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A SEMI-EMPIRICAL METHOD TO DETERMINE THE SPEED OF SOUND AND ATTENUATION FACTOR IN A CONFINED FLUID

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ABSTRACT

A simple semi-empirical method is presented for calculating the speed of sound and attenuation factor for an unknown medium enclosed in a pipe. This method is based on the calculation of four-pole matrix parameters. It can be used in connection with transient or steady state excitation. The material presented is limited by the fact that no ideal measurement process exists. This limitation is significant when gathering attenuation factor and speed of sound data over a wide frequency range. However, the results, which are easily obtainable, are more than sufficient in solving daily engineering problems.

INTRODUCTION

Any linear elastic [8], here an acoustic system which has a single input point and a single output point, can be described by a pair of simple linear equations:

$$\begin{aligned} P_{in} &= A \cdot P_{out} + B \cdot V_{out} \\ V_{in} &= C \cdot P_{out} + D \cdot V_{out} \end{aligned} \qquad \textbf{(1)}$$

where**Ps** are the dynamic pressures and **Vs** are the dynamic volume velocities. Coefficients **A**, **B**,**C** and **D** are the four-pole parameters of the system. The advantage of such a representation is that the four-pole parameters permit considerable simplification of the algebraic complexities involved in the analysis of acoustic systems which can be modeled as a series of discrete regions/elements. Hence, such piping elements as series-connected, parallel-connected, loops, volumes, dead legs and other piping elements are represented by a set of coefficients which depend only on the properties of the region/element. In addition, this set of coefficients relates the dynamicpressure and the dynamic volume velocity at each inlet of the region/element to those quantities ateach exit of the region/element. To represent a complex system by this method all the region/elements have to be 'grouped' into an overall region/element. This is usually done as follows:

Inspecting the four-pole equations (1) allows us to write the four-pole equations in matrix form:

$$\begin{bmatrix} P \\ V \end{bmatrix}_{in=port1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} P \\ V \end{bmatrix}_{out=port2} \qquad \textbf{(2)}$$

The above square matrix (2) is called the transfer matrix. Coupling **n** two-ports of one matrix in a way that the output of it is an input of the next matrix (cascade connection) easily creates the equations that relate the pressure and flow variables of the first and the last region/element in a complex system [2, 3, 4, 6]. A general form of overall matrix is as follows:

$$\begin{bmatrix} P \\ V \end{bmatrix}_1 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{1,n} \begin{bmatrix} P \\ V \end{bmatrix}_n = \begin{bmatrix} P \\ V \end{bmatrix}_1 = [M_1] \cdot [M_2] \cdot [M_{i-1}] \cdot \begin{bmatrix} P \\ V \end{bmatrix}_{i-1} = [M_{OVR}] \cdot \begin{bmatrix} P \\ V \end{bmatrix}_n \qquad \textbf{(3)}$$

Assuming that the analyzed system obeys the reciprocity theorem [7, 9], i.e. four-pole parameters are related by the following equation:

$$A\,D - C\,B = 1 \qquad \textbf{(4)}$$

we could observe that it takes only three of these quantities to completely describe the behavior of any two-port station, regardless of the internal complexity of any single region/element or an overall matrix. It is possible to calculate the **As**from the set of equations that describe the two-port system. In this work, we are primarily interested in direct measurement of these parameters.

RATIONALE FOR DEVELOPMENT OF THE METHOD

The oil and gas industry deals with an increasing number of chemical compounds. Theoretical evaluation of accurate thermodynamic properties for these substances—for example, speed of sound or acoustic damping—is expensive, time consuming and not always convenient.

This method allows determination of the speed of sound and the acoustic damping in an enclosed system, for example in piping. Acoustic damping models have been extensively studied in the past [1, 5]. The equations developed, in many cases, stilldo not give factual values. Therefore, it is sometimes necessary to determine or verify such information in the field.

