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# MODELING OF LIQUID BRIDGE AND SURFACE WETTING ON TWO RIGID SPHERICAL PARTICLES

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## ABSTRACT:

Spreading and drainage of liquid over particle surface in a liquid bridge between two particles has been modeled to study the influence of temperature of particle and initial kinetic energy of liquid mass remain attached between two particles on impingement of droplet to form liquid bridge on liquid spreading. The spreading and drainage of liquid bridge is a key factor for particle agglomeration, which is commonly encountered in many industrial processes such as petroleum refinery, spray coating and flocculation which involve the collision of droplet and solids particles. A model based on mass and energy conservation of liquid trapped between two particles has been proposed to estimate the surface area of particle wetted by the liquid drainage and the thickness of liquid film under the influence of heat transfer to the liquid film from hot solid particles and initial kinetic energy of spreading liquid. The Laplace-Young model of liquid bridge is solved with discrete method to obtain the volume and surface change of liquid bridge due to spreading and drainage effect. The proposed model is capable of determining transient surface coverage of spreading liquid, liquid film thickness and its temperature. The influence of some key parameters, such as initial kinetic energy of spreading liquid, initial temperature of particle on the wetting thickness and wetting area is also investigated.

**Keywords:** liquid Bridge, Wetting, Drainage, Spreading

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## INTRODUCTION

Agglomeration of solid particles is found in many industrial processes, such as petroleum refinery, spray coating and flocculation where wetted particle collide and stay together to form agglomerates. In the agglomeration process of solids particles, wetting and spreading of liquid over particle surfaces plays a vital role. When high velocity spray droplets collide with hot particles, some liquid remain attached with solids particles and remaining bounce off on colliding with solid particle. Depending on the kinetic energy of attached mass of liquid on solid and temperature of particle, it will spread and forms wetted solids particles. As wetted solids particles collide with each other, depending on their kinetic energy they may stay together to form liquid bridge. The amount and the shape of liquid bridge are mainly determined from the amount of liquid attached on solids particle surfaces, the surface tension and contact angle of liquid, and the size of solids particles. If the solids particles are much hotter than liquid a droplet and liquid bridge liquid has some initial kinetic energy, which is a normal case in many industrial processes, the surface tension, viscosity and contact angle of liquid will keep on changing due to the continuous heat transfer from hot solid particle to liquid which in turn will influence the shape of liquid bridge. The excessive liquid will spread along the surface of the particles, which may enhance the size of agglomerate by attaching more particles on wetted site.

Many research efforts have been made to study the equilibrium shape & strength of the static liquid bridge (Mazzone *et al*., 1986, Lian *et al.*, 1993, Rossetti & Simons, 2003) and rupture criteria for liquid bridges (Pitois et. al., 2001) between two isothermal particles. Murase *et* al., 2004 studied experimentally and numerically the shapes and the tensile strength of a static liquid bridge formed among three spheres. The shape of liquid bridge can be described by Laplace-Young’s equation, which normally has different expression under different conditions (Rynhart *et al*., 2002). Unfortunately, the Laplace-Young’s equation for liquid bridge between two sphere particles cannot be solved directly due to its nature of expression (Lian *et al*., 1993, Rynhart *et al*., 2002). One way to solve it is discrete method by a truncated Taylor series to obtain recurrence equation and then modified Euler method for numerical solution of ordinary differential equation (Lian *et al*., 1993). The dynamic behavior of the droplet impacting on flat isothermal surface has been investigated experimentally and by numerical simulation by many researchers (Mao et al., 1997, Yarin, 2006). But there is no research literature on the investigation of spreading and wetting dynamics of liquid bridge between two rigid and hot spherical particles. Most studies on the spreading of liquid under the influence of temperature gradient are on hot flat surfaces (Ehrhard & Davis, 1991, Anderson & Davis, 1995) which do not consider the influence of initial kinetic energy of spreading liquid, surface curvature on contact angle of liquid with surface and spreading.

