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AN EXPERIMENTAL INVESTIGATION OF THE SKIN FRICTION IN A PLANE TURBULENT WALL JET OVER SMOOTH AND ROUGH SURFACES

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ABSTRACT

Estimation of the skin friction in a turbulent wall jet flow over smooth and rough surfaces was studied experimentally. Wall jet flows can be found in many engineering applications in which knowledge of the skin friction behavior is essential for predicting the drag force as well as the heat transfer rate at the wall. Although there are many studies which consider a wall jet on a smooth surface, only a few experiments have examined wall jet flows on a rough surface. This paper reports on an experimental investigation which used a two-component laser Doppler anemometry (LDA) system to measure the mean velocity field in a plane turbulent wall jet on both smooth and transitionally rough surfaces. The Reynolds number based on the slot height and exit velocity of the jet was approximately $Re = 7500$*. A glass plate was used for the smooth surface, while the rough surface consisted of a 36-grit sheet glued to the glass plate. The momentum-viscosity scaling originally introduced by Narasimha et al. (1973) and revisited by Wygnanski et al. (1992) can be used to construct a similarity profile for a wall jet on a smooth surface, which together with the momentum integral equation leads to a convenient expression for the friction velocity and hence skin friction coefficient* C_f*. This approach has been used to process the experimental results, which gives values of* C_f *which are consistent with the results of other methods and some existing empirical correlations. However, for rough wall flow, the friction at the wall is not only governed by viscosity, but also by surface roughness. Hogg et al. (1997) suggested that for a fully rough surface, the viscosity be replaced by the roughness parameter* $U_o k_e$*, where* U_o *and* k_e *are the initial velocity and roughness length, respectively. Here, this approach is applied to our recent velocity measurements in a wall jet on a transitionally rough surface, where both viscous and roughness effects are present. The present results indicate that for an equivalent sand-grain roughness range of* $40 < k_s^+ < 70$*, the momentum-viscosity scaling is able to capture the skin friction behavior compared to that obtained from the logarithmic and power laws. The results also show that the scalings proposed by Hogg et al. (1997) and Wyg-*

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nanski et al. (1992) both result in similar values for the friction velocity. However, the values of C_f estimated by both scalings are considerably larger (approximately 47%*) than those obtained from the logarithmic and power laws.*

INTRODUCTION

A turbulent wall jet is a "shear flow directed along a wall where, by virtue of an initially supplied momentum, at any downstream location the streamwise velocity over some region within the flow exceeds that in the external stream" [1]. A definition sketch of this flow is shown in Figure 1. In this figure, x and y denote streamwise and wall-normal distances, respectively; U and V are respectively the streamwise and wall-normal components of the mean velocity; $U_\circ$ is the jet exit velocity; H is the slot height; U_m is the maximum velocity; y_m and $y_{1/2}$ are respectively the wall-normal locations where U_m and $U_\mathrm{m}/2$ occur.

In most wall jets, the flow is exhausted through a nozzle into quiescent surroundings creating a shear layer between the high momentum fluid discharging from the nozzle and the surrounding fluid. As this shear layer develops with downstream distance from the jet exit, the surrounding fluid is entrained by the shear layer. A boundary layer region is also formed between the wall and the fluid discharging through the nozzle. When these two layers grow to meet each other, the wall jet is considered to be nominally fully developed. However, it may take much longer for these two layers to reach an equilibrium such that self-similar profiles are sustained in the streamwise direction. The location where the wall jet becomes fully developed is open to debate. Narasimha et al. [2] suggested that this location is $30H$ for a uniform nozzle flow, while Hall and Ewing [3] suggested $20-25H$ downstream from the jet exit.

In a wall jet, the flow field is conventionally divided into two regions, i.e. an inner layer and outer layer. The inner layer which has many of the characteristics of a turbulent boundary layer, extends from the wall up to $y = y_\mathrm{m}$, while the outer region which is structurally similar to a free plane jet, stretches from $y = y_\mathrm{m}$ to the outer edge of the flow. The interaction between the small-scale turbulence within the inner layer and the large-scale turbulence which dominates the outer layer, makes the wall jet a complex flow for turbulence research.

