Proceedings of the ASME 2010 3rd Joint US-European Fluids Engineering Summer Meeting and 8th International Conference on Nanochannels, Microchannels, and Minichannels
FEDSM-ICNMM2010
August 1-5, 2010, Montreal, Canada

FEDSM-ICNMM2010-30279

LINEAR STABILITY OF A THIN LIQUID FILM FLOWING ALONG AN INCLINED SURFACE

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ABSTRACT

The formation of the waves on a thin liquid water film was analytically investigated by studying its shear mode stability. The inclined angle of the substrate is limited to 8°. The purpose of analytical solution is to determine the maximum growth rate of the generated wave as well as its corresponding wave number, which is realized by solving the Orr-Sommerfeld equations for both gas and liquid phases with the corresponding boundary conditions. The results of wave formations on a surface with a thin liquid film of de-icing are validated by previous experimental data as well as compared with Yih's theoretical analysis [7]. Studies have also conducted on the effect of surface tension or liquid film depth on the stability of a thin liquid film flowing along a solid substrate.

INTRODUCTION

The studies of a thin liquid film flowing down an inclined plane have been an important subject for many years due to its various engineering and industrial applications, such as in the manufacturing of photographic film or computer chips, coating, spraying, painting, ink-printing and water shedding on an airfoil. The investigation of a thin liquid film is to determine the uniformity and completeness of wetting on a substrate, which mainly depends on wave formations on the interface between liquid and gas, and therefore depends on the instability of a flowing thin liquid film. The instability can cause variations in both wave amplitude, and wave speed, and then leads to the breakup of a continuous flow. Reviews for the stability of a thin liquid film driven by gravity are given by Hocking [1] and Lopez et al. [2]; and for surface wave formation on a continuous film , both isothermal and temperature driven, by Oron et al. [3], Joo et al. [4], Liu et al. [5] and Chang [6]. Only a few investigations are found to be related to the stability of a thin liquid film driven by shear stress. Among them, the most important one is by Yih [7]. The wave formation of a thin liquid layer used for the de-icing of airplane wings was investigated by studying the stability of air flowing over a layer of de-icing coating with a zero angle of attack. By solving the Orr-Sommerfeld equations with its corresponding boundary conditions, an equation describing the relation between the wave number and the growth rate of a flow was obtained, which is also called the dispersion equation. However, this dispersion equation can be only used for the flow with very high viscosity ratio of liquid to gas, up to more than half million. The result of this work was also limited to a thin liquid film flowing on a horizontal plane. Therefore, the purpose of this work is to derive the dispersion equation for a thin liquid film driven by shear stress flowing down an inclined substrate. It can be also considered an extension of Yin's results without the limitations of high viscosity ratio and zero angle of attack.

NOMENCLATURE

ρ	density
μ	viscosity
σ	surface tension
m	viscosity ratio of liquid to gas
r	density ratio of liquid to gas
α	wave number
c_r	phase speed
c_i	growth rate
d	depth of a liquid film
b	dimensionless boundary layer thickness of gas phase
$\hat{U}_0$	free stream velocity of gas phase
$\hat{U}_1$	linear velocity for the boundary layer of gas phase
$\hat{U}_2$	linear velocity for the liquid phase
$\hat{U}_s$	velocity at the interface between liquid and gas
a_1	coefficient of dimensionless linear velocity for the boundary layer of gas phase
a_2	coefficient of dimensionless linear velocity for liquid phase
Re	Reynolds number defined as $\mathrm{Re}=\hat{U}_0 d\rho_1/\mu_1$