This study presents a mechanistic model to investigate the spreading and wetting phenomenon over spherical particle under the influence of initial kinetic energy of spreading liquid and heat transfer from hot solid particles to spreading liquid. The proposed model is based on control volume approach and governing equations of conservation of mass and energy are written for spreading liquid. In the model, the volume and surface area of liquid bridge is calculated with a discrete method of Laplace-Young equation. The wetting thickness and spreading area is then be determined by solving coupled governing equations. The proposed study presents the formation mechanisms of wetting and spreading of liquid on spherical particles which is a key phenomenon for understanding of agglomeration process.

## MODELING APPROACH

Figure 1 show the geometry of the liquid bridge between hot spherical particles of same size, which are in contact with each other. The mass of liquid trapped between to particle to form a liquid bridge is assumed to be analogous with mass of liquid remain attached on flat surface due to impingement of high velocity liquid droplet. Previous studies on liquid droplet impingement on flat surface shows that the attached mass of liquid on flat surface has some initial kinetic energy, which helps bridge liquid to drain over particle surface. The volume and shape of the liquid bridge are governed by Laplace-Young an equation which is function surface tension and contact angle of the liquid which are the functions of temperature. If initially the particles are very hot compared to the cold liquid, there will be heat transfer from the particles to the liquid and hence the temperature of liquid will increases, which reduces the surface tension of liquid and enhance the liquid spreading along with initial kinetic energy of bridge liquid, until it reaches to the thermal equilibrium . Due to the change of liquid temperature, the surface tension and contact angle will change continuously, which in turn will change the shape and volume of bridged liquid. The coverage of surface area of the drained liquid is governed by the balance of kinetic energy of spreading liquid, surface energy (potential energy) and the energy dissipation at moving liquid-solid surface interface. To reduce the mathematical complexity, we assume that the initial solid-liquid-gas interface line will remain unchanged on spreading of liquid, which is shown as point A, B, C, and D in figure 1.. The spreading of the liquid under the influence of kinetic energy of liquid and heat transfer from solid will reduce the mass of liquid bridge by same amount and hence change the shape of liquid bridge. The excessive liquid will flow out of liquid bridge and spread on the particle surfaces as shown in Figure 1. The final thickness and wetting area of drainage is governed by the initial kinetic energy of bridge liquid and initial temperature of the particle. Glycerin of 99.0% purity has been used as bridge liquid in this study because of its high boiling point temperature and its surface tension is sensitive with temperature.

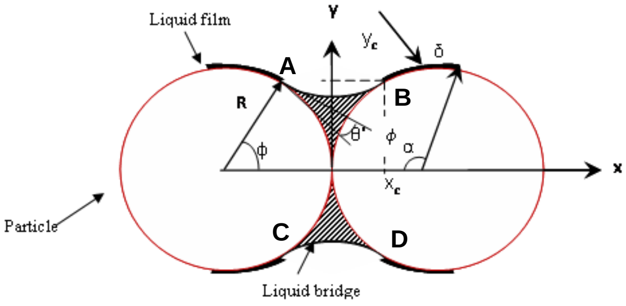


**Figure 1 Liquid bridge with particle wetting**

Consider the small volume of the liquid film over the particle as shown in figure 1.1. Assuming infinite heat capacity of particle and the temperature of the particle is much higher than the bridge liquid and below the boiling temperature of liquid, the mass, momentum and energy equation for the volume of fluid can be written as given below;

Assuming there is no evaporation of liquid film, the mass conservation of liquid film can be written as;

$$\frac{d}{dt}\left( \int_{v(t)} \rho dV \right) = 0 \tag{1}$$

In absence of gravity force, as the volume of bridge liquid is very small, the momentum equation for liquid film can be written as;

$$\frac{d}{dt}\left( \int_{v(t)} \rho \vec{v} dV \right) = \int_{s(t)} \vec{\tau} \cdot \vec{n} ds + \int_{s(t)} \sigma ds \tag{2}$$