Turbulent wall jets have been studied for many years due to the fact that this type of flow can be found in many engineering and industrial applications. Applications of wall jets include separation control which leads to effective modification of lift and drag forces on airfoils, film cooling of the walls of combustion chambers in gas turbine engines, and space heating and air conditioning flows in buildings [4,5]. In these applications, knowledge of the skin friction behavior is essential for predicting the drag force as well as the wall heat transfer rate.

The turbulent plane wall jet on a smooth surface has been relatively well studied, including some recent "benchmark" studies. Eriksson et al. [6] investigated both the mean and turbulent velocity fields of a wall jet on a smooth surface. They obtained high quality experimental data using high spatial resolution laser Doppler anemometry (LDA). George et al. [7] proposed a similarity theory for the turbulent plane wall jet based on power laws. They compared their theoretical results with the experimental data of Eriksson et al. [6]. They also proposed a theoretical relation for the skin friction coefficient, C_f. More recently, Barenblatt et al. [8] proposed a theory of incomplete similarity for the entire flow field of the wall jet. In contrast to George et al. [7], they concluded that there is no single self-similar structure in the wall jet to which one can apply the scaling laws. Wygnanski et al. [9] studied a wall jet flow over a smooth surface using hot-wire anemometry (HWA) and analyzed the results using the momentum-viscosity scaling proposed by Narasimha et al. [2]. They found that use of a similarity profile in the momentum integral equation provided reasonable estimates of the local friction velocity.

In contrast to the smooth wall case, only a few studies have looked at the plane wall jet on a rough surface. Rajaratnam [10] investigated wall jet flows over surfaces with deterministic roughness patterns using Pitot-tube measurements. Tachie et al. [11] used LDA measurements to study a wall jet flowing over sand grain roughness. They used the power law formulation proposed by George et al. [7] to determine the friction velocity. Recently, Smith [12] studied the effect of different roughness on a wall jet in a wind tunnel using HWA measurements. He investigated both the mean and turbulent velocity fields. Hogg et al. [13] proposed scaling laws for a rough wall jet based on the work of Wygnanski et al. [9] for a smooth wall jet. They proposed that the characteristics of the jet depend weakly upon the roughness length scale associated with the surface.

Notwithstanding the many studies of plane turbulent wall jets, there is not yet any general consensus on a correlation for the skin friction, especially for a wall jet over a rough surface. To this end, this paper reviews some methods proposed for estimating the skin friction for a smooth wall jet and extends them to the estimation of the skin friction coefficient for a transitionally rough wall jet based on a new experimental data set obtained using LDA.

EXPERIMENTAL APPARATUS

The experiments were carried out in a water tank with a length, width and height of 4.16 m, 1.28 m and 1.7 m, respectively. The water flow was supplied by a pump which discharged through a rectangular slot at a jet exit velocity of $U_\circ = 1.21$ m/s. The slot had a width of 750 mm and height of $H = 6$ mm, so that the width-to-height ratio was large enough to consider the jet to be two-dimensional. Velocity measurements were carried out at different streamwise positions measured from the jet exit up to $x = 80H$. The Reynolds number of the wall jet, based on

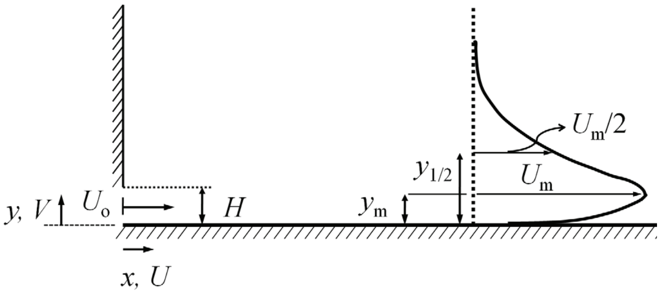

FIGURE 1. SCHEMATIC OF A TURBULENT WALL JET.

the jet exit velocity (obtained from the integral of velocity profile across the slot at the exit) and the slot height, was approximately $Re = 7500$. All measurements were made at a water temperature of 22°C. A glass plate was used for the smooth surface, while the rough surface consisted of a 36-grit sheet glued to the glass plate using contact cement. This sheet was manufactured by Gator Grit® and had a nominal grain size of $k_g = 0.53$ mm, creating a transitionally rough flow.

