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SHEAR-IMPROVED SMAGORINSKY MODELING OF TURBULENT CHANNEL FLOW USING GENERALIZED LATTICE BOLTZMANN EQUATION

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ABSTRACT

Generalized Lattice Boltzmann Equation (GLBE) was used for computation of turbulent channel flow for which Large Eddy Simulation (LES) was employed as a turbulence model. The subgrid-Scale turbulence effects were simulated through a Shear-Improved Smagorinsky Model (SISM) which is capable of predicting turbulent near wall region accurately without any wall function. Computations were done for a relatively coarse grid with shear Reynolds number of 180 in a parallelized code. Good numerical stability was observed for this computational framework. Results of mean velocity distribution across the channel showed good correspondence with Direct Numerical Simulation (DNS) data. Negligible discrepancies were observed for computed turbulent statistics between present computations and those reported from DNS. Three-dimensional instantaneous vorticity contours showed complex vortical structures appeared in such flow geometries. It is concluded that such framework is capable of predicting accurate results for turbulent channel flow without adding significant complication and computational cost to the standard Smagorinsky model.

INTRODUCTION

Lattice Boltzmann Method (LBM) has attracted much attention as a promising alternative for simulation of fluid flows with complex physics in the last two decades [1-3]. LBM is a method based on the solution of Boltzmann equation on a lattice with discrete velocity field. It was shown that basic conservation equations of fluid flow (Navier-Stockes equations) recover from Boltzmann equation [4]. Solution of Boltzmann equation provides a velocity distribution function from which macroscopic fluid properties, such as density, velocity and pressure can be obtained. Some advantages of using LBM in computing fluid flow problems as compared to CFD are: the lack of convective term in Boltzmann equation and simple pressure computation using an equation of state [2]. Moreover, the streaming-and-collision computational procedure of LBM, which is a local operation in computation, makes it an excellent candidate for parallel computing [5, 6].

Turbulent flow occurs in many engineering applications. Its computation suffers from two main restrictions: inability to solve the wide range of all scales especially at high Reynolds numbers and accurate modeling of eddies in unresolved subgrid scale. While Direct Numerical Simulation (DNS) of turbulent flow is the most accurate method for turbulent flow computations, its high computational cost made it unreachable in many situations. Large Eddy Simulation (LES) has been considered as an alternative to DNS due to its ability to compute large scales directly while modeling universal small scales with an appropriate subgrid scale (SGS) modeling. In fact, LES is an affordable means for simulation of turbulent flow as compared to DNS. An important advantage of LES is its ability to approach DNS with improving computational facilities and more accurate SGS modeling.

LES computations can be done using LBM, for which different configurations have been considered recently [5-9]. The simplest SGS model used in LES is Smagorinsky model. Fernandino et al. [7] performed a LES of free surface duct flow using LBM in which Smagorinsky sub-grid scale (SGS) model was used. Their results showed that the simple SGS model could be used as a possible tool for the simulation of free surface duct flow. In another study, Lammers et. al. [8] showed that a high resolution DNS of plane-channel turbulent flow at $\mathrm{Re}_\tau = 180$ using LBM is capable of producing statistics of the same quality as pseudo-spectral methods, at resolutions comparable to, and in fact overall better than, those of the pseudo-spectral runs.

Premnath et. al. [10] presented a framework for LES using GLBE with forcing term, for wall-bounded flows in which, the forcing term shows the effect of external forces, such as constant body forces representing pressure gradient in a periodic domain. Assessment of their method was done for two geometries: fully-developed free surface channel flow and shear-driven flow in a cubical cavity. Free surface channel flow studies were reported for Reynolds number of 183.6 based on friction velocity and channel half-width. Their results showed good agreement with DNS and experimental data.

Recently Lèvêque et. al. [11] proposed a shear-improved Smagorinsky model (SISM) in which the Smagorinsky eddy viscosity is computed from the difference between the magnitude of mean shear and that of the instantaneous resolved strain-rate tensor. Their results for LES of channel flow showed excellent agreement with DNS and dynamic Smagorinsky model. Moreover, no wall function is needed for this model and its computational cost was reported to be lower than DNS. This model was employed for the present computations of channel flow.

