Proceedings of the ASME 2010 3rd Joint US-European Fluids Engineering Summer Meeting and 8th International Conference on Nanochannels, Microchannels, and Minichannels
FEDSM-ICNMM2010
August 1-5, 2010, Montreal, Canada

FEDSM-ICNMM2010-30404

APPLICATION OF A ZONAL $K-\varepsilon$ MODEL ON SIMULATION OF TURBULENT MIXED CONVECTION

Mehdi Shahraeeni*
School of Engineering
University of British Columbia Okanagan
Kelowna, BC V1V 1V7
Email: mehdi.shahraeeni@ubc.ca

Mehrdad Raisee
School of Mechanical Engineering,
College of Engineering,
University of Tehran,
P.O.Box: 11155-4563,
Tehran, IRAN
Email:mraisee@ut.ac.ir

ABSTRACT

Fluid flow and heat transfer in a vertical tube under constant heat flux have been studied and the effect of buoyancy force on the heat transfer coefficient is investigated. The finite volume method is used to study turbulent flow in both upward and downward directions. For the turbulence modeling, a zonal k-e model is employed and the numerical results are compared with available experimental data. The results of the simulation show that for the downward flow, heat transfer is enhanced and for strong buoyancy force, flow reversal is observed. In contrast, for the heated upward flow, heat transfer can be either impaired or enhanced by the buoyancy force depending on its strength. Partial laminarization is caused by the buoyancy in the case of modest buoyancy force. For the condition of stronger buoyancy force, a sudden decrease in the fully-developed Nusselt number is evident in the experimental data and well predicted by the numerical solution. In general, the quantitative agreement between the numerical results and the experimental data is satisfactory.

INTRODUCTION

Extensive practical applications of mixed convection including the cooling of nuclear reactors and electronic components, internal cooling system of turbine blades and compact heat exchangers caused a huge attention to be directed on the subject. Although the geometry involved in most applications are relatively simple, predicting heat transfer phenomena of such flows is very complex. The complexities are associated with the behavior of fluid flow in the near-wall region. In the case of laminar mixed convection, the near-wall velocity is increased in buoyancy-aided flows and decreased in buoyancy-opposed flows. Thus, heat transfer is enhanced and impaired respectively. In contrast for turbulent flow, the interaction between the velocity field and the rate of turbulence production in the near-wall region determines the impairment or enhancement of the rate of heat transfer. While the buoyancy force reduces advection for downward flow, the higher level of turbulence production in the near-wall region always improves the wall heat transfer. In the buoyancy-aided case, advection in the near-wall region is increased. However, the turbulence production is reduced due to the decreased level of shear stress in the same region. The net result is impairment of the wall heat transfer. A complete condition of laminarization is achieved when the shear stress in the near-wall region falls as a result of increased buoyancy force. Therefore, any further increase of buoyancy force raises the rate of turbulence production and results in heat transfer enhancement.

Several studies have been conducted on the implementation of various turbulence closures to take into account mixed convection phenomena. A comprehensive review of these models up to the ninetieth is presented by Jackson et al. [1]. Recently, Kenjeres et al. [2] proposed a quasi-linear model for turbulent heat

*Address all correspondence to this author.