The velocity measurements were made using a two-component LDA system with a burst mode processor supplied by Dantec Inc. The LDA system was powered by a 750 mW argon ion laser. The measurement volume sizes were 0.184×3.88 mm and 0.194×4.09 mm for the streamwise and wall-normal velocity components, respectively. Version 4.10 of the BSA Flow software was used for data collection and reduction. Hollow glass beads with an average diameter of 10 μm were used to uniformly seed the flow. To reduce the velocity bias in the LDA measurements due to the high turbulence intensity levels, the raw data were corrected using the analytical techniques of McLaughlin and Tiederman [14] and Zhang [15] for different turbulence intensity levels.

It should be noted that one of the characteristics of the present wall jet apparatus is the use of a special nozzle configuration which produced a remarkably uniform velocity profile at the slot exit. This facilitated a precise measurement of the exit momentum, $M_\circ$, used in the scaling analysis. The turbulence intensity in the central region of the jet at the exit plane was less than 1%.

RESULTS AND DISCUSSION

Skin friction coefficient for smooth wall jets

A summary of some relations for the skin friction coefficient is given in the somewhat dated review article by Launder and Rodi [1]. The work of Sigalla [16] is referenced as one of the first to study predictions of the skin friction coefficient. Assuming that the region below the velocity maximum was analogous to a classical turbulent boundary layer, he correlated his data as follows,

$$C_f = 0.0565 {Re_m}^{-0.25}, \tag{1}$$

where $Re_m = U_m y_m/\nu$ is the local Reynolds number and ν is the kinematic viscosity of the fluid. Bradshaw and Gee [17] measured the skin friction using a Preston tube and proposed the following correlation,

$$C_f = 0.0315 {Re_m}^{-0.182}. \tag{2}$$

Hammond [5] derived the following correlation for the skin friction coefficient in a plane wall jet,

$$C_f = 0.0667 {Re_m}^{-0.258}. \tag{3}$$

Eriksson et al. [6] proposed an empirical fit to their own LDA data of the following form,

$$C_f = 0.0179 {Re_m}^{-0.113}. \tag{4}$$

George et al. [7] also derived a theoretical relation for the skin friction coefficient in a turbulent wall jet based on the power law,

$$\sqrt{C_f/2} = \frac{C_o}{C_i}\left(y^+_{1/2}\right)^{-\gamma}, \tag{5}$$

where C_o, C_i and γ are power law constants and are functions of the local Reynolds number, ${y_{1/2}}^+ = y_{1/2}u_\tau/\nu$. Here u_τ is the friction velocity which is closely related to the skin friction coefficient, i.e. $C_f = 2(u_\tau/U_\mathrm{m})^2$. Based on the analysis of George and Castillo [18] and LDA data of Karlsson et al. [19], the following relations were recommended for the coefficients used in Equation (5):

$$\gamma = 0.0362 + \frac{1.334}{\left(\ln {y_{1/2}}^+\right)^{1.46}} \tag{6}$$

and

$$\frac{C_o}{C_i} = 0.023 exp\left[\frac{4.234}{\left(\ln {y_{1/2}}^+\right)^{0.46}}\right]. \tag{7}$$

Wygnanski et al. [9] provided their own skin friction relation based on the momentum integral equation and similarity of the mean velocity profiles. They proposed the following relation for the skin friction coefficient:

$$\frac{C_f}{2} = A\left(\frac{M_\circ}{\nu U_\mathrm{m}}\right)^2\left(\frac{xM_\circ}{\nu^2}\right)^\alpha \tag{8}$$

where $M_\circ$ is the initial momentum flux. Their relation is an especially convenient method for estimating the skin friction coefficient since A and α are obtained from the decay rate of the maximum velocity and the spread rate of the wall jet. For Equation (8), Wygnanski et al. [10] reported the values of $A = 0.146$ and $\alpha = -1.07$ for a smooth wall jet; the present study obtained values of $A = 0.161$ and $\alpha = -1.054$, which differ by approximately 10% and 2%, respectively.

Figure 2 compares the values of the skin friction coefficient obtained from the present study using Equations (5) and (8) proposed by George et al. [7] and Wygnanski et al. [9], respectively, to the correlations given by equations (1) - (4), as well as the experimental data of Tachie et al. [11]. Figure 2 indicates that the proposed expression by Wygnanski et al. [9] results in acceptable skin friction values for $Re_\mathrm{m} > 3000$ corresponding to $x/H \geq 50$. The collapse of the present data with other data at $x/H \geq 50$ might imply that the present wall jet approaches fully developed behaviour in a region beginning at $x/H = 50$. There is a difference of approximately 4% between the C_f values obtained from the proposals of George et al. [7] and Wygnanski et al. [9] for $Re_\mathrm{m} > 3000$.