Where, the first and second term on the right hand side of equation represents viscous shear and surface tension force respectively. The viscous shear tensor in the above equation can be written as;

$$\vec{\tau} = -\mu[\nabla\vec{v} + (\nabla\vec{v})] \tag{2-1}$$

The kinetic and thermal energy are the major driving force for the spreading of the liquid over particles surface and hence the energy equation for the liquid film with viscous energy dissipation over surface can be written as;

$$\frac{d}{dt}\left( \int_{v(t)} \frac{1}{2} \rho \vec{v}\vec{v} dV \right) + \frac{d}{dt}\left( \int_{s(t)} \sigma dS \right) = -\mu \nabla^2 \vec{v} \left( As.\delta \right) \tag{3}$$

The temperature of the film is governed by the convection heat transfer from particle, hence the temperature of the film in terms of the convection heat transfer from the particle to the liquid film can be written as;

$$\frac{d}{dt}\left( \int_{v(t)} \rho c_p \vec{T} dV \right) = \int_{s(t)} h_{pl} As \nabla \vec{T} dS - \int_{s_\alpha(t)} h_{l\alpha} As_\alpha \nabla \vec{T} dS_\alpha \tag{4}$$

Where, the first and second term on right hand side of above equation represents respectively, the convective heat transfer from hot particle to liquid film and convective heat loss from liquid film to atmosphere.

The liquid volume of the liquid is conserved during spreading process; hence any liquid drained from initial liquid bridge will spread over the particle surface under the influence of kinetic and thermal energy hence;

$$V_0 = V + V_s \tag{5}$$

Where $V_0$, V and $V_s$ represents initial bridge liquid volume, film volume and mass remained in parent liquid bridge.

As long as the initial conditions of liquid bridge and the initial and final properties of liquid are known, the above equations can be solved for the volume and surface area of spreading liquid. The thickness of spreading liquid, δ, if which is assumed to be uniform, can be obtained directly from;

$$\delta = {2V}\big/{A_s} \tag{6}$$

According to Laplace-Young equation, the shape of liquid bridge between two spherical particles with same diameter can be expressed (Lian et. al, 1993) as’

$$\left( \frac{\ddot{Y}}{\left(1+\dot{Y}\right)^{3/2}} - \frac{1}{Y\left(1+\dot{Y}^2\right)^{1/2}} \right) = H^* \tag{7}$$

Where, $H^* = \dfrac{\Delta P.R}{2\sigma}$, $\sigma$ is surface tension of liquid, $\Delta p$ is pressure difference across the curved interface and $Y$ is a function describing the liquid-bridge neck height that is made dimensionless with the particle radius R.

By use of discrete method, the equation (7) can be solved for $\dot{Y}$ and $\ddot{Y}$. A truncated Taylor series can be used to obtain recurring expression of Y(X), which can be expressed as;

$$Y_{i+1} \cong Y_i + \left(X_{i+1} - X_i\right)\dot{Y}_i + \frac{1}{2}\left(X_{i+1} - X_i\right)^2 \ddot{Y} \tag{8}$$

where $i = 0,1,2,...$

and

$$\dot{Y}_i = \sqrt{\left(\frac{Y_i}{C - H^* Y_i^2}\right)^2 - 1}\ , \tag{9-a}$$

$$\ddot{Y}_i = \frac{1+\dot{Y}_i^2}{Y_i} + 2H^*\left(1+\dot{Y}_i^2\right)^{\frac{3}{2}} \tag{9-b}$$

$$C = \sin\phi \sin\left(\theta+\phi\right) + H^* \sin^2\phi \tag{10}$$

The above equations can be solved for proper boundary conditions. The details of solution method can be found in Lian et al., 1993. After solving the Laplace-Young equation, the volume and surface area of liquid bridge can then be written as

$$V_s = R^3 \int_0^{X_c} 2\pi Y^2 dX - \frac{2\pi}{3}\left(1-\cos\phi\right)^2\left(2+\cos\phi\right) \tag{11}$$

$$As_\alpha = R^2 \int_0^{X_c} 2\pi Y \sqrt{1+\dot{Y}^2}\, dX + 2\pi R^2\left(1-\sin\theta\right) \tag{12}$$