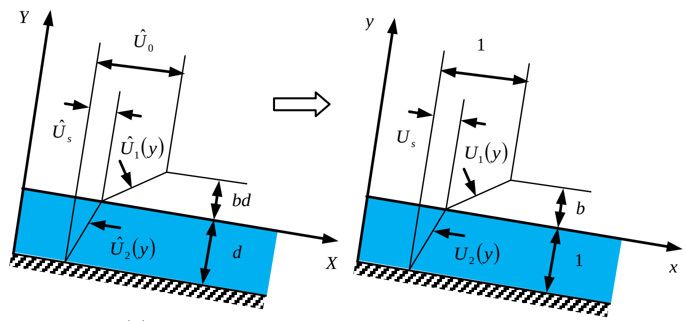
Subscript

1	gas phase
2	liquid phase

METHODOLOGY

This study focuses on a small inclined angle ($\leq 8°$). Although, the flow in general is three-dimensional, it has been proved by Schmid and Henningson [8] that only two-dimensional disturbances are sufficient to obtain the minimum critical Reynolds number, the maximum instability growth rate and its corresponding wave number using Squire's theorem [9]. The purpose of the analytical solution is to determine the maximum growth rate, of the generated wave as well as its corresponding wave number, α.

In this work, the gas phase (e.g. air) is considered as the primary flow and the liquid phase (e.g. water) is the secondary flow. The primary and the secondary flow are represented by subscription 1 and 2, respectively. In order to simplify the problem, it is assumed that both the liquid and the gas are Newtonian, and the physical properties such as surface tension σ and viscosity (μ_1, μ_2) are constant. The primary flow is considered as Blasius flow [10] over an inclined flat plate at zero angle of incidence, shown in Figure 1. In addition, $\hat{U}_1$, $\hat{U}_s$, $U_1(y)$ and $U_2(y)$ represent the air free-stream velocity, the interfacial velocity, the dimensionless linear velocities of gas phase and liquid phase, respectively. The term d is the water film thickness, and b is the dimensionless boundary layer thickness of the gas phase (i.e. the boundary layer thickness is bd).



(a) (b)

Figure 1 Schematic sketch of runback flow on a solid substrate (a) dimensional; (b) dimensionless

The dimensionless parameters in Figure 1b are defined as follows:

$$x = X/d \quad y = Y/d \quad U_s = \hat{U}_s/\hat{U}_0$$
$$U_1(y) = \hat{U}_1(Y)/\left(\hat{U}_0 * d\right) = a_1 y$$
$$U_2(y) = \hat{U} \qquad \left(\hat{U}_0 * d\right) = a_2 y$$

Governing Equations

The governing equations in the dimensionless form are written as follows.

For the gas phase,

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u_1}{\partial t} + \frac{u_1(\partial u_1)}{\partial x} + \frac{v_1(\partial u_1)}{\partial y} = -\frac{\partial p_1}{\partial x} + \mathrm{Re}^{-1}\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v_1}{\partial t} + \frac{u_1(\partial v_1)}{\partial x} + \frac{v_1(\partial v_1)}{\partial y} = -\frac{\partial p_1}{\partial y} + \mathrm{Re}^{-1}\left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2}\right) \tag{3}$$

For the liquid phase,

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \tag{4}$$

$$\frac{\partial u_2}{\partial t} + \frac{u_2(\partial u_2)}{\partial x} + \frac{v_2(\partial u_2)}{\partial y} = -\frac{\partial p_2}{\partial x} + F_x + \frac{r}{m}\mathrm{Re}^{-1}\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2}\right) \tag{5}$$

$$\frac{\partial v_2}{\partial t} + \frac{u_2(\partial v_2)}{\partial x} + \frac{v_2(\partial v_2)}{\partial y} = -\frac{\partial p_2}{\partial y} + F_y + \frac{r}{m}\mathrm{Re}^{-1}\left(\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial y^2}\right) \tag{6}$$

where $F_x = \dfrac{gd\sin\theta}{\hat{U}_0^2}$, $F_y = \dfrac{gd\cos\theta}{\hat{U}_0^2}$ and $\mathrm{Re} = \hat{U}_0 d\rho_1/\mu_1$

Orr-Sommerfeld equations

When velocity and pressure perturbations, i.e. u', v', p', are introduced into the primary flow, the flow is resolved into its primary and perturbation parts. For the gas flow, the perturbed parameters are

$$u = U_1 + u', \quad v = v', \text{ and } \quad p = P + p'$$

in which P is the dimensionless pressure for the primary flow and is assumed independent of x. In order to simplify the governing equations, a stream function ψ is introduced with the following definitions:

$$u' = \frac{\partial \psi}{\partial y} \qquad v' = -\frac{\partial \psi}{\partial x} \tag{7}$$