According to Figure 2, considerable variation in the skin friction values versus Reynolds number can be seen between the various correlations. The present C_f values obtained from the theoretical relation of George et al. [7] are in better agreement with the correlation of Bradshaw and Gee [17] than other correlations. Some of the inconsistencies between the data sets these correlations are based on can be attributed to the possible lack of two-dimensionality in the wall jet, small inner layer thickness and inherent limitation of different experimental techniques [9].

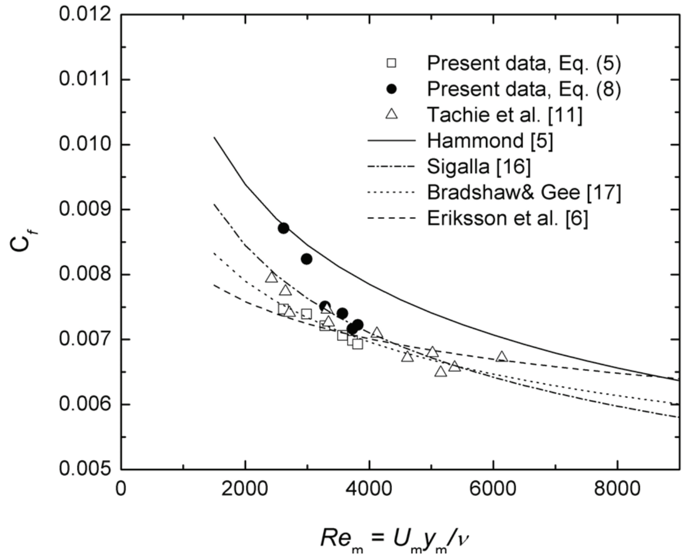

FIGURE 2. SKIN FRICTION COEFFICIENT AS A FUNCTION OF REYNOLDS NUMBER FOR A SMOOTH TURBULENT WALL JET.

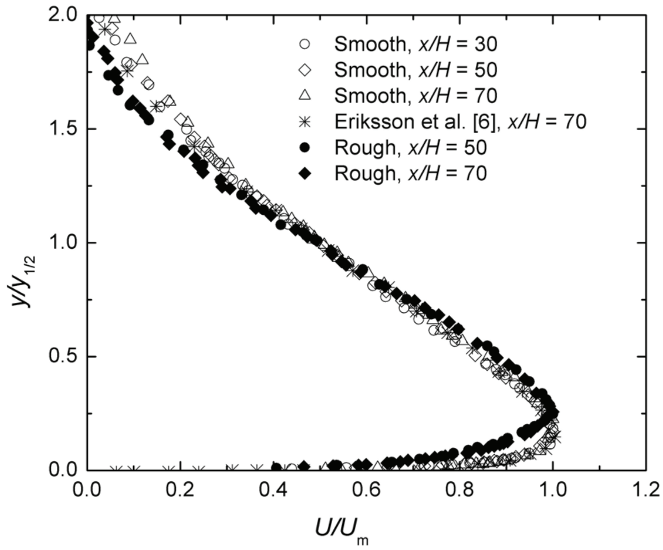

FIGURE 3. MEAN VELOCITY PROFILES OF SMOOTH AND ROUGH WALL JETS IN OUTER COORDINATES.

Skin friction coefficient for rough wall jets

Wall jet measurements were next carried out over a rough surface with a nominal grain size of $k_g = 0.53$ mm. For the rough wall jet, the mean velocity field is compared to the smooth wall jet as well as the experimental data of Eriksson et al. [6] in Figure 3. According to this figure, a self-similar behaviour is observed for the velocity profiles at different streamwise distances for both the smooth and rough surfaces. The maximum velocity of the smooth wall jet occurs at approximately $y/y_{1/2} = 0.16$ which is in agreement with the values of 0.16 and 0.17 proposed by Rajaratnam [10] and George et al. [7], respectively. For the rough wall jet, the present results show that the maximum velocity occurs at approximately $y/y_{1/2} = 0.28$ which is in the range of 0.25 to 0.40 suggested for different roughness by Rajaratnam [10].