The surface tension usually depends on the scalar fields in the system (e.g., electrical fields and temperature Fields), as well as on the concentration of foreign materials on the interface. Since the temperature field is the focus of this study, an equation of state must be considered for the surface tension. The most common approach is to characterize this dependence by a linear law as follows (Ehrhard & Davis, 1991);

$$\sigma(T) = \sigma_0 - \gamma(T - T_0) \tag{13}$$

Where, $\sigma_0$ is the reference surface tension at the reference temperature $T_o$.

Where, $\gamma$ represent positive constant of liquid. The contact angle of liquid with particle surface is function of temperature, for known correlation of contact angle and temperature for given liquid and surface the above equations can be solved to find out wetting area and wetting film thickness. We have six variables namely, T, V, δ, Vs, As, and V and six governing equations 1-4, 6, 11 and hence the problem is closed.

## RESULTS AND DISCUSSIONS

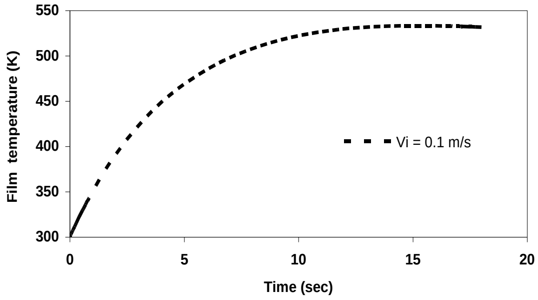
The model predictions of particle surface wetting, liquid film thickness and liquid film temperature at different initial temperature of particle and initial kinetic energy spreading liquid have been discussed in this section. The parametric results of initial liquid volume and initial particle temperature are also discussed in this section. All the results of model predictions for wetting area of particle and film thickness are presented in non-dimensional form. In this study the non-dimensional wetting area is defined as ration of particle area covered by spreading liquid to particle surface area, while the non-dimensional liquid film thickness is defined as thickness of spreading liquid film to the radius of particle. The particle radius is ½” in this study.

### Surface Wetting, Liquid Film Thickness and Film Temperature

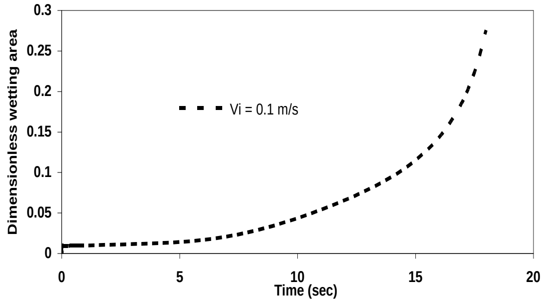
Figure 2(a) shows the model predictions of liquid film temperature for initial particle temperature 550° K. The result shows that the film temperature initially increases with time and asymptotically approaches to the particle temperature, which is due to the assumption of infinite heat capacity of the particle.

Figure 2 (b) shows the result of model predictions for wetting surface area of particle by drained liquid from bridge in terms of non-dimensional wetting area. The model predictions are calculated for initial particle temperature to be 550° K and assuming that the initial velocity of bridge liquid is 0.1 m/s. The result shows that the wetting surface area of particle by drained liquid increases with time which is due to the heat transfer from hot particle to the bridge liquid and its initial kinetic energy. The temperature of the liquid film increases with time which reduces the surface tension forces of liquid and hence along with initial kinetic energy of the bridge liquid, it also increases the spreading of liquid over particle surface.

