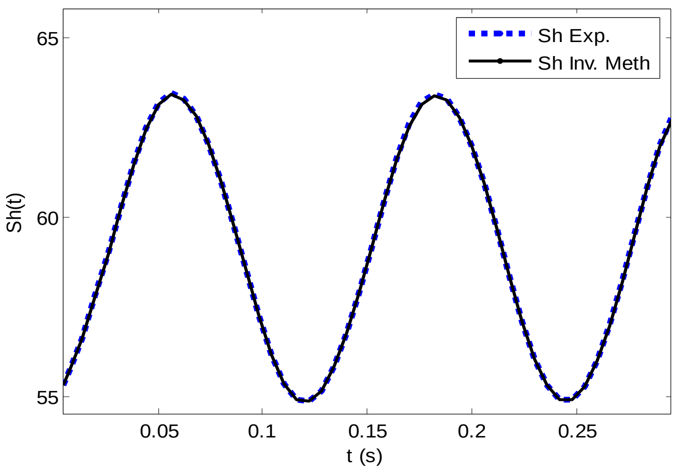

Figure 7 a: Time-evolution of the mass transfer obtained by the inverse method and compared to the experimental one for θ =90°, Re* = 2.07, F = 8Hz, A = 60%, Pe = 1.5 10^5

The time - evolutions of the dimensionless wall shear stress on the upper disk of the sliding rheometer are shown on Figures 7b and 8b for different directional angles, i.e. θ respectively equals to 90° and 180°, for a significant inertia effect (*Re* =2.07>1*), and for a non significant diffusion effect i.e. Peclet number $Pe = 1.5\ 10^5$. We present on the same figures the numerical time evolution of the wall shear stress obtained from the inverse method. The validation of the inverse method on the case of significant presence of secondary flow on the sliding rheometer is thus illustrated.

When the directional angle increases to 90° (Figure 7.b), we show that the inverse method is a little shifted and has approximately the same amplitude of oscillation as the experimental wall shear stress. This can be due to the angle flow effect on the frequency response of the double probe or to the insulating gap effect not taking into account on the numerical method. In fact, the comparison with Figure 7.b highlight that when the directional angle was equal to zero and the inertia effect was neglected, both the phase and the amplitude of the time-evolution of the numerical wall shear stress signals are well predicted.

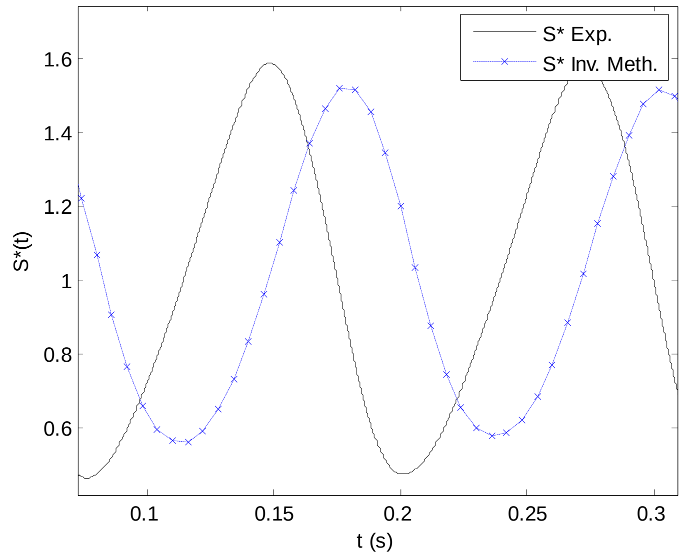

Figure 7 b: Time-evolution of the wall shear stress obtained by the inverse method and compared to the experimental one for θ =90°, Re* = 2.07, F = 8Hz, A = 60%, Pe = 1.5 10^5