The classical logarithmic law for a smooth-wall turbulent flow can be expressed as

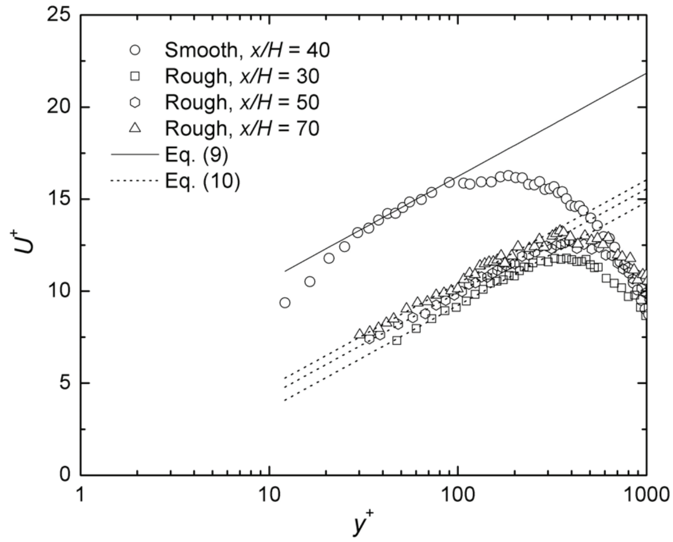

FIGURE 4. MEAN VELOCITY PROFILES OF A ROUGH TURBULENT WALL JET IN INNER COORDINATES.

$$U^+ = \frac{U}{u_\tau} = \frac{1}{\kappa}\ln y^+ + B, \quad (9)$$

where κ and B are the log law constants and are assumed to be universal and independent of Reynolds number. For the present study, the values of $\kappa = 0.41$ and $B = 5.0$ are adopted. For a rough wall turbulent flow, the mean velocity profile in inner coordinates can be written as

$$U^+ = \frac{1}{\kappa}\ln y^+ + B - \Delta U^+, \quad (10)$$

where so called roughness shift ΔU^+ represents the vertical displacement between the smooth-wall and rough-wall velocity profiles on a semi-logarithmic plot. The value of ΔU^+ is often assumed to be a function of the roughness Reynolds number, ${k_s}^+$. The mean velocity profiles of the rough wall jet at different downstream locations of $x/H = 30$, 50 and 70 are shown in Figure 4 using inner coordinates. For comparison, the smooth wall jet data obtained at $x/H = 40$ is also included. Figure 4 indicates that the present rough wall data collapse well with a log-law profile in a narrow overlap region. This is in agreement with the results of Smith [12], however he obtained the value of $\kappa = 0.548$ for his hot-wire anemometry (HWA) data.

George et al. [7] proposed a power law for the overlap region of the wall jet assuming similarity in the inner and outer layers of the wall jet in the limit of infinite Reynolds numbers. Their

TABLE 1. SUMMARY OF FLOW CONDITIONS AND SKIN FRICTION COEFFICIENT FOR A ROUGH WALL JET.

x/H	U_m (m/s)	$y_{1/2}$ (m)	u_τ (m/s)	$C_f \times 10^3$	ΔU^+	${k_s}^+$
30	0.714	0.0211	0.0608	14.47	7.01	70
40	0.640	0.0258	0.0518	12.73	6.57	62
50	0.575	0.0311	0.0455	12.29	6.29	55
60	0.536	0.0361	0.0404	11.34	5.96	48
70	0.487	0.0411	0.0372	11.50	5.80	45
80	0.454	0.0462	0.0334	10.86	5.71	43

proposal is in the form of

$$U^+ = C_i(y^+ + a^+)^\gamma \quad (11)$$

where C_i and γ are the same power law constants given in Equation (5). The parameter $a^+ = -16$ adopts the value recommended by George and Castillo [18]. Both logarithmic and power laws were used to estimate the skin friction coefficient on the rough surface. A summary of the experimental coefficients is shown in Table 1. This table also includes the values of inner and outer scales for a rough wall jet at different streamwise locations. According to Table 1, the surface roughness created a transitionally rough flow since the roughness Reynolds number is in the range of $5 < {k_s}^+ < 70$.

