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STABILITY OF A SHEAR-THINNING FILM ON A POROUS SUBSTRATE

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NOMENCLATURE

$d_s = \left(\frac{\eta_0 Q}{\rho g \sin\theta}\right)^{\frac{1}{3}}$ Length scale
g Acceleration due to gravity
K Permeability of the porous medium
$L = \frac{\delta Q}{d_s^2}$ Dimensionless shear-thinning parameter
n Dimensionless parameter (Carreau model)
Q Flow rate
$Re = \frac{\rho Q}{\eta_0}$ Reynolds number
u Longitudinal velocity
v Vertical velocity
$\dot{\gamma}$ Shear rate
$\hat{\alpha}$ Dimensionless parameter dependent on the structure of the porous medium
$\beta = \frac{\sqrt{K}}{\hat{\alpha} d_s}$ Dimensionless parameter describing the influence of permeability
δ Characteristic time (Carreau model)
η Viscosity
η_0 Viscosity at null shear rate (Carreau model)
η_∞ Viscosity at infinite shear rate (Carreau model)
ρ Fluid density
θ Angle of inclination

Introduction

A significant feature of gravity-driven film flows of Newtonian and rheologically complex fluids down an inclined/vertical substrate is the instability of the free surface which manifests as surface waves having wavelengths much larger than the film thickness. There are a number of applications which can be modeled as thin film flow systems on porous substrates. Pascal [1] investigated the stability of a falling power-law film on an inclined porous substrate. This model for the fluid predicts a viscosity that goes to infinity as the shear rate approaches zero. There is a need to employ a more appropriate model to examine the effects of non-Newtonian rheology on the dynamics and stability of thin film free surface flows down inclined or vertical rigid/porous substrates. The four-parameter Carreau model predicts a viscosity that remains finite as the shear rate approaches zero and is given by

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = [1 + (\delta\dot{\gamma})^2]^{\frac{n-1}{2}}. \quad (1)$$

Weinstein [2] and Rousset et al. [3] have considered the Carreau model and have examined the temporal stability of a film flow down an impermeable rigid inclined substrate. The authors

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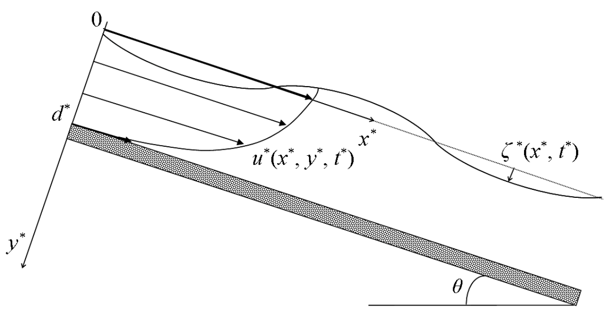

FIGURE 1. DEFINITION SKETCH.

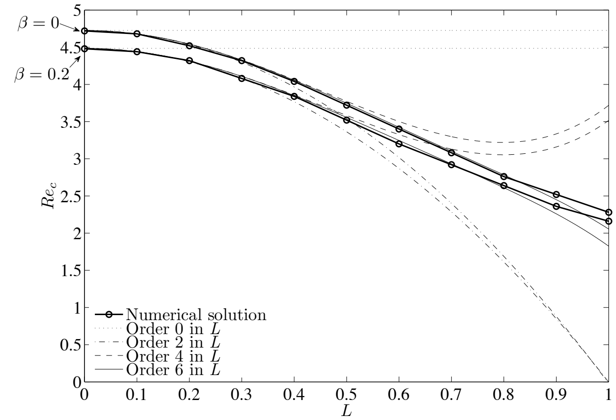

FIGURE 2. INFLUENCE OF PERMEABILITY PARAMETER β ON CRITICAL REYNOLDS NUMBER ($\theta = 10°$, $n = 0.5$, $\frac{\eta_\infty}{\eta_0} = 10^{-3}$)

show that a shear-thinning Carreau fluid film on a rigid impermeable substrate is more unstable than a Newtonian film. This calls for an analysis that includes both the effects of Carreau non-Newtonian rheology and bottom permeability and the present study reports such an investigation of a Carreau non-Newtonian film on a porous inclined substrate.

Mathematical formulation and results

The two-dimensional laminar flow of a Carreau fluid layer along the surface of a porous medium inclined at an angle θ with the horizontal is considered as displayed in Fig. 1. The porous medium is assumed to be saturated with the same fluid and the flow through it is governed by modified Darcy's law. The boundary conditions at the interface of the fluid layer and the porous medium are:

$$\frac{\partial u}{\partial y} = -\frac{\hat{\alpha}}{\sqrt{K}}u, \tag{2}$$

$$v = 0. \tag{3}$$

The linear stability of the flat film solution with regards to small disturbances is examined through a two-dimensional normal-mode analysis. A spectral-Tau collocation method based on Chebyshev polynomials is used for the discretization of the generalized eigenvalue problem. Moreover, an asymptotic solution of the generalized Orr-Sommerfeld equation is obtained for very long waves and for a weakly non-Newotonian behavior (i.e. for shear-thinning parameter L in the vicinity of 0).

Numerical and asymptotic results up to order 6 in L for very long waves are shown on Fig. 2. The evolution of the critical Reynolds number is displayed for two different values of the permeability parameter β as a function of the shear-thinning parameter L. As shown by Rousset et al. [3], shear-thinning properties cause flow destabilization. Moreover the critical Reynolds number is lower when the flow takes place on a porous medium: there is also a destabilizing effect due to the porosity of the wall. For a shear-thinning fluid flowing on a porous substrate, both phenomena combine to destabilize the flow. The numerical investigation shows that the same conclusions hold at arbitrary wavelength. We also note that the asymptotic development converge to the numerical solution, showing the validity of numerical calculations.

An energy balance shows that the destabilization induced by shear-thinning and porosity are both due to an increase in the perturbation shear stress work at the free surface. More precisely, the flow destabilization induced by a porous substrate as well as the flow destabilization induced by a shear-thinning fluid is due to an increase in longitudinal perturbation velocity magnitude at the free surface.

Conclusion

It has been shown both numerically and analytically that there is a destabilizing effect due to the porosity of the wall. Moreover in the case of a non-Newtonian fluid the destabilization is due to both porosity and shear-thinning. Through an energy balance performed on the perturbation fields, it has been demonstrated that the terms involved in the destabilization are linked to the viscous shear work rate on the free surface.

REFERENCES

[1] J. P. Pascal, *J. Non-Newton. Fluid Mech.* **133**, 109 (2006).
[2] S. J. Weinstein, *AIChE* **36**, 1873 (1990).
[3] F. Rousset, S. Millet, V. Botton and H. BenHadid, *J. Fluids Eng.* **129**, 913 (2007).