The present work is focused on the application of SISM [11] in LES computation of turbulent flow which is carried out through GLBE. A benchmark problem of wall-generated turbulent flow, i.e. a fully-developed turbulent channel flow at shear or friction Reynolds number of 180 was considered as a test case for evaluating the above-mentioned computational procedure. There is an extensive experiment and DNS data available for assessment and comparison of the detailed structure of turbulent statistics for this geometry [10, 12]. It is worth mentioning that all of the previous applications of SISM were reported using computational methods based on CFD. This model which is capable of revealing near-wall flow characteristics without any wall-damping function was not used in MRT LBM before. Computational results showed that this model is also applicable to a coarse grid with good numerical stability and possibility of doing parallel processing.

Generalized Lattice Boltzmann method with forcing term

Lattice Boltzmann Method computes the evolution of a distribution function of particles as they move and collide on a lattice grid. The collision process considers their relaxation to their local equilibrium values, and the streaming process describes their movement along the characteristic directions given by a discrete particle velocity space represented by a lattice [1-3]. Generalized Lattice Boltzmann Equation, GLBE, computes collision in moment space, while the streaming process is performed in the usual particle velocity space [13, 14]. GLBE with forcing term [10] which incorporates an additional forcing term represents the effect of external forces as a second-order accurate time-discretization in moment space. Using multiple relaxation times in GLBE enhances the numerical stability. GLBE with forcing term can be written in the following form [10]:

$$\begin{aligned}&\mathbf{f}(\vec{x}+\vec{e}\,\delta t, t+\delta t) = \mathbf{f}(\vec{x},t) - \\ &M^{-1}.\hat{S}.\left[\mathbf{m}-\mathbf{m}^{eq}(\rho,\vec{u})\right](\vec{x},t) + \\ &M^{-1}.(I-\frac{1}{2}\hat{S}).\mathbf{S}(\vec{x},t)\end{aligned} \quad (1)$$

where ρ and $\vec{u}$ are the macroscopic density and velocity respectively, the bold face symbols such as $\mathbf{f}$ stands for 19-component vectors, 19 is the number of discrete velocities. In this equation $\mathbf{f}$ is the 19-component vector of the discrete distribution functions, $\mathbf{m}$ and $\mathbf{m}^{eq}$ are 19-component vectors of moments and their equilibria, $\mathbf{S}$ is the 19-component vector of the source terms in moment space, M is the transformation matrix and $\hat{S}$ is the diagonal matrix of relaxation rates.

The collision and source term are expressed in moment space in this equation. Here M is an orthogonal transformation matrix with 19×19 elements, mapping velocity distribution vector $\mathbf{f}$ to moment vector $\mathbf{m}$ in the moment space. The collision matrix in velocity space, Λ, is related to $\hat{S}$ in Eq. (1) through the relation $\hat{S} = M \Lambda M^{-1}$ such that $\hat{S}$ is a diagonal matrix. The elements of M are obtained in a suitable orthogonal basis as combinations of monomials of the Cartesian components of the particle velocity $\vec{e}_\alpha$ through the standard Gram-Schmidt procedure, which are provided by d'Humieres et al. [14]. Components of moments, $\mathbf{m}$, their equilibria, $\mathbf{m}^{eq}$, and source term, S, are mentioned in [10].

The last term in Eq. (1) shows the effect of external force field on the evolution of distribution function. While different external force fields may exist such as gravity, Lorentz or Coriolis forces, pressure gradient in a periodic domain may also consider as an external force field, a technique which is used in the present computation.

Fig. 1 shows the three-dimensional, nineteen particle-velocity-lattice (D3Q19) model which has widely and successfully been used for simulating three-dimensional flows.
The macroscopic density and momentum on each lattice node are calculated using the following equations:

$$\rho = \sum_{\alpha=0}^{18} f_\alpha \qquad (2)$$

$$\vec{j} = \rho\vec{u} = \sum_{\alpha=1}^{18} \vec{e}_\alpha f_\alpha \qquad (3)$$