The skin friction can also be determined from the spread and decay rates assuming similarity. Following the momentum-viscosity scaling of Narasimha et al. [2], Wygnanski et al. [9] proposed the following power law relations for the spread rate and maximum velocity:

$$\frac{y_{1/2}M_\circ}{\nu^2} = A_1\left(\frac{xM_\circ}{\nu^2}\right)^{\alpha_1}; \quad (12)$$

$$\frac{U_m\nu}{M_\circ} = A_2\left(\frac{xM_\circ}{\nu^2}\right)^{\alpha_2}. \quad (13)$$

In Equations (12) and (13), A_1, A_2, α_1 and α_2 are power-law constants; their values may depend on the initial conditions such as the source Reynolds number and characteristics of the velocity profile at the exit plane. In Equation (8), the constants α and A were obtained as $\alpha = 2\alpha_2 + \alpha_1 - 1$ and $A = -\lambda(\alpha+1)A_1{A_2}^2$, where λ can be considered a shape factor for the wall jet velocity profiles and is obtained from the self-similarity of the mean velocity profile, i.e.

$$\lambda = \int_0^\infty \left(\frac{U}{U_m}\right)^2 d\left(\frac{y}{y_{1/2}}\right). \tag{14}$$

Hogg et al. [13] hypothesized that for a fully rough wall jet, the dimensional variables which govern the flow are the initial momentum flux ($M_\circ$) and a measure of the turbulent viscosity associated with the roughness elements given by $U_\circ k_e$, where k_e is the roughness length. They reasoned that since the roughness elements prevent the establishment of a viscous boundary layer, the fluid viscosity does not ultimately influence the flow and therefore the turbulent viscosity based on roughness length is the relevant parameter to scale a fully-rough wall jet. Substituting $U_\circ k_e$ for the kinematic viscosity (ν) in Equations (12), (13) and (8) proposed by Wygnanski et al. [9], Hogg et al. [13] obtained the following relationships in non-dimensional, power-law form:

$$\frac{y_{1/2} M_\circ}{{U_\circ}^2 {k_e}^2} = B_1 \left(\frac{x M_\circ}{{U_\circ}^2 {k_e}^2}\right)^{\beta_1}; \tag{15}$$

$$\frac{U_m U_\circ k_e}{M_\circ} = B_2 \left(\frac{x M_\circ}{{U_\circ}^2 {k_e}^2}\right)^{\beta_2}; \tag{16}$$

$$\frac{C_f}{2} = B \left(\frac{M_\circ}{U_\circ k_e U_\mathrm{m}}\right)^2 \left(\frac{x M_\circ}{{U_\circ}^2 {k_e}^2}\right)^{\beta}. \tag{17}$$

Here, B_1, B_2, β_1 and β_2 are the coefficients which can be obtained from the spread rate and maximum velocity decay rate of the rough wall jet, i.e. Equations (15) and (16). According to Hogg et al. [13], coefficient β is calculated as $\beta = 2\beta_2 + \beta_1 - 1$. However, Hogg et al. [13] did not provide a detailed explanation of how they determined constant B.

Table 2 summarizes the coefficients including the constants and exponents in Equations (8), (12), (13), (15), (16) and (17) for the present study, as well as the experiments of Wygnanski et al. [9] and Hogg et al. [13]. According to this table, the exponent β was obtained as -1.12 in the present study, while Hogg et al. [13] proposed the value of approximately -1.11, a difference of approximately 1%.

In the present study, the nominal grain size ($k_g = 0.53$ mm) of the rough surface as determined by the manufacturer was used as the roughness scale in above equations. By applying the similarity approach of Wygnanski et al. [9] in the relations proposed by Hogg et al. [13], the value of 0.29 was obtained for constant B: $B = -\lambda(\beta+1)B_1{B_2}^2$ based on the momentum balance analysis and the self-similarity assumption proposed by Wygnanski et al. [9]. The level of the skin friction coefficient was found to be especially sensitive to the value of this constant.

TABLE 2. COEFFICIENTS OF MEAN-FLOW DEVELOPMENT EQUATIONS OBTAINED FROM DIFFERENT STUDIES.