flux. They considered a buoyancy-extended stress-strain model coupled with five equations ($k-\varepsilon-v^2-f-\theta^2$) and applied it to a wide range of natural and mixed convection flows. The results were in fairly good agreement with experimental data. Direct numerical simulation (DNS) has also been used to study mixed convection in the vertical channels. One of the earliest studies was conducted by Kasagi and Nishimura [3]. They primarily fixed the Reynolds number (based on the channel half-width and the friction velocity) at 150, while the Grashof number varies from 0 to 1.6×10^6. They found that the opposing and aiding buoyancy forces affect the turbulent statistics and the quasi-coherent structures in much the same way as the wall injection/suction and the Lorenz force. As a result the near-wall force balance is modified causing heat transfer enhancement or impairment. You et al. [4] studied turbulent mixed convection in a heated vertical tube using DNS. The fluid properties were assumed to be uniform and Boussinesq's approximation was used. They confirmed the validity of the log laws of the mean velocity and the temperature profiles for downward heated flow. The same research group in Bae et al. [5] conducted a DNS study of mixed convection heat transfer of carbon dioxide at supercritical pressure. They reported deformation of the mean velocity profile into an M-shaped one for upward flow, and the variation of velocity fluctuation both in sign and magnitude causing impairment of heat transfer. Kim et al. [6] presented an assessment of the performance of a variety of turbulence models in simulating buoyancy-aided turbulent mixed convection in vertical tubes. They compared the prediction of RANS-based models with available DNS results. They showed that indirect influence of the buoyancy force on the turbulence in a heated vertical tube is the dominant mechanism which causes laminarization and deterioration of heat transfer.

In the present study, the zonal modeling approach is employed to investigate the influence of the buoyancy force on the fluid flow and heat transfer. The zonal $k-\varepsilon$ model is applied for both aiding and opposing turbulent flows inside a vertical tube with constant heat flux. This modeling approach allows the resolution of the mean motion across the sub-layer region without the need for very fine grid resolutions associated with low-Reynolds number models, in which the dissipation rate equation is integrated up to the wall [7]. Therefore, the model is computationally more economical. Moreover, the use of the zonal $k-\varepsilon$ model for the prediction of mixed convection in vertical tubes has not been previously investigated.

FLOW GEOMETRY

In order to validate the numerical results presented in this paper, the experimental results of Li and Jackson [8] are employed. Figure 1(a) shows the general arrangement of the test configuration for the upward flow with an unheated development section. A blower delivers laboratory air through a flexible duct to the entry box with a honeycomb arrangement inside to straighten the

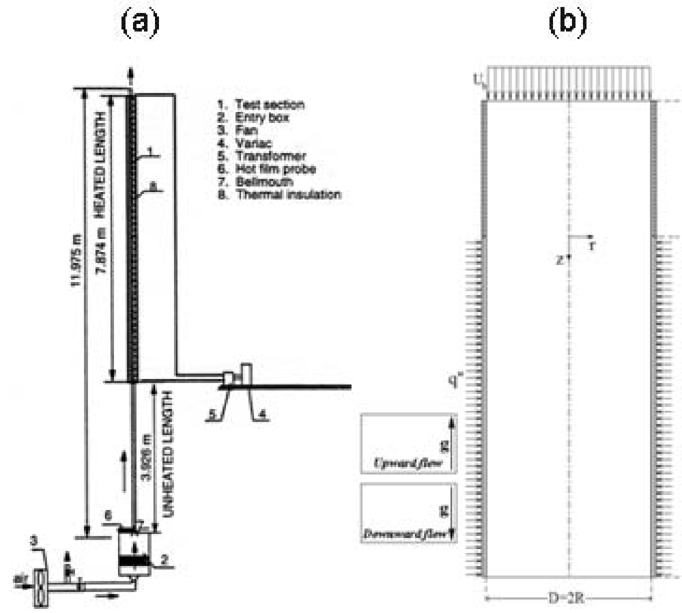

FIGURE 1. (a) GENERAL ARRANGEMENT OF Li AND Jackson (1999) EXPERIMENT, (b) GEOMETRY MODEL.

flow. A long unheated development section makes the flow to be fully-developed at the inlet of the heated section. After passing the heated section, air is finally exhausted through a flow metering nozzle. Full details of the experiment and its setup can be found in Li and Jackson [8]. The schematic of 2D axi-symmetric flow geometry considered for analysis of upward and downward flows is shown in Figure 1(b). The only difference between the two flows is the sign of the gravitational force which is in the same direction of the main flow for downward case, while it opposes the main flow for upward case.

