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APPLICATION OF STOCHASTIC OPTIMIZATION METHODS IN THE GAS TURBINE PROJECTION PROBLEMS

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ABSTRACT

When projecting the gas turbine an important problem is an ensuring the high values of gas turbine parameters and required gas turbine operating characteristics on the different operating conditions. These requirements can be reached by engine function units system perfecting on base of multicriterion stochastic optimization problems solution.

Three stochastic optimization problems definitions were formulated. Each problem has own features and can be used for different application solution. These applied problems are: M-problem can be used on the technical system unit conceptual design stage; V-problem can be used for the problem solution of tolerancing during the technical system unit production; P-problem can be used for interval analysis of technical system functional unit.

The multicriterion stochastic optimization problem rational decision is realized by the evolutional method. This method makes it possible to find the solution with given accuracy by attraction the less information recourses than standard methods. In the stochastic optimization problems definitions the input data random character is taken into account. It makes it possible to find the optimal values of desired parameters. These parameters ensure the maximal probability of finding the objective function in given range.

INTRODUCTION

When the complex technical systems (e.g. gas turbine engine) are projected and developed an important problem is the ensuring the high values of gas turbine parameters and required gas turbine operating characteristics on the different operating conditions. These parameters and features can be reached by engine function units system perfecting on base of multicriterion stochastic optimization problems solution. [1]

However it take into consideration that on the one hand the gas turbine engine is complex technical system with multiechelon hierarchy, therefore modification to perfecting the its one element can lead to the integral system parameter deterioration in whole. [2,3] On the other hand at a serial production departures of scantling nominal value (in the limit of production tolerance) appear when gas turbine production; so its technical features real value can substantially differ from nominal value.

At present paper the stochastic optimization models and methods were considered for complex technical system with multiechelon hierarchy (gas turbine engine).

An object of paper is evolutional method preparation. This method bases on the classic genetic algorithm and allows solving the defined problems with less computer resources as compared with the classic genetic algorithms and other structural-parametric optimization methods.

STOCHASTIC OPTIMIZATION PROBLEM STATEMENT

Stochastic optimization problem are defined as $f(x) \rightarrow min$, where f is target function (TF), its type depends on concerned problem specification, and x is a random variate with given distribution law. In present paper x is considered with Gaussian law or uniform law. In the case of Gaussian law x is set uniquely by specifying its average $M[x] = x_c$ and variance $D[x] = \sigma_x^2$. It should be note if multiparametric optimization problem is considered then variable x and its influential values x_c and σ_x are the vectors; its dimension corresponds to current problem dimension.

In consequence of variable x random nature target function $f(x)$ also is random variate. For random target function definition it needs to specify its average $M[f(x)] = f_c$ and variance $D[f(x)] = \sigma_f^2$. For calculation the expectancy of hitting a target function values to given interval the vector family $\bar{x} = \{x_1, x_2, x_3 \ldots x_n\}$ is generated randomly for given values of x_c and σ_x, appropriate target function values f_i

($i = 1...n$) is calculated, and number of elements i_f satisfied to requirement $f_{min} \le f(x_i) \le f_{max}$ is found. Received number is normalized by division into *n*.

Often several variables enter into the target function expression. Let us assume target function has *J* variables. In this case the convolution of target function variables $\hat{f}$ is used. For calculation the expectancy of hitting a target function values to given interval the vector $f_i = \{f_{1,i}, f_{2,i} \dots f_{J,i}\}$ is calculated for element family. In that case the own contingencies are defined for every variables. And instead of a requirement $f_{min} \le f(x_i) \le f_{max}$ disjunction of requirements $f_{j\,min} \le f_j(x_i) \le f_{j\,max}$, $j = 1...J$ is defined.

In the stochastic optimization problem class the three decision models are assigned. These models are M-problem (minimal target function average search), *V*-problem (minimal target function variance search) and *P*-problem (minimal or maximal target function achievement probability). Let us consider the every of these problems.

M-problem: it necessary to find such x_c, which ensures the minimal target function value f_c by given σ_x. For modification problem solution it is right to reduce this problem to minimization problem of function such as $|f_c - f^*|$, where f^* is given target function value. This problem is the nearest to classic type of optimization problems. If the stochastic nature of variable x and target function is disregarded, the problem is reduced to classic problem.

V-problem: it necessary to find such x_c, which ensures the minimal spread in values σ_f by given σ_x. Conversation the optimization problem into the modification problem can be carried out similarity to *M*-problem conversation.

P-problem: it necessary to find such x_c in order to the probability $P(f_{min} \le f(x) \le f_{max})$ reach extremum by given σ_x.

It should be note often in stochastic optimization problem it necessary to observe the average and variance changing simultaneously and not allow exceeding the given values. For this problem solving the models with mixed conditions are considered.