Author(s)	A_1	α_1	A_2	α_2	A	α
Present (smooth)	9.31	0.791	0.662	-0.422	0.16	-1.53
Present (rough)	7.51	0.804	1.443	-0.462	1.37	-1.12
Wygnanski et al. [9]	1.445	0.881	1.473	-0.472	0.146	-1.07
Author(s)	B_1	β_1	B_2	β_2	B	β
Present (rough)	0.584	0.804	2.38	-0.462	0.29	-1.12
Hogg et al. [13]	0.21	0.84	2.3	-0.475	-	-1.11

Figure 5 shows the values obtained for the skin friction coefficient for the wall jet over a transitionally rough surface. The classic logarithmic law together with the power law proposed by George et al. [7], the momentum balance analysis proposed by Wygnanski et al. [9] and the scaling proposed by Hogg et al. [13] were used to estimate the skin friction coefficient for the rough wall case. According to Figure 5, a similar trend can be seen in the behaviour of C_f for all three methods. The momentum-viscosity scaling appears to be able to capture the skin friction behavior compared to that obtained from the logarithmic and power laws. However, there is a significant difference (approximately 47%) in magnitude between the C_f values obtained from the log- and power-laws and from the scaling proposed by Hogg et al. [13] showing the sensitivity of the C_f value to the value of constant B. Fitting the data obtained from the method of Hogg et al. [13] to the data obtained from logarithmic and power laws, a value of $B = 0.2$ is proposed for Equation (17). Variation of C_f for the present data obtained from the scaling proposed by Hogg et al. [13] with the adopted value of $B = 0.2$ is shown in Figure 5. This figure also shows that the scaling proposed by Wygnanski et al. [9] for smooth wall jets and that proposed by Hogg et al. [13] for fully rough wall jets result in almost identical values for the skin friction coefficient. An experimental uncertainty of approximately $\pm 8\%$ was calculated for the estimation of C_f obtained from the logarithmic and power laws.

CONCLUSIONS

An experimental investigation of the skin friction in a turbulent plane wall jet over both smooth and rough surfaces using LDA is reported in the present study. The present results for the skin friction coefficient (C_f) of a smooth wall jet show that the scaling proposed by Wygnanski et al. [9] is an effective method for estimating the skin friction without resolving the inner velocity scale but knowing only the streamwise development of the velocity field. A difference of approximately 4% between the

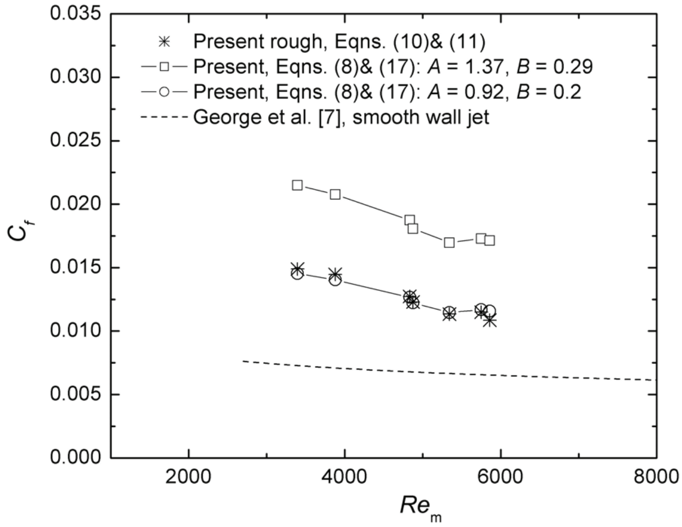

FIGURE 5. SKIN FRICTION COEFFICIENT FOR A ROUGH TURBULENT WALL JET.

present skin friction values obtained from the scaling of Wygnanski et al. [9] and those from the theoretical skin friction relation proposed by George et al. [7] was obtained for $x/H > 50$. For the rough wall jet, the mean velocity profile in the inner layer was fitted to logarithmic and power law relations using inner coordinates that then led to an estimation of the skin friction coefficient. The scalings proposed by Wygnanski et al. [9] and Hogg et al. [13] were also used to determine the skin friction values for a rough wall jet. Although, these two scalings use different turbulent viscosities, they result in almost identical values for C_f. However, these values of C_f differed by approximately 47% from those obtained from the logarithmic and power laws. This comparison suggests that the value of the constant B warrants further investigation. A final observation is that a complete theory for scaling transitionally rough wall jets would require considerations of both viscous and roughness length scales.

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