The following assumptions for ψ ,p' are made in order to represent all the spectrum of perturbations .

$$(\psi,\ p') = \{\phi(y), f(y)\} \exp i\alpha(x - ct) \tag{8}$$

where ϕ and f represent the initial wave and pressure perturbation amplitude, respectively. Then, the Orr-Sommerfeld equation for the gas phase can be obtained upon linearization of the Navier-Stokes equations using the stream function ψ.
For air

$$\phi^{iv} - 2\alpha^2\phi'' + \alpha^4\phi = i\alpha \operatorname{Re}(U_1 - c)(\phi'' - \alpha^2\phi) \tag{9}$$

Similarly, the Orr-Sommerfeld equation for the liquid phase can be derived by replacing ϕ with X, and retaining the meaning of *Re* and *f*. The Orr-Sommerfeld equation for the liquid phase is summarized as follows.

$$\mathrm{X}^{iv} - 2\alpha^2\mathrm{X}'' + \alpha^4\mathrm{X} = i\alpha \operatorname{Re} m^{-1} r(U_2 - c)(\mathrm{X}'' - \alpha^2\mathrm{X}) \tag{10}$$

Boundary Conditions

In order to obtain the boundary conditions for the gas phase, ϕ is denoted by ϕ_0 in the free stream and ϕ_1 for within the boundary layer, and η is used to represent the displacement of the artificial lower boundary of the free stream when the flow is disturbed at $y=b$.

The boundary conditions for the gas phase are summarized as follows.

$$\phi_0 \to 0 \quad as\ y \to \infty \tag{11}$$

Kinematic condition,

$$\frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial x} = -i\alpha\phi(b) \exp i\alpha(x - ct) \tag{12}$$

Continuity of velocity,

$$\phi_0(b) = \phi_1(b) \qquad (v_{y=b^+} = v_{y=b^-}) \tag{13}$$

$$\phi_0'(b) - \phi_1'(b) = \frac{\phi_0(b)}{c-1} \quad (u_{y=b^+} = u_{y=b^-}) \tag{14}$$

Continuity of shear stress,

$$\phi_0''(b) + \alpha^2\phi_0(b) = \phi_1''(b) + \alpha^2\phi_1(b) \qquad (\tau_{y=b^+} = \tau_{y=b^-}) \tag{15}$$

Continuity of normal stress,

$$\begin{aligned}&\{-\alpha \operatorname{Re}(c-1)\phi_0' - \phi_0''' + \alpha^2\phi_0'\} + 2\alpha^2\phi_0' = \\ &\{-i\alpha \operatorname{Re}[(c-1)\phi_1' + \alpha\phi_1] - \phi_1''' + \alpha^2\phi_1'\} + 2\alpha^2\phi_1'\end{aligned} \tag{16}$$

The boundary conditions for the liquid phase are described as follows.
No-slip boundary on the wall,

$$\mathrm{X}(-1) = 0 \quad \mathrm{X}'(-1) = 0 \tag{17}$$

Continuity of velocity at the interface,

$$\phi(0) = \mathrm{X}(0) \qquad (v_{y=0^+} = v_{y=0^-}) \tag{18}$$

$$\phi'(0) - \mathrm{X}'(0) = \frac{\phi(0)}{c'}(a_2 - a_1) \quad (u_{y=0^+} = u_{y=0^-}) \tag{19}$$

Continuity of shear stress,

$$\phi''(0) + \alpha^2\phi_0 = m(\mathrm{X}''(0) + \alpha^2\mathrm{X}(0)) \quad (\tau_{y=0^+} = \tau_{y=0^-}) \tag{20}$$

Continuity of normal stress,

$$\begin{aligned}&-i\alpha \operatorname{Re}(c'\phi' + a_1\phi) - (\phi''' - \alpha^2\phi') + 2\alpha^2\phi' + ir\alpha \operatorname{Re}(c'\mathrm{X}' + a_2\mathrm{X}) + \\ &m(\mathrm{X}''' - \alpha^2\mathrm{X}') - 2\alpha^2 m\mathrm{X}' = i\alpha \operatorname{Re}(F^{-2} + \alpha^2 S)\phi/c'\end{aligned} \tag{21}$$

where

$$F^{-2} = (r-1)F_y^{-2}, \quad S = T/(\rho u_0^2 d), \quad c' = c - a_2$$

and a_1, a_2 are the slopes of the linear velocities for the gas phase and the liquid phase, respectively.