THEORY

The following method determines both the speed of sound and acoustic damping in an enclosed acoustic element (e.g. piping). This method is based on four-pole parameters given for a fluid system. These are derived from the Navier Stoke's equation, with the assumption that mean flow is equal to zero (i.e. reciprocity theorem holds) [7]. For cylindrical pipe, the derived parameters in the transfer matrix form are given as follows:

$$\begin{bmatrix} P \\ V \end{bmatrix}_{x=0} = \begin{bmatrix} \cosh(\gamma L) & Z\sinh(\gamma L) \\ \frac{1}{Z}\sinh(\gamma L) & \cosh(\gamma L) \end{bmatrix} \cdot \begin{bmatrix} P \\ V \end{bmatrix}_{x=L} \quad \textbf{(5)}$$

where**Z = Zo/S** is called the acoustic impedance and γ is function f(**α, k**) where **α** is a function f**(f, ρ, c, d)**.

The speed of sound in a one-phase, bubble or condensation free fluid enclosed within a pipe can be calculated by using a simple testing approach and applying the theory of plane-wave acoustics. Consider the apparatus illustrated in Figure 1. Its dynamic behavior can be described in mathematical terms by relating the complex Fourier transformed variables, **P** and **V**, at the apparatus input and output stations, 1 and 2. The relationships among variables at station 1 and 2 are as follows:

$$\begin{bmatrix} P \\ V \end{bmatrix}_{1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{1,2} \cdot \begin{bmatrix} P \\ V \end{bmatrix}_{2} \quad \textbf{(6)}$$

From equation (6) the pressure is obtained by matrix calculation as follows:

$$P_1 = P_2 \cdot \left(A_{1,2} + \frac{V_2}{P_2} \cdot B_{1,2} \right) \quad \textbf{(7)}$$

Since there is a rigid end at station 2, there is no flow and **V2** is equal to **0**. Consequently, equation (7) reduces to:

$$\frac{P_1}{P_2} = A_{1,2} \quad \textbf{(8)}$$

Using equations (5) and (8) the matrix parameter **A12** is derived as follows:

$$A_{1,2} = \cosh(\gamma L) = \cosh(\alpha L + ikL) \quad \textbf{(9)}$$

Recalling that there is no mean flow in the system and α << 1, equation (9) becomes:

$$A_{1,2} = \cosh(ikL) + \alpha L \sin(kL) \quad \textbf{(10)}$$

Using definitions of hyperbolic functions, equation (10) reduces to:

$$A_{1,2} = \cos(kL) + i\alpha L \cdot \sin(kL) \quad \textbf{(11)}$$

Equation (11) constitutes a base to measure and record the **A12** transfer matrix parameter. This can be done by using a duel channel analyzer. The pressure spectra and their phases or matrix parameter **A12** can be easily collected and subsequently recorded. The data is in complex number form and can be written as follows:

$$\begin{matrix} |P_1|, \theta_1 \\ |P_2|, \theta_2 \end{matrix} \quad \text{for f =f 1} \qquad \text{or} \quad \frac{|P_1|}{|P_2|} = e^{i(\theta_1 - \theta_2)} \quad \textbf{(12)}$$

Consequently,

$$A_{1,2} = |A_{1,2}| e^{i\beta}, \text{ where } \beta = (\theta 1 - \theta 2) \quad \textbf{(13)}$$

$$\text{Re (A12)} = \cos(kL) \quad \textbf{(14)}$$

$$\text{Im (A12)} = \alpha L \sin(kL) \quad \textbf{(15)}$$

In summary, the speed of sound is directly determined from equation (14) and the dynamic damping factor is found by solving equation (15).

PROCEDURE

To take accurate spectra or transfer function measurements, several conditions have to be fulfilled. The most important is to minimize the fluid-pipe interaction. This can be accomplished by using an experimental setup in which mechanical and acoustical system resonances are separated. To achieve this, the apparatus has to be made 'rigid'. The best way of stiffening it is to put several sand bags under and on top of the rig. Next, the pressure transducer sensitivities have to be carefully chosen so that the signal will be strong enough and at the same time ensure that the coherence function is as close as possible to 1 in the frequency band of interest. The choice of pressure transducer depends of the static pressure of the medium and the strength of the excitation signal. Finally, the boundary conditions are important. The procedure can be divided into two distinct parts: (i) the equipment test, (ii) the actual test. The first part is a basic preparation for the test in which the apparatus is calibrated. The second one describes the actual test and data collection procedure. Both are described in detail in the appendix.