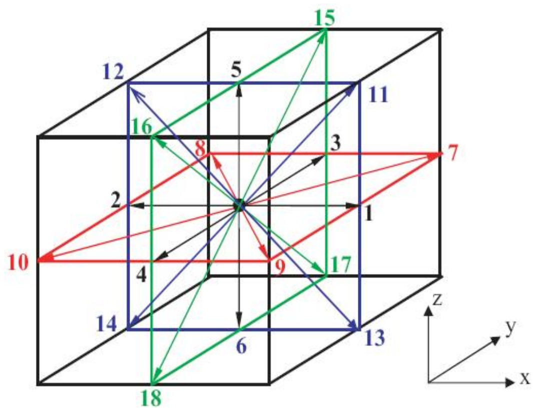

Fig. 1 D3Q19 lattice Model

Pressure can be obtained from an equation of state which is similar to the one for an ideal gas, i.e. $P = \rho c_m^2$. The speed of sound, c_s, in D3Q19 model is written as $c_m = c/\sqrt{3}$. Through a multiscale analysis based on the Chapman-Enskog expansion [15] applied to the GLBE with relaxation time scales augmented by an eddy viscosity, it can be shown that the grid-filtered weakly compressible Navier-Stokes equations with external force can be recovered. It should be noted that all quantities, i.e., ρ, $\vec{u}$ and other moments of $\mathbf{f}$, are filtered quantities.
The diagonal matrix $\hat{S}$ of relaxation rates $\{s_i\}$ is given as:

$$\hat{S} = diag(s_0, s_1, ..., s_{18})$$

where some of the relaxation times S_α in this diagonal matrix, i.e. those corresponding to hydrodynamic modes can be related to the transport coefficients and modulated by eddy viscosity due to SGS model as follows[10]: $s_1^{-1} = \frac{9}{2}\zeta + \frac{1}{2}$, where ζ is the molecular bulk viscosity, and $s_9 = s_{11} = s_{13} = s_{14} = s_{15} = s_\nu$, where $s_\nu^{-1} = 3\nu + \frac{1}{2} = 3(\nu_0 + \nu_t) + \frac{1}{2}$, with ν_0 being kinematic viscosity and ν_t the eddy viscosity determine from the Shear Improved SGS model which is discussed in more detail in the next section. The other relaxation rates are usually indicated through von Neumann stability analysis of the linearized GLBE [14] as s_1=1.19, $s_2 = s_{10} = s_{12} = 1.4$, $s_4 = s_6 = s_8 = 1.4$ and $s_{16} = s_{17} = s_{18} = 1.98$.
The source terms in moment space are functions of external force $\vec{F}$ and velocity fields $\vec{u}$. The driving force in the present channel flow computation is pressure gradient which is related to τ_w and therefore shear velocity as: $\vec{F} = -\frac{dp}{dx}\vec{x} = \frac{\tau_w}{H}\vec{x} = \frac{\rho u_\tau^2}{H}\vec{x}$.

Subgrid Scale Model

The shear-improved Smagorinsky model (SISM) which was proposed by Lèvêque et. al. [11] is a model for computing subgrid scale stresses used in LES computations of the present work. It is based on the fact that SGS eddy viscosity should encompass two types of interactions: (i) between the mean velocity gradient and the resolved fluctuating velocities (the rapid part of the SGS fluctuations) and (ii) among the resolved fluctuating velocities themselves (the slow part of the SGS fluctuations). The rapid part is related to the large-scale distortion, while the slow part is associated with the Kolmogorov's energy cascade. These developments end up with a shear improved Smagorinsky model (SISM) [11] for the SGS eddy-viscosity, in which it appears that the shear should be subtracted from the magnitude of the resolved rate-of-strain tensor. This improvement accounts for the large-scale distortion in regions of strong shear (e.g., near a solid boundary) and, at the same time, allows us to recover the standard Smagorinsky model in regions of locally homogeneous and isotropic turbulence (at grid scale). The SISM does not call for any adjustable parameter nor ad hoc damping function; it does not use any kind of dynamic adjustment either. Results concerning a plane-channel flow [11] and a backward-facing step flow [16] have shown good predictive capacity of this model, essentially equivalent to the dynamic Smagorinsky model [17], but with a computational cost and manageability comparable to the original Smagorinsky model. SISM Eddy viscosity is obtained by subtracting the magnitude of the shear from the instantaneous resolved rate of strain:

$$\nu_T^{SISM}(x,t) = (C_S\Delta)^2\left(\left|S_\Delta(x,t)\right| - S(x,t)\right) \qquad (12)$$

Here $S(x,t)$ denotes the shear at the position x and time t, $\left|S_\Delta(x,t)\right|$ shows the magnitude of instantaneous resolved rate of strain at position x and time t, C_S is the Smagorinsky constant for homogeneous and isotropic turbulence