NUMERICAL METHOD AND BOUNDARY CONDITIONS

The general form of governing equations in the polar-cylindrical coordination system for a dependent variable is written as:

$$\frac{\partial}{\partial z}(\rho r_c U\Psi)+\frac{\partial}{\partial z}(\rho r_c V\Psi)=\frac{\partial}{\partial z}(\Gamma^{\Psi} r_c\frac{\partial\Psi}{\partial z})+\frac{\partial}{\partial r}(\Gamma^{\Psi} r_c\frac{\partial\Psi}{\partial r})+r_c S^{\Psi} \quad (1)$$

where Ψ represents velocity components, temperature and turbulent quantities (k and ε), z and r are the coordinates in the axial and radial directions respectively. Γ^{Ψ} is the effective diffusion coefficient and S^{Ψ} denotes the source term in each transport equation. For the momentum equation, S^{Ψ} includes the pressure gradient as well as the buoyancy force. To take into account the effect of buoyancy, the pressure source term in the axial momentum equation is modified as follows:

$$S^P = -\frac{\partial P}{\partial z} - \rho g_e \beta (T - T_{ref}) \tag{2}$$

in which, $\partial P/\partial z$ is the pressure gradient and $\rho g_e \beta (T - T_{ref})$ is the force due to buoyancy. For the upward flow, where the buoyancy force is aiding the main flow, $g = -g_e$, while for the downward flow, $g = +g_e$.

TURBULENCE MODELING

In the zonal $k-\varepsilon$ model, the computational domain is divided into two parts: the fully-turbulent region and the low-Re near-wall region. In the fully-turbulent region, the standard high-Re version of $k-\varepsilon$ model is used, while in the near-wall region a low-Re version of one-equation model is employed. This approach allows the resolution of mean flow across the viscous sublayer without the need to use a very fine near-wall grid [7]. The Reynolds stresses $(\rho\overline{u_i u_j})$ and the turbulent heat fluxes $(\rho C_p \overline{u_i t})$ are obtained via the well-known effective viscosity and effective diffusivity approximations. Full details of this approach is presented in [9].

RESULTS AND DISCUSSION

Buoyancy parameter (Bo) is used to characterize the strength of buoyancy force in the flows under study. Two different cases of low and high buoyancy-influenced flows are studied in this paper.

LOW BUOYANCY FORCE

For the Reynolds number of $Re = 15023$ and Grashof number of $Gr = 2.163 \times 10^8$, the buoyancy parameter is $Bo = 0.1124$. The buoyancy parameter, as discussed earlier, represents the effect of buoyancy force in the flow with respect to the other existing forces such as inertia and viscosity. Thus, the effect of buoyancy force for this case should not be significant. However, the difference between upward and downward flows should be appreciated.

The computed velocity profiles normalized with the bulk fluid velocity (U_b) at various stations along the tube are presented in Figure 2. At the inlet of the tube, due to negligible effect of the buoyancy force, the corresponding profiles are almost identical for both directions. Further downstream as the near-wall fluid warms up, the buoyancy force accelerates the near-wall fluid in upward flow. In contrast, for downward flow, the buoyancy force acts in the opposite direction of the main stream and slows down the near-wall fluid. As flow approaches the end of the tube ($z/D = 60$), differences in the velocity profiles become more visible and finally both flows reach to the fully-developed condition.

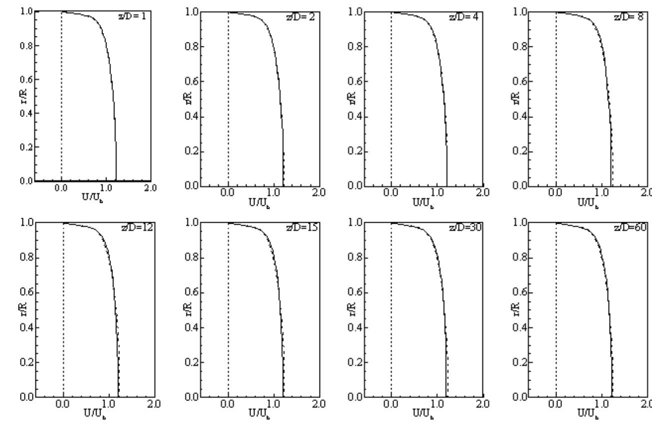

FIGURE 2. NORMALIZED VELOCITY PROFILES FOR UPWARD (SOLID LINE) AND DOWNWARD (DASHED LINE) FLOWS AT DIFFERENT CROSS SECTIONS ($Bo = 0.1124$).