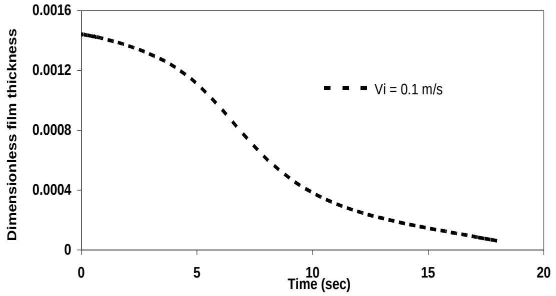
Figure 2(c) shows the results of model predictions for wetting film thickness in terms of non-dimensional wetting film thickness. The model predictions are calculated for initial particle temperature to be 550° K and assuming that the initial velocity of bridge liquid is 0.1 m/s. The result shows that the thickness of wetting film decreases with time. Initially the decease in film thickness is less which is mainly contributed from initial kinetic energy of bridge liquid, but with time the heat transfer from hot particle will increase the film temperature and hence reduces the film thickness faster and reaches to steady state thickness.



(a) Film temperature



(b) Wetting Area

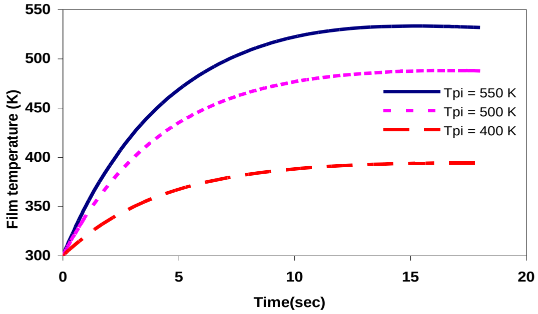


(c) Film Thickness

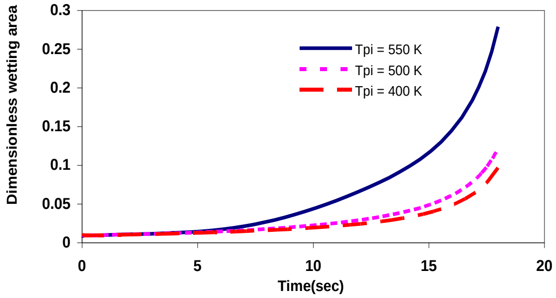
**Figure 2 Model predictions of (a) Film temperature (b) Wetting area (c) Film thickness at particle initial temperature 500° K and 0.1 m/s initial velocity of bridge liquid**

## Parametric Studies

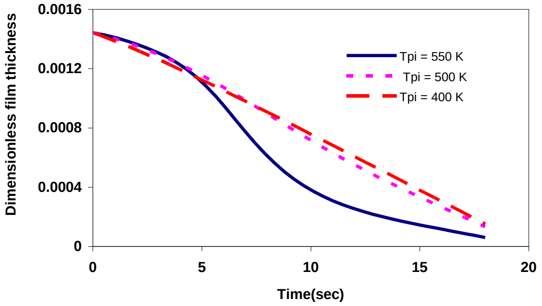
In this section we extended our model predictions and discussion into parametric effect of initial bridge liquid volume and initial particle temperature on wetting surface coverage, film thickness and film temperature, keeping fixed initial kinetic energy of bridge liquid.