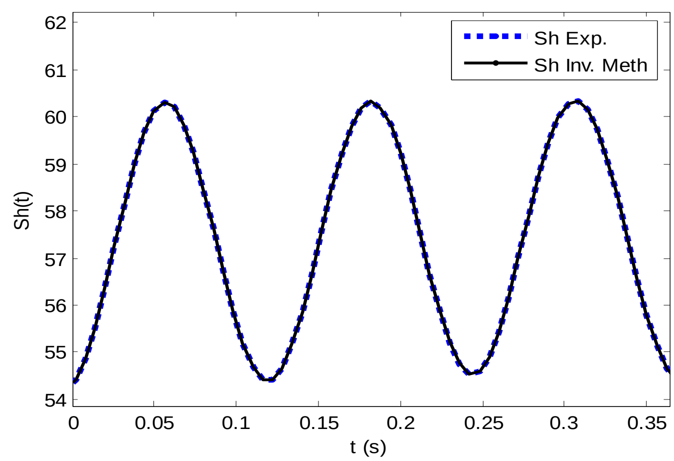

Figure 8 a: Time-evolution of the mass transfer obtained by the inverse method and compared to the experimental one for θ =180°, Re* = 2.07, F = 8Hz, A = 40%, Pe = 1.5 10^5

While the directional angle increases to the double, i.e. θ=180° (Figure 8.b), the flow direction is the opposite comparing to the reference one, i.e. θ = 0° (Figure 6.b). The inverse method, within the non significant insulating gap effect assumption, gives a good agreement with experimental ED results. We prove that the inverse method well predicts the instantaneous wall shear stress. For instance, the comparison of Figure 7.b (i.e. θ=90°) to Figure 8.b (i.e. θ=180°) demonstrates that the little shift detected on the case of directional angle θ=90° disappears while the directional angle increases to θ=180°.

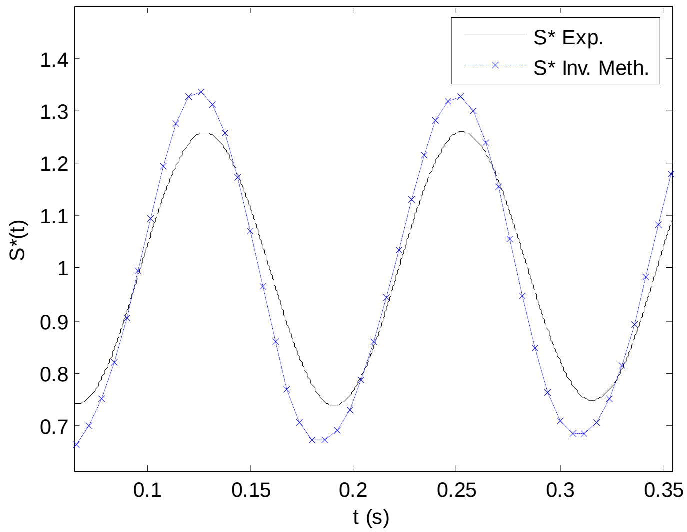

Figure 8 b: Time-evolution of the wall shear stress obtained by the inverse method and compared to the experimental one for

θ =180°, Re* = 2.07, F = 8Hz, A = 40%, Pe = 1.5 10^5

Indeed, we can see that the inverse method leads to well follow the shear stress at the wall of the upper disk of the sliding rheometer while the directional angle remains equal to 180°, under the experimental conditions of high frequencies of oscillations, high modified Reynolds numbers (significant inertia effect), high Peclet number (non significant diffusion effect). This proves that the shift source of error for the directional angle 90° case is probably due to the sensitivity of the double probe to such an angle. In fact, the double probe is sensitive to the angle flow and can give errors for some critical angles [24].

6. CONCLUSION

The frequency response of a sandwich probe to sinusoidal harmonic fluctuations of a single phase flow has been studied in order to test the robustness of the inverse method. This numerical method, based on the inversion of the convection diffusion equation, has been validated under the hypothesis of a neglecting insulating gap effect. In the absence of the edge effect, i.e. our gap between the disks is thin, several cases are presented to demonstrate the possible effect of non-negligible inertial effect due to the centrifugal forces (presence of a distortion on the tangential flow) as well as the diffusion effect due to the ED inertia effects (the capacitive filter) on the robustness of the inverse method. For non-significant inertial effect, for high and low frequencies and different amplitude tested, the inverse method is in a good agreement with the ED results, i.e. the wall shear stress is predicted correctly. Nevertheless, for significant inertial effect, high frequencies of oscillations of tortional flows on parallel plate rheometer, it still needs to be ameliorated for some critical directional angles tested. To conclude, these results are uncourageous for the development of the inverse method for a 3D mass transfer problem. An eventual study using three-segment probe is with a big interest to a well interpretation of the inertia effect and the flow direction flow effects and to a good test of the robustness of the inverse method for the determination of the "real" wall shear stress. The development of the inverse method taking into account the insulating gap effect as next step of the study presents a topic interest.

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