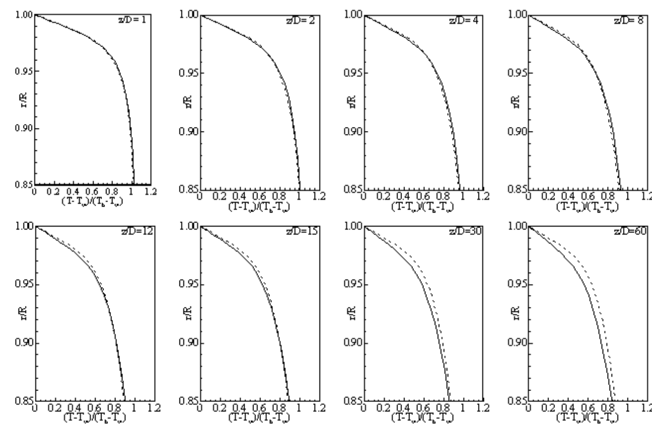

FIGURE 3. NORMALIZED TEMPERATURE PROFILES FOR UPWARD (SOLID LINE) AND DOWNWARD (DASHED LINE) FLOWS AT DIFFERENT CROSS SECTIONS ($Bo = 0.1124$) .

Although one would expect higher heat transfer rates for upward flow compared to the downward flow due to higher near-wall velocity, the non-dimensionalized temperature profiles, shown in Figure 3, exhibit exactly opposite behavior. Similar to the velocity profiles, the temperature profiles are identical at the entrance region. Further downstream, the difference between the profiles increases. The normalized temperature profiles for the downward flow change marginally while for the upward flow at similar sections, they vary significantly. The variations in temperature profiles explain the distribution of Nusselt numbers.

To explain the differences in the levels of heat transfer for the upward and downward flows, the near-wall turbulent kinetic energy profiles are presented in Figure 4. The square-root of turbulent kinetic energy is normalized using the inlet velocity in

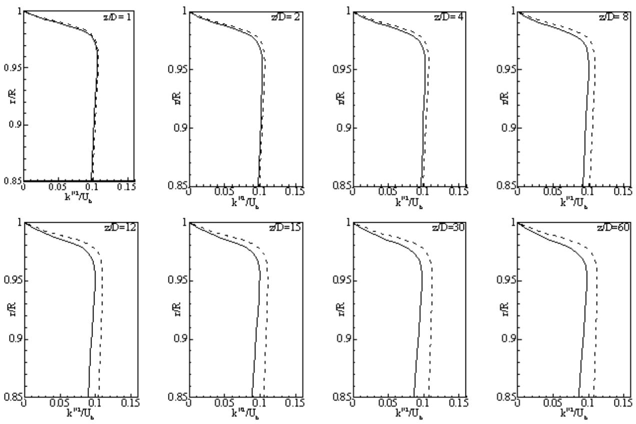

FIGURE 4. NORMALIZED TURBULENT KINETIC ENERGY PROFILES FOR UPWARD (SOLID LINE) AND DOWNWARD (DASHED LINE) FLOWS AT DIFFERENT CROSS SECTIONS ($Bo = 0.1124$) .