(a) Film temperature



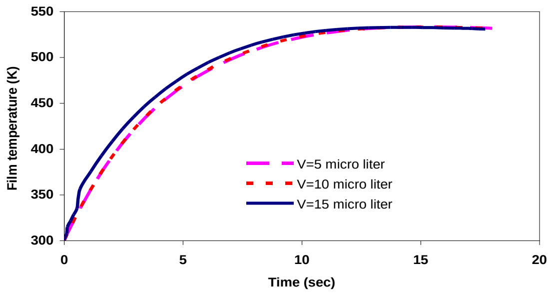
(b) Wetting area



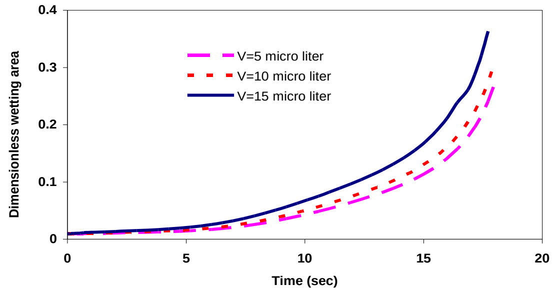
(c) Film thickness

**Figure 3 Parametric study of initial particle temperature on (a) Film temperature (b) Wetting area (c) Film thickness**

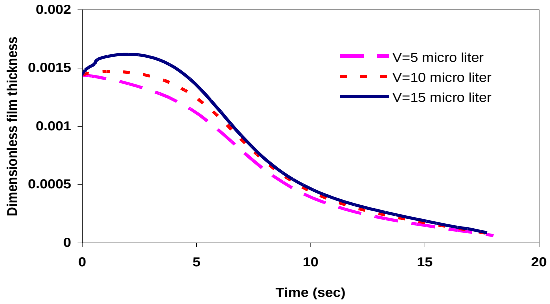
Figure 3 (a) shows the parametric effect of initial particle temperature on film temperature at fixed initial velocity of bridge liquid say, 0.1 m/s. The result shows that, with high initial particle temperature, the liquid film will achieve high final temperature, which is due to high heat transfer rate from particle. The trends of the film temperature rise curve are similar for all the case under the study. Figure 3 (b) and (c) shows the parametric effect of initial temperature of particle on wetting area and film thickness. The results show that, with high initial particle temperature, the liquid tends to spread more over the particle surface and hence wetting of particle surface, while the film thickness is remaining almost constant. This study shows that, the tendency of particle agglomeration increases with collision among high initial temperature of particles due to large particle surface coverage of drained liquid.



**(a) Film Temperature**



**(b) Wetting area**



**(c) Film thickness**

**Figure 4 Parametric study of initial bridge liquid volume on (a) Film temperature (b) Wetting area (c) Film thickness**

Figure 4(a) shows the parametric effect of volume of bridge liquid on film temperature at fixed initial temperature of particle and initial velocity of bridge liquid 550° K and 0.1 m/s respectively. The result shows a little difference in temperature of the liquid film for initial time period and then reaches to the steady state temperature for different bridge volume. This result shows that for a range of bridge liquid volume selected in this study, the liquid film will reach to same final steady state temperature. Figure 4 (b) & (C) shows the parametric effect of bridge liquid volume on wetting area and film thickness. The result shows that wetting area increases with increasing the volume of bridge liquid while the film thickness initially increases with increasing bridge liquid volume but reaches to minimum thickness at the end of spreading process.

## CONCLUSION

A model has been proposed to simulate the wetting and spreading of liquid on particle surface from liquid bridge, due to heat transfer from hot particles to bridged liquid and initial kinetic energy of spreading liquid. The model is based on the mass, momentum and surface energy conservations of bridged liquid. The shape and volume of the liquid bridge are obtained from the solution of Laplace-Young equation with discrete method. The wetting thickness and spreading area are then obtained by solving coupled governing differential equations with appropriate initial boundary conditions. The model can predict wetting thickness and spreading area under different thermal conditions of liquid and solids particles, which is a key issue for the understanding of agglomeration process in a lot of industrial applications.

## NOMENCLATURE

| | |
|---|---|
| $\sigma$ | Surface tension |
| R | Radius of the spherical particle |
| T | Temperature |
| V | Volume |
| $\delta$ | Thickness of liquid file |
| $\theta$ | Contact angle |
| $\varphi$ | Half bridge angle |

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