DISCUSSION

There are several possible sources of error while using this method. First, error can be introduced by unreliable measurement. This deficiency is easily detected by the coherence function and erroneous data can be immediately eliminated while testing the system. Another source of error can be generated by inaccurate length measurements of the rig, or other rig dimensions. This error can be minimized since there is broad access to highly accurate dimensional measurement tools. The relative error due to erroneous geometry can be limited to 0.1%. Other errors can be generated by inaccurate determination of the resonant frequency. As seen in Figure 2, the cosine function is distorted close to the resonant frequencies due to acoustic damping. For higher frequencies, the acoustic damping is higher, and so is the distortion. Thus, for high frequencies, the resonant peaks are obscured by relatively large damping. On the other hand, for low frequencies, the equipment's relative accuracy is not as good, e.g. if the analyzer is set up for 1 Hz measurement accuracy, there is 2% error at 100Hz and 0.4% error at 500Hz, etc. Hence, the frequency bandwidth in which one wants to determine the accurate resonant frequency has to be carefully chosen. From the above, it can be concluded that the speed of sound and damping factor values can be measured to within 1% error by using the described method.

The additional advantage of this method is the possibility of immediate data verification. There are two ways of realizing it. The first is based on frequency domain calculations, which gives a high margin of confidence for collected data, and the second is a simple measurement of the time which elapses when a wave travels over a known distance (transducer locations) inside a pipe.

From equation (8), one can see that the acoustic resonance occurs when **P2** becomes large compared to **P1**. Setting **A12** equal to zero gives the frequency equation that can be solved for **k.** Since the dominant term in equation (11) is a cosine term, the solution of this equation will have a form as in Figure 2. Note that for **α** equal to zero, matrix parameter **A12** is equal to cos(kL). In this manner, a quick evaluation of the speed of sound can be done.

The second estimation of the speed of sound is time-domain based, and should give results very close to the actual wave propagation speed. However, in this test, the absorption coefficient cannot be determined. Moreover, in this method, the dependency of the speed of sound on frequency is not easily determinable.

NOMENCLATURE

A, B, C, D: Elements A11, A12, A21, A22 of a 2x2 transfer matrix

c: Speed of sound
d: Internal diameter
f: Frequency
i: Integer or $i = \sqrt{-1}$
k: Wave number
L: Length of the test element
P: Laplace-transformed pressure
S: Cross-section of pipe ID
V: Laplace-transformed volume velocity
Z: Acoustic impedance
Zo: Acoustic characteristic impedance
α: Acoustic attenuation factor
γ: Propagation constant
ρ: Density of the fluid

Since a convenient method to include the dissipation of acoustic energy is to express some of the above parameters as complex quantities, we re-defined the following parameters:

$\bar{k} = k - i\alpha$: Complex wave-length

$\bar{c} = \dfrac{\varpi}{\bar{k}}$: Complex speed of sound

$\bar{\gamma} = \alpha + ik$: Complex propagation constant

$\bar{Z} = \dfrac{\rho \cdot \bar{c}}{S}$: Complex impedance

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FIGURES

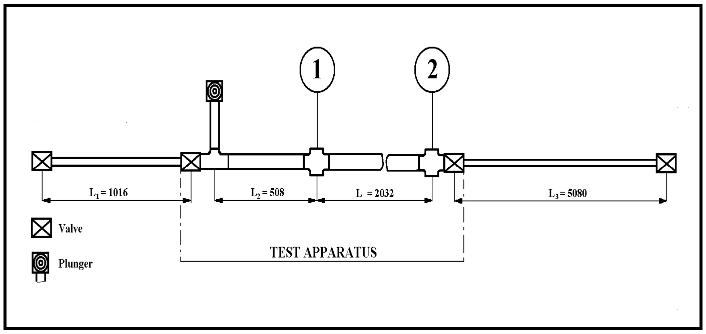

Figure 1: Test Apparatus with Marked Test Points (1) and (2)