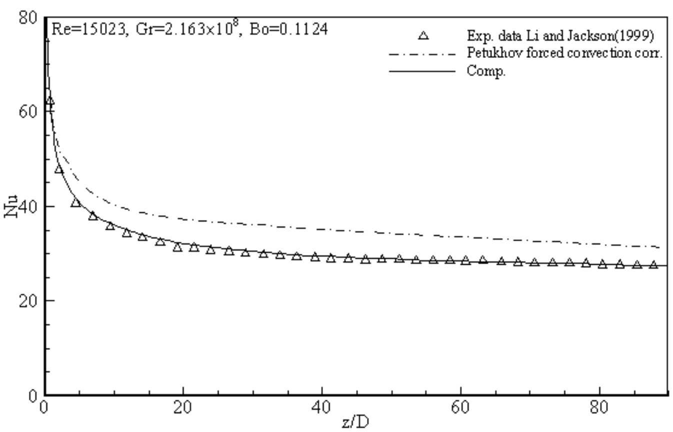

FIGURE 5. COMPARISON OF PREDICTED NUSSELT NUMBER FROM THE CURRENT MODEL WITH EXPERIMENTAL DATA FOR UPWARD FLOW.

this figure. Similar to what has been found for the velocity and temperature profiles, the profiles of turbulent kinetic energy are almost identical for both directions in the entry region. However, as the upward flow develops, the turbulent kinetic energy slightly decreases. As a result, the turbulent diffusion is impaired for the corresponding stations and consequently the Nusselt number decreases (Figure 5).

In contrast for downward flow, the buoyancy force opposes the main stream flow in the regions close to the wall. Eventually, the turbulent kinetic energy increases as the buoyancy effect established (Figure 6). Thus, the rate of heat transfer is enhanced compared to the forced convection rate as can be seen in Figure 10. While there are some discrepancies between the predicted

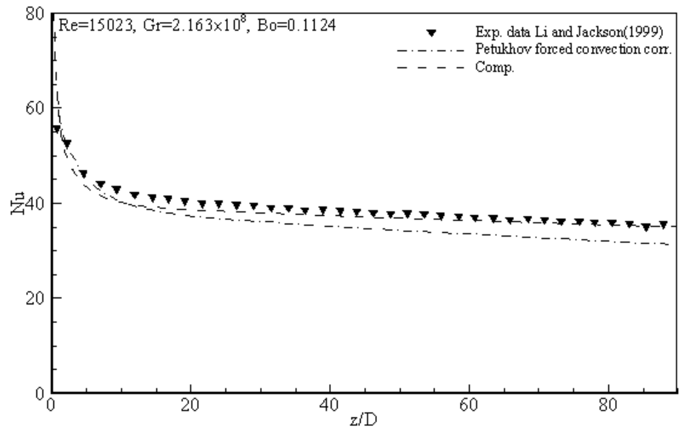

FIGURE 6. COMPARISON OF PREDICTED NUSSELT NUMBER FROM THE CURRENT MODEL WITH EXPERIMENTAL DATA FOR DOWNWARD FLOW.

local Nusselt numbers of the zonal $k-\varepsilon$ model and the experimental data, the predictions are in acceptable agreement with the data of Li and Jackson(1999).

HIGH BUOYANCY FORCE

The buoyancy force in this case is such strong that in the near-wall region for both upward and downward flows, it becomes the dominant force. The flow parameters are $Re = 3049$, $Gr = 1.039 \times 10^8$ making the buoyancy parameter $Bo = 12.7079$. For the upward flow, the buoyancy force acts in the flow direction. Thus, the velocity of the near-wall fluid increases slightly as the fluid warms up. The velocity profile obtains its maximum inflated shape at $z/D \sim 15$. Afterwards, the profile is getting smoother due to the development of thermal boundary layer as well as the propagation of buoyancy effect in the radial direction. However, the velocity maintains its M-shape profile through the end of the tube unlike the downward flow (see Figure 7, $z/D = 30$ and 60).