($C_S \approx 0.18$) and $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$ is width of the grid filter and, Δx , Δy and Δz are the local grid spacing in x, y and z direction, respectively. It is assumed that the flow is well enough resolved in the direction of the shear, so that

$$S(x,t) \approx \left| \langle S_\Delta (x,t) \rangle \right| \tag{13}$$

In flow regions where the fluctuating part of the rate of strain is much larger than the shear, i.e. $\left| S'_\Delta \right| \geq S$, width of the grid filter $\Delta \leq L_S$ by assuming that $\left| S'_\Delta \right| = u' / \Delta$. In that case, turbulence can be considered as homogenous and isotropic at scale comparable to Δ . The SIMS then reduces to the original Smagorinsky model, which is known to perform reasonably well. In flow regions where $\left| S'_\Delta \right| \leq S$, width of the grid filter $\Delta \geq L_S$ and therefore shear effects are significant at scales comparable to Δ . In that case, the SIMS yields a SGS energy flux of order $\Delta^2 S \left(\left| S'_\Delta \right|^2 \right)$, which is fully consistent with the SGS energy budget that can be derived from the Navier-Stokes equations in the case of a locally homogenous flow [11].
In present work, spatial averaging over x and y directions (homogenous directions in channel flow) is used to compute $\left| \langle S_\Delta (x,t) \rangle \right|$ and equations (5)-(11) are employed to obtain the magnitude of instantaneous resolved rate of strain. Therefore equation (12) can be written in the following form:

$$\nu_T^{SISM} (x,t) = (C_S \Delta)^2 \left(\left| S \right| - \left| \langle S_\Delta (x,t) \rangle \right| \right) \tag{14}$$

ν_T^{SISM} obtained from above equation is used as eddy viscosity, ν_t , in $s_\nu^{-1} = 3\nu + 0.5 = 3(\nu_0 + \nu_t) + 0.5$.

Computational details

The geometry for fully-developed channel flow is shown in figure 2. The linear dimensions of domain are 6H and 3H in the stream-wise and span-wise directions respectively, where H is the channel half height. The flow is assumed to be homogeneous in both span-wise and stream-wise directions. There are no mean gradients (other than pressure) in the stream-wise direction, therefore the flow is considered as a fully-developed one. Reynolds number based on the shear velocity and channel half-width is considered as $\mathrm{Re}_\tau = u_\tau H/\nu = 180$, for which there is some previous published data [12].
Periodic boundary condition was considered in the stream-wise and span-wise directions due to the assumption of homogeneity in these directions. A bounce-back boundary condition was applied for the bottom and top solid walls of the channel.
A uniform grid was used for the present computations. The number nodes in stream-wise, span-wise and cross-stream directions were selected as 240 by 120 by 80 respectively which is correspond to a mesh with a resolution of 6 wall units in each direction. Although the total number of 2304000 grid points corresponds to a coarse grid distribution, the accuracy of

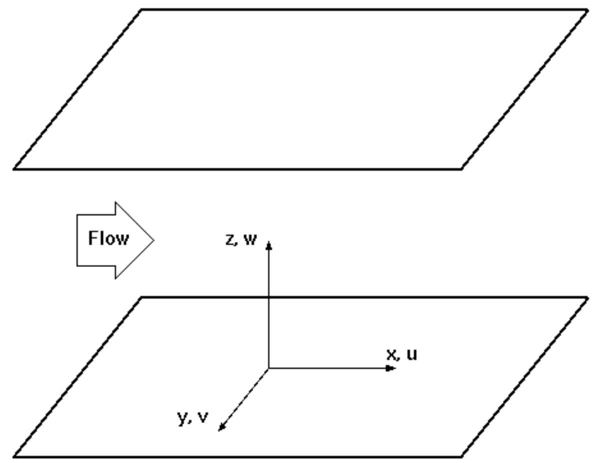

Fig. **2** Simulation geometry

the computational results has not affected significantly as will be discussed in the results part of the paper. The computer code was parallelized with MPICH2 parallel algorithm in which the domain was divided in slices along the stream wise direction.
The initial mean velocity is specified to satisfy the 1/7 power law, while initial perturbations satisfying divergence-free velocity field. A suitable initial condition can significantly decrease the number of iterations needed for convergence of the solution to a statistically steady state condition, and the GLBE computations was implemented until reaching statistically steady state.