MATHEMATICAL DECISION MODEL FOR STOCHASTIC OPTIMIZATION PROBLEM WITH MIXED CONDITIONS

There is need to satisfy the several conditions simultaneously for practical problem solution. For example, there is maximization the probability of hitting a target function values to given interval by minimal target function probabilistic average deviation from given value. The denoted conditions of two criterion problems can be solved in following type.

1. Convolution of target functions on base of power average concept by Kolmogorov –

$$\hat{f} = \frac{\Delta_R^2 + \Delta_C^2}{\frac{i_{\alpha,f}}{n_\alpha} + \gamma} + \beta \left[\sum_{m=1}^{M_k} \Delta_{x,m}^2 + \frac{1}{n_\alpha} \sum_{m=1}^{M_k} \left| \chi_{x,m}^2 - n_\alpha \right| \right], \tag{1}$$

where $\Delta_R = \frac{M[R] - R^*}{\sigma_R^*}$; $\Delta_C = \frac{M[C] - C^*}{\sigma_C^*}$, $R^*, C^*, \sigma_R^*, \sigma_C^*$ are desired target function values and its mean-square deviations; $i_{\alpha,f}$ is number of points from n_α, which are in the given interval; $\Delta_{x,m} = \frac{M_\alpha[x_m] - x_{m,0}}{\sigma_m^*}$; $\chi_{x,m}^2 = \frac{n_\alpha M_\alpha\left[(x_m - M_\alpha[x_m])^2\right]}{(\sigma_m^*)^2}$, $x_{m,0}$ are variables x_m values for prototype; σ_m^* is an upper limit of control variable definitional domain.

2. Convolution of target functions on base of maximum likelihood concept –

$$\hat{f} = \frac{1}{2}(\Delta_R^2 + \Delta_C^2) + \frac{\beta}{n_\alpha} \left\{ \frac{1}{2} \sum_{m=1}^{M_k} q_{x,m}^2 + n_\alpha \sum_{m=1}^{M_k} \ln(\sigma_{x,m}) + \frac{1}{2} \sum_{m=1}^{M_k} \left[\chi_{\sigma,m}^2 + (n_\alpha - 3) \ln(\chi_{\sigma,m}^2) \right] - n_\alpha \ln\left(\frac{i_{\alpha,f}}{n_\alpha} + \gamma \right) \right\}, \tag{2}$$

where $q_{x,m}^2 = \frac{n_\alpha M_\alpha\left[(x_m - x_{m,0})^2\right]}{(\sigma_m^*)^2}$ and $\chi_{\sigma,m}^2 = \frac{n_\alpha (\sigma_m)^2}{(\sigma_m^*)^2}$.

EVOLUTIONAL METHOD OF STOCHASTIC OPTIMIZATION PROBLEMS SOLUTION

The evolutional method (EM) is defined as classic genetic algorithm (GA) modification for optimization problems solution [4]. In present paper novelty elements of available algorithm description is offered as compared with classic GA.

Usually for every individual the target function is calculated one time by computation in determine definition. If one of the described problems is solved, a mini-population with given center and variance is generated randomly (Gaussian law) instead of one individual. Then subject to defined problem the probabilistic average (M-problem) or the variance (V-problem) or probability of target function staying in given interval for mini-population (P-problem) are calculated. The mini-population size is selected on base of efficiency balance computation and the correctness ensuring of stochastic problem solution and is not less 200 elements.

In order to protection the population from nonoptimal chromosome dominance and also prevention the early GA convergence the fitness function is used:

$$Fitness = 1 - e^{-Mf_c}, M > 1.$$

The evolutional method was developed on base of GA for calculation accuracy improvement and for more rational GA advantage usage.

The special feature of introduced algorithm is adaptation procedure application. The essence of this algorithm is in following. After the first running the required problem parameters with the best target function of present GA settings are given in initial definitional domain D_0 by low iterations number. Then the given individual is taken as the new definitional domain D_1 center, that has a less dimension than D_0 (if condition $D_1 \subset D_0$ is respected), and the next GA running is performed. And due to search domain reduction with usable individuals number keeping it is possible to improve the GA calculation accuracy on the regular iteration in proportion to definitional domain decrease level.

As GA settings in EM the following values were used: maximal number of epochs (iterations) was from 20 to 200, crossover probability was 0.9, mutation probability was 0.1, population size was from 50 to 100 individuals (mini-populations), algorithm stopping criterion was maximal iterations number, parent individuals selection principle was the tape with elite selection and mutation and crossover were standard one-point.

The introduced EM allows solving the defined problems with less computer resources as compared with the classic genetic algorithms.