For the downward flow, on the other hand, as the near-wall fluid warms up, it is decelerated by the buoyancy force and the flow reversal occurs. The reversed flow develops at the first few stations of the heating zone according to the velocity profiles shown in Figure 7 ($z/D = 1, 2$ and 4). Further downstream, due to the increase in the average fluid temperature, flow is rapidly recovered. Consequently, the velocity profiles at the last four sections ($z/D = 12, 15, 30$ and 60) do not exhibit any significant changes and the flow obtains fully-developed condition.

The corresponding temperature profiles for upward and downward flows are compared in Figure 8. For the upward flow, as thermal boundary layer grows, heat transfer between the wall and the adjacent fluid is impaired. Further downstream, as the effect of buoyancy appears, the fluid accelerates in the near-wall

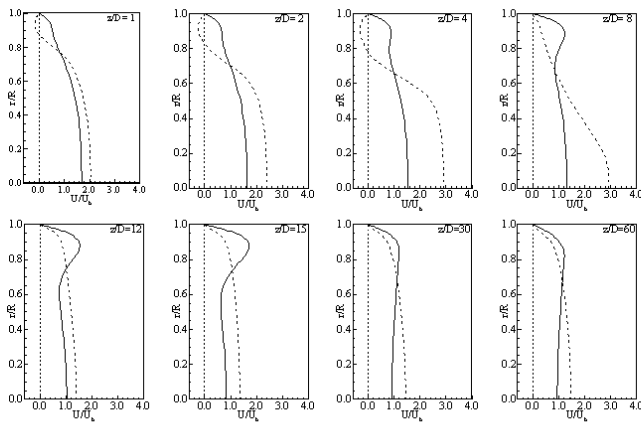

FIGURE 7. NORMALIZED VELOCITY PROFILES FOR UPWARD (SOLID LINE) AND DOWNWARD (DASHED LINE) FLOWS AT DIFFERENT CROSS SECTIONS ($Bo = 12.7079$).

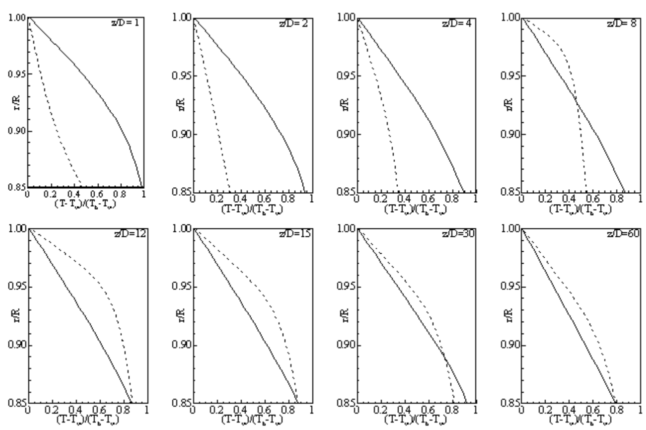

FIGURE 8. NORMALIZED TEMPERATURE PROFILES FOR UPWARD (SOLID LINE) AND DOWNWARD (DASHED LINE) FLOWS AT DIFFERENT CROSS SECTIONS ($Bo = 12.7079$).

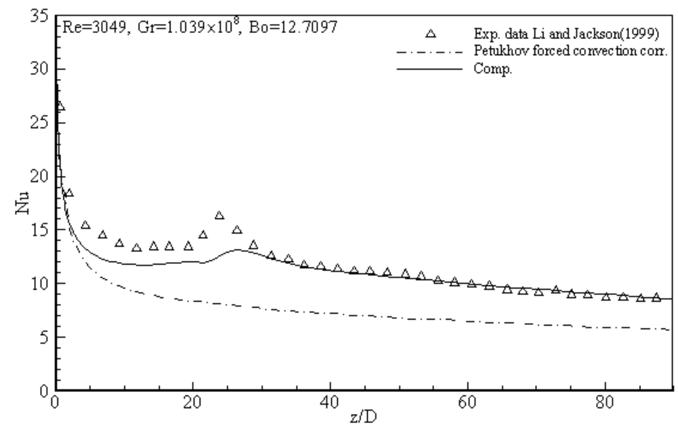

FIGURE 9. COMPARISON OF PREDICTED NUSSELT NUMBER FROM THE CURRENT MODEL WITH EXPERIMENTAL DATA FOR UPWARD FLOW.