RESULTS AND DISCUSSION

Fully developed turbulent channel flow is a simple flow geometry which may be considered as a benchmark problem in assessing various computational procedure and turbulent models. This flow geometry has been studied by various authors, e.g. DNS and LES based on Navier-Stokes equations [11, 12], LES and DNS based on SRT-LBE [8] and MRT-LBE [10]. Recently, a SISM of LES computation of channel flow [11] showed its accuracy as compared to dynamic LES models with lower computational cost. The present results were presented with the aim of evaluation of SISM of LES based on GLBE computations for a relatively coarse grid. The flow Reynolds number considered in the present study was selected as Re_τ=180 based on shear velocity and half channel width. The first computational result presented here is the mean velocity profile normalized by the shear velocity u_τ versus distance from the wall in terms of wall units, i.e., $z^+ = z/\delta$ where δ is the viscous length scale, see figure 3. The DNS data of Kim et. al. [12] was presented in this figure for comparison. It is observed that present mean velocity profile corresponds to DNS data relatively well, especially for the near wall region. It should be noted that Kim et. al. [12] used a grid which was more finer than the one used in the present computations. The small differences observed in the regions far from the wall, may be due to the turbulence model used in the present work as compared to DNS data.

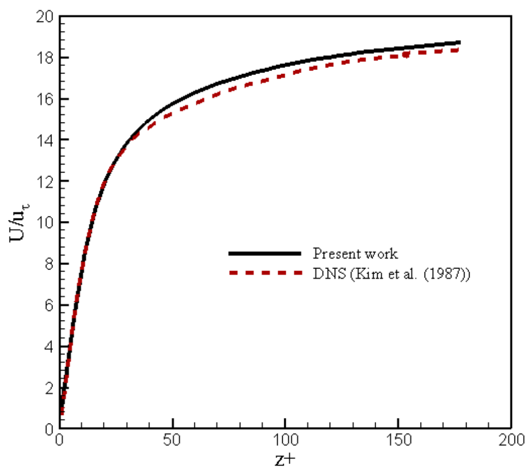

Fig. **3** Comparison of mean velocity profile with DNS data

Figures 4, 5 and 6 show variations of root-mean-square (rms) of stream-wise, span-wise and cross-stream velocity fluctuations, respectively along with the data from DNS based on NSE (Kim et al. [12]). Reasonable agreement is observed between the present work and previous DNS data.

A sample of instantaneous velocity fields at different cross-sections are shown in Figures 7, 8 and 9. As it is illustrated in Figure 7, flow field in y-z plane shows a random pattern. Moreover, this figure shows the movement of near-wall eddies toward and away from the wall. Figure 8 displays the velocity vector plot in x-z plane. The random deviations from the expected profile are obviously shown. In figure 9, the velocity field in the x-y plane, at $y = H/5$ from bottom wall is displayed which clearly shows the stream wise velocity is predominant in this direction. Furthermore, the random pattern of flow field in this plane is obvious.

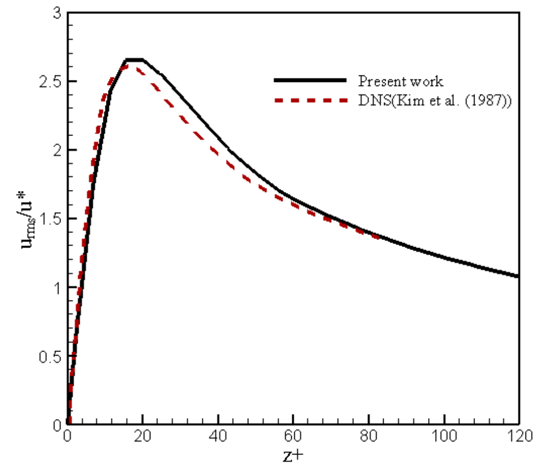

Fig. **4** Root-mean-square, stream-wise velocity fluctuations normalized by the wall shear velocity

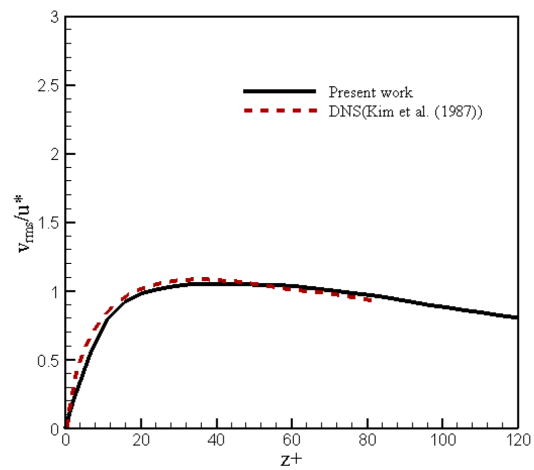

Fig. **5** Root-mean-square, span-wise velocity fluctuations normalized by the wall shear velocity