DERIVATION OF DISPERSION EQUATION

For the free stream (the gas phase), the solution for the Orr-Sommerfeld equation can be reduced to two terms, which only includes the invisicid solution due to the given four boundary conditions.

$$\phi_0 = A_{01} \exp(-\alpha y) + A_{02} \exp(-\beta(y - b))$$

where

$$\beta^2 = i\alpha \operatorname{Re}(1 - c) + \alpha^2$$

$$\phi_1 = A_{11} \exp(-\alpha y) + A_{12} \exp(\alpha y) + A_{13} \exp(\beta(y - b))$$

This is only an approximate solution, but still sufficiently accurate because the term $(\beta(y-b))$ decreases toward 0 as y decreases from b only a small distance.

By applying the above equations to the boundaries given as equations (13), (14), and (15), it can be shown that if A_{01}, A_{11}, and A_{12} are of *O(1)* in magnitude, A_{02} and A_{13} are both of order $O(Re^{-0.5})$. Furthermore, it can also be found that the ϕ' and ϕ_1' are of O(1) by to substituting ϕ_0 and ϕ_1 to equation (15). Thus equation 13 becomes, upon division by $i\alpha \operatorname{Re}$ and ignoring terms of O ($\operatorname{Re}^{-0.5}$), at y=b,

$$(c-1)\phi_0' = (c-1)\phi_1' + a_1\phi_1 \tag{22}$$

in which all viscous solutions (i.e. those with coefficient A_{02} and A_{13}) are dropped. Similarly, in equation (23), only the inviscid solution is needed.

$$\phi_0(b)=\phi_1(b) \tag{23}$$

Construction of the Eigenfunctions

The simplification of the boundary condition at $y=b$ to described by equations (22) and (23), allows us to construct ϕ by forming its inviscid part first, and then adding to the viscous part to satisfy the interfacial conditions at $y=0$.

In order to obtain the inviscid part, let

$$\phi_0 = A_0 \exp(-\alpha y)$$
$$\phi_1 = A_1 \exp(-\alpha y) + A_2 \exp(-\alpha y)$$

Using equations (22) and (23) result in the following solution for the inviscid part.

$$A_1 = \frac{1}{2\alpha b(1-c)}\exp(-2\alpha b)A_0$$

$$A_2 = \left(1-\frac{1}{2\alpha b(1-c)}\right)A_0$$

Then, $\phi_1(0) = A_1 + A_2 = \frac{1}{1-c}(1-\lambda-c)A_0$

$$\phi_1'(0) = \alpha(A_1 - A_2) = \left\{\frac{1}{b(1-c)} - \alpha\left(\frac{1}{1-c}+\lambda\right)\right\}A_0$$

In which

$$\lambda = \frac{1}{2\alpha b}\left(1-\exp(-2\alpha b)\right) \quad (0 \le \lambda \le 1)$$

Assuming that c is small compared to 1, one can rewrite $\phi_1(0)$ and $\phi_1'(0)$ as

$$\phi_1(0) = (1-\lambda)A_0 \tag{24}$$

$$\phi_1'(0) = \left\{\frac{1}{b} - \alpha(1+\lambda)\right\}A_0 \tag{25}$$

Next, the viscous solution, $A_3\phi_3$, is added to the inviscid solution, ϕ_1. Thus,

$$\phi = \phi_1 + A_3\phi_3 \tag{26}$$

In which ϕ_3 is given by equation (27) (Lin [11])

$$\phi_3 = \int_\infty^\eta d\eta \int_\infty^\eta H_{\frac{1}{3}}^{(1)}\left\{\frac{2}{3}(i\eta)^{\frac{3}{2}}\right\}\eta^{\frac{1}{2}} d\eta \tag{27}$$

Where $\eta = \frac{y-y_c}{\varepsilon}$, $\varepsilon = (\alpha \operatorname{Re} U_1')^{-\frac{1}{3}} = \left(\frac{b}{\alpha \operatorname{Re}}\right)^{\frac{1}{3}}$

, and y_c is the value of y at which $U_1 = c$. Since c is expected to be very small, we can henceforth take $y_c = 0$.