GAS TURBINE MODIFICATION PROBLEM SOLUTION EXAMPLE

The numerical solution of system perfection problem for three-shaft turbojet bypass engine D-36 was given. The integral model was used for flow characteristics definition, in this model subsystem phase variable values were assigned as initial data. In calculation result the target functions were defined such as fuel rate C_{FR} and specific impulse R_{SI}. The convolutions of target function (e.g. (1) or (2)) were used as effective target functions. The fan efficiency value η_f, low-pressure compressor efficiency value η_{LPC} and high-pressure compressor efficiency value η_{HPC} were chosen as control variables. For prototype variables C_{FR} and R_{SI} values were equal to $C_{FR} = 0.0694$, $R_{SI} = 102.3 \quad N \cdot s./kg$. Its desired values were chosen like that $C^*_{FR^*} = 0.0687$, $kg/N \cdot h \quad R^*_{SI} = 103.36 \quad N \cdot s./kg$. The lower limit of control variable changing interval was taken equal to nominal prototype characteristics values ($\eta_f = 0.859$; $\eta_{LPC} = 0.879$; $\eta_{HPC} = 0.844$), and the upper limit was taken equal to 0.9 for all control variables.

On the figures 1a and 1b there are computed solutions of C_{FR}, R_{SI} and its mean-square deviations. The solid line is labeled the efficiency-averaged solution and the symbols "×" are computed solutions, which are the closest to solutions averaged by Cartesian length. Selected points are result of rational engine parameters and its mean-square deviations choice problem solution.

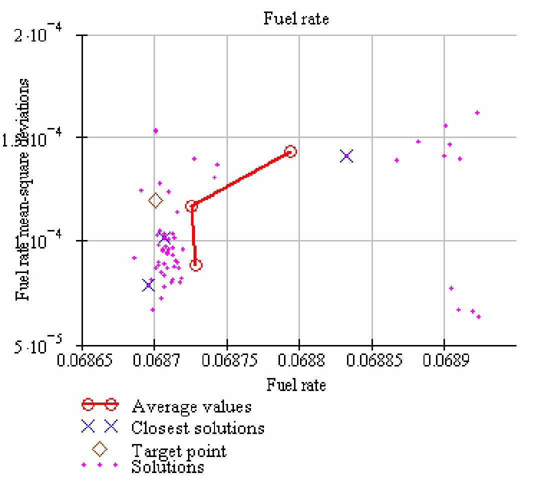

Figure 1a. The computed solutions of C_{FR}

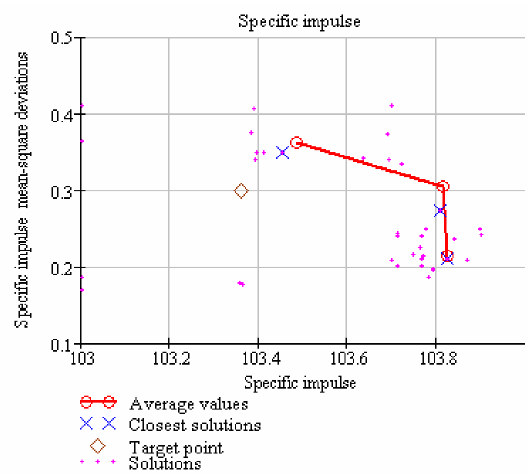

Figure 1b. The computed solutions of R_{SI}

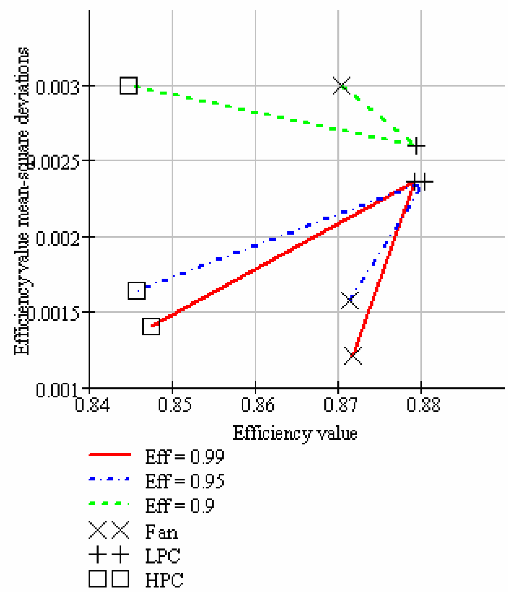

Figure 2. The level lines of solution probability

On the figure 2 there are the level lines, which are according to the computed rational engine parameters probability and its mean-square deviations. In accordance with the computed values it can draw a conclusion that for the engine parameters mean-square deviations decrease it is necessary to decrease the mean-square deviations of fan and high-pressure compressor characteristics. The low-pressure compressor parameters mean-square deviations have not a material effect to the engine parameters mean-square deviation in whole.

Thus the operating tolerance detection problem was solved on the basis of desired target function values and given tolerance of its deviation for system in whole.

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