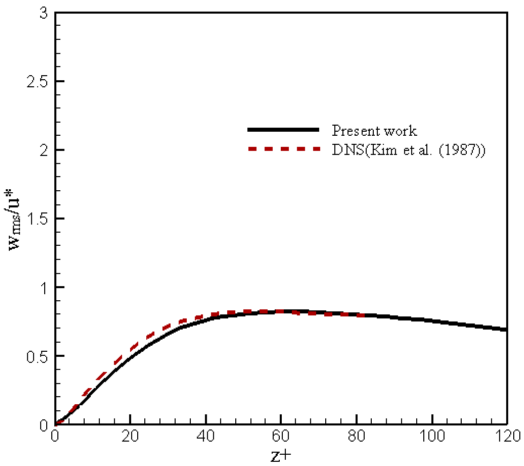

Fig. 6 Root-mean-square, wall-normal velocity fluctuations normalized by the wall shear velocity

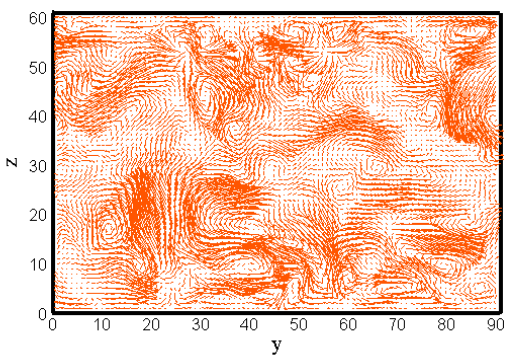

Fig. 7 sample velocity vector plot in y-z plane

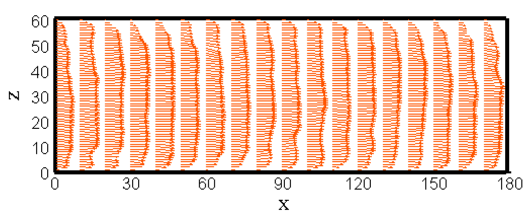

Fig. 8 sample velocity vector plot in x-z plane

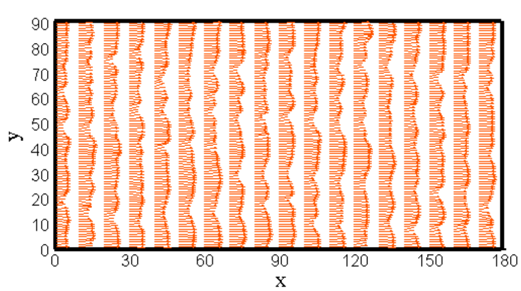

Fig. 9 sample velocity vector plot in x-y plane

CONCLUSION

A Generalized Lattice Boltzmann Equation (GLBE) using multiple relaxation times along with a forcing term was employed for simulation of turbulent channel flow at $Re_\tau = 180$. Turbulence simulation was done through LES with a recently-proposed subgrid scale model called Shear-Improved Smagorinsky Model (SISM) [11]. This subgrid scale model has shown reasonable results with a lower computational cost compared to turbulent flow simulation using CFD methods.
Results of turbulence statistics obtained from SISM LES were shown to be comparable to those obtained from Smagorinsky model while lower grid points were employed in the present computations. Using SISM LES in GLBE reveals its ability to predict accurate results in computational framework of LBM.
Computational results for the mean turbulent quantities such as mean velocity distribution across the channel height, showed good correspondence with DNS data. Comparison of the results for turbulence statistics such as rms velocity fluctuations with DNS and those obtained from Smagorinsky subgrid scale model reveals the accuracy of the present computational results which are obtained using a smaller number of grid points. This computational framework predicts Reynolds stresses with a reasonable accuracy as compared to a high-resolution DNS data reported earlier. Various instantaneous velocity vectors also showed complicated 3D vortical structures of turbulent channel flow. Based on the present computational results, it seems that GLBE with SISM-LES has the capability of predicting turbulent flow characteristics for turbulent wall flows with a relatively coarse grid as compared to DNS.

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