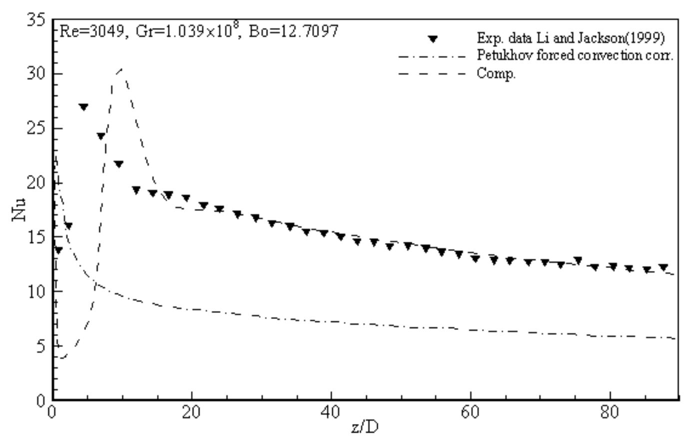

FIGURE 10. COMPARISON OF PREDICTED NUSSELT NUMBER FROM THE CURRENT MODEL WITH EXPERIMENTAL DATA FOR DOWNWARD FLOW.

region as discussed earlier. The inflation in the velocity profile strengthens the convection mechanism leading to enhancement of heat transfer which is observed in the distribution of Nusselt number at $z/D \sim 15$ (Figure 9). The rate of heat transfer slightly decreases as the flow develops. As opposed to the previous test case ($Bo = 0.1124$), the levels of Nusselt number are higher than that of pure forced convection confirming that heat transfer can be either impaired by buoyancy or enhanced. This is the established picture of buoyancy-influenced heat transfer in vertical tubes for the buoyancy-aided case.

Figure 10 represents the distribution of Nusselt number for downward flow. At the onset of heating zone, non-uniformity in the distribution is evident. This unusual behavior is due to the effects of thermal boundary layer and strong buoyancy force acting in combination. The motion of fluid in the near-wall region is retarded by the buoyancy force at the beginning of the heating zone where such influences are significant. This decreases the convection but also leads to an increase in the production of turbulence. As Figure 11 shows the turbulent kinetic energy grows rapidly at the first few stations ($z/D = 2, 4$ and 8). The net effect is that the thermal boundary layer develops more rapidly. Thus, the Nusselt number falls drastically at first and as the thermal layer grows in thickness, the production of additional turbulence due to the buoyancy force becomes considerable. Consequently, the Nusselt number increases and then stabilizes once the thermal development is completed. The value of the local Nusselt number achieves its maximum at the same location where the value of the turbulent kinetic energy is maximized (compare the

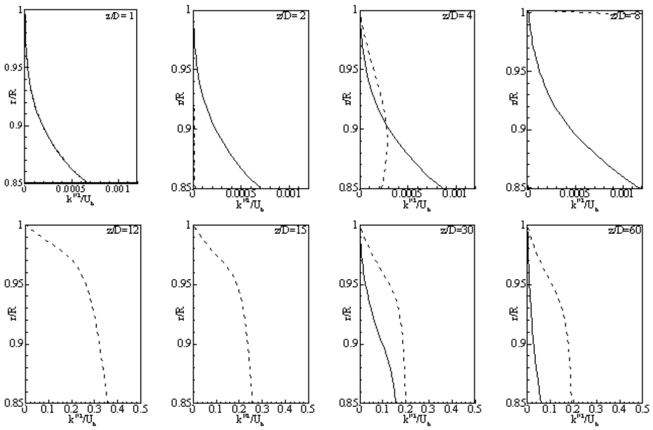

FIGURE 11. NORMALIZED TURBULENT KINETIC ENERGY PROFILES PROFILES FOR UPWARD (SOLID LINE) AND DOWNWARD (DASHED LINE) FLOWS AT DIFFERENT CROSS SECTIONS ($Bo = 12.7079$).

sections of $z/D \sim 12$ in Figures 10 and 11).