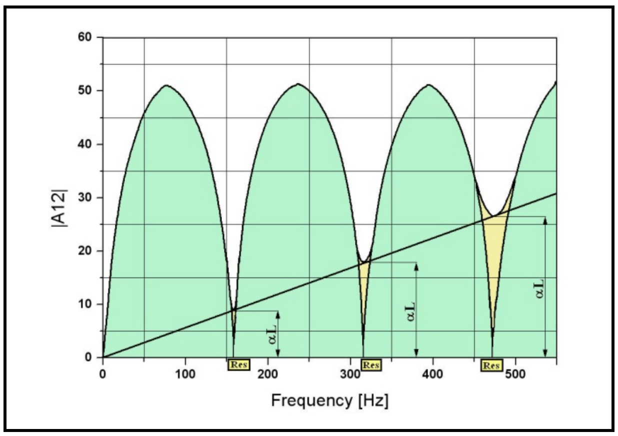

Figure 2: Four - Pole Parameter A12

APPENDIX

A1 Assumptions

1. Sound propagation is in the form of plane waves
2. Non-viscous flow through a constant area
3. A homogenous medium; no temperature gradient or humidity change through the system
4. Gravitational forces within the medium are neglected
5. Changes in density of the medium are small
6. The sound pressures are small in comparison with the average equilibrium pressure in the system

A2 Apparatus

The actual test apparatus is shown in Figure A1. It consists of two sections of NPS1/2 Sch. XS pipe, one NPS ½ x ¼ reducing tee, two cross fittings, and three block (isolation) valves. The plunger is connected to the test apparatus by a high pressure hose. The additional flexible steel tubing and block valves are necessary to fill and empty the test apparatus. The necessary equipment includes the spectrum dynamic analyzer, oscilloscope, two piezoelectric pressure transducers, hammer with force or accelerometer transducer and three power supplies.

All parts of the apparatus need to be assembled first as shown in Figure A1. Next, the equipment and the actual tests can be carried out.

First, both pressure transducers should be placed in each of the cross fittings (1) and (2) at the assembled test apparatus, refer to Figure 1. Special attention should be paid so that the transducers are flash mounted with the tees walls as best as practical. All valves should be open and the system should be flushed with the fluid to be tested. It is critical that all air bubbles be removed from the system. Then, the outlet block valve (1) at RHS, see Figure A1, needs to be closed and the system pressurized. When the system is filled up and still connected to the reservoir, slowly open/crack valve (1) to evacuate any air bubbles from the apparatus. One can lift slightly up an end of the entire rig to improve the bubble evacuation process. While doing so, be warned that an abrupt opening could cause extensive cooling, and inaccurate measurement will follow. In addition, freezing at the valve could block the system, making its filling or flushing impossible. After bleeding air from the apparatus, close all block valves from the right to the left, see Figure A1. Set up the spectrum analyzer and strike the plunger with a triggering hammer. For each test, plot two pressure spectra, their phases, the coherence function, as well as the transfer function with its phase. Ensure that the transfer function is approximately equal to unity and its phase is equal to zero when the coherence function is equal to one. If this is not the case, check circuits and possibly exchange transducers and repeat the test.

Prior to the actual test, depressurize the system and install the transducers in the upstream and the downstream cross fittings. Repeat flushing and pressurize the system as before. When the probability of air presence in the system is low, prepare to collect the data. Strike the plunger with a triggering hammer and adjust the amplifiers to the appropriate levels of excitation. Repeat the striking and collect the data. Repeat the flushing and the test itself. The collected results should be very similar to the initial results. If it not, there is likely air still left in the system. Repeat the entire procedure. Always make sure that there are no leaking valves or faulty connections in the system.

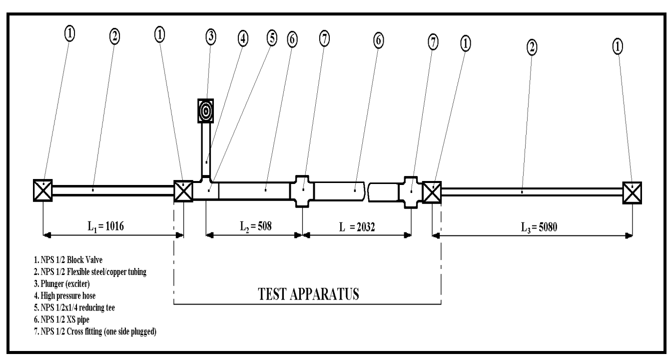

Figure A1: Detailed Design Information about Test Apparatus