CONCLUSION

In this study, numerical computations are performed for turbulent mixed convection in vertical tubes. The flow is studied in both upward and downward directions using the zonal $k-\varepsilon$ model. In general, the numerical results are in good agreement with the experiment of Li and Jackson (1999). For downward flow, the buoyancy force acts in opposite direction of the main fluid flow and regardless of the magnitude of the force, buoyancy enhances the rate of heat transfer compared to forced convection in the same condition. Previous studies for laminar flow [10] showed that for downward flow, due to the reduction of the near wall velocity, the rate of heat transfer is impaired. In the same sense, for turbulent flow, the buoyancy force decelerates the near-wall fluid which weakens convection in this region. For the condition of strong buoyancy force, a flow reversal is also observed. However, the rate of turbulent production is increased in the same region which results in the enhancement of heat transfer.

In contrast to the downward flow, there is no general conclusion for upward flow in terms of enhancement or impairment of heat transfer due to buoyancy. While for downward flow, the buoyancy force always enhances the rate of heat transfer, depending on its magnitude, the buoyancy force may enhance or impair heat transfer.

The results show that for all buoyancy parameter the zonal $k-\varepsilon$ model is accurate for the prediction of turbulent mixed convection in vertical tubes. However, for higher values of the buoyancy parameters, the model only reproduces the Nusselt number for the fully-developed region accurately. For high values of buoyancy parameter, the model is successful to predict the occurrence of a maximum in the Nusselt number distribution, though there are some discrepancies for the location of this point compared to the experimental data.

REFERENCES

1. Jackson, J.D., Cotton, M.A., Axcell, B.P., 1989. Studies of mixed convection in vertical tubes. Int. J. Heat and Fluid Flow. 10, 2-14.
2. Kenjeres, S., Gunarjo, S.B., Hanjalic, K., 2005. Contribution to elliptic relaxation modelling of turbulent natural and mixed convection, Int. J. Heat Fluid Flow. 26, 569-586.
3. Kasagi, N., Nishimura, M., 1997. Direct numerical simulation of combined forced and natural turbulent convection in a vertical plane channel. Int. J. Heat Fluid Flow. 18, 88-99.
4. You, J., Yoo, J.Y., Choi, H., 2003. Direct numerical simulation of heated vertical air flows in fully developed turbulent mixed convection. Int. J. Heat Mass Transfer. 46, 1613-1627.
5. Bae, J.H., Yoo, J.Y., Choi, H., 2005. Direct numerical simulations of turbulent supercritical flows with heat transfer. Phys. Fluids. 17, 105104.
6. Kim, W.S., He, S., Jackson, J.D., 2008. Assessment by comparison with DNS data of turbulence models used in simulations of mixed convection. Int. J. Heat Mass Transfer. 51, 1293-1312.
7. Raisee, M. 1999. Computation of flow and heat transfer through two- and three-dimensional rib-roughened passages. Ph.D. Thesis, Department of Mechanical Engineering, UMIST.
8. Li, J., Jackson, J.D., 1999. Buoyancy-influenced variable property turbulent heat transfer to air flowing in a uniformly heated vertical tube. 2nd EF Conference on Turbulent Heat Transfer, Manchester.
9. Shahraeeni, M., Raisee, M., 2010, Investigation of turbulent mixed convection of air flow in vertical tubes using a zonal turbulence model. Int. J. Heat Fluid Flow. 31(2), 179-190.
10. Raisee, M., Shahraeeni, M., 2007. Numerical simulation of laminar mixed convection of air flow through uniformly heated vertical tubes. ASME-JSME Therm. Eng. and Summer Heat Transfer Conf.,Vancouver, BC, Canada.