Construction of the Eigenfunction

The Orr-Sommerfeld equation for the liquid phase is as follows.

$$X^{iv} - 2\alpha^2 X'' + \alpha^4 X = i\alpha \operatorname{Re} m^{-1} r(U_2 - c)(X'' - \alpha^2 X)$$

The eigenvalue of the above equation can be presented by,

$S=[0, r_2, r_3, r_4]$

Then the general solution is described as,

$$X = A + B\exp(r_2 y) + C\exp(r_3 y) + D\exp(r_4 y)$$

By applying the shear stress and no-slip boundary conditions to the general solution, the coefficients of the above general solution can be determined, and then $X(0)$, $X'(0)$, and $X'''(0) - 3\alpha^2 X'(0)$ can be expressed as follows.

$$X(0) = A + B + C + D \tag{28}$$

$$X'(0) = r_2 B + r_3 C + r_4 D \tag{29}$$

$$X''(0) - 3\alpha^2 X'(0) = (r_2^3 - 3\alpha^2 r_2)B + (r_3^3 - 3\alpha^2 r_3)C + (r_4^3 - 3\alpha^2 r_4)D \tag{30}$$

Wave Growth Rate

Upon neglecting a_2 in the brackets on the right-hand side of equation (16), eliminating $\varphi(0)$ between equations (15) and (16) results in the following,

$$-bc\phi'(0) = X(0) - bc'(0) \tag{31}$$

Let

$$\phi_3(0) = \delta \qquad \left(\frac{d\phi_3}{d\eta}\right)_{\eta=0} = \gamma$$

where $\delta = -0.8660 + 0.2320i$ and $\gamma = 1.1154 + 0.2989i$ (Lin [11]).

Equation (31) then takes the following form,

$$-bc'\left\{\left[b^{-1} - \alpha(1+\lambda)\right]A_0 + \varepsilon^{-1}\gamma A_3\right\} = \frac{4\beta\sinh(\alpha)\cosh(\beta) - 4\alpha\sinh(2\beta) + 2\alpha\sinh(\beta)\cosh(\alpha)}{\beta\cosh(\beta) - \beta\exp(\alpha) + \alpha\sinh(\beta)}A - bc'\frac{4\alpha\beta - 4\alpha\beta\cosh(\alpha)\cosh(\beta) + 2(\alpha^2+\beta^2)\sinh(\alpha)\sinh(\beta)}{\beta\cosh(\beta) - \beta\exp(\alpha) + \alpha\sinh(\beta)}A \tag{32}$$

Equations (21) and (23) enables us to write equation (15) as

$$(1-\lambda)A_0+\delta A_3=$$
$$\frac{4\beta\sinh(\alpha)\cosh(\beta)-4\alpha\sinh(2\beta)+2\alpha\sinh(\beta)\cosh(\alpha)}{\beta\cosh(\beta)-\beta\exp(\alpha)+\alpha\sinh(\beta)}A \tag{33}$$

Elimination of A_3 between equations (32) and (33), and based on the fact that c' is small and $\beta >> \alpha$, equation (33) can be simplified as follows.

$$c'\{-1+\alpha b(1+\lambda)+b\varepsilon^{-1}\gamma\delta^{-1}(1-\lambda)\}A_0= \frac{A}{\alpha+\beta}\{(2\beta+\beta^2)\sinh(\alpha)+\alpha\exp(\beta)\} \tag{34}$$

Since $\gamma\delta^{-1}=-(1.1155+0.6440i)$, equation (31) can be rewritten as

$$bc'(P_r+iP_i)A_0=A \tag{35}$$

where

$$P=P_r+iP_i=\frac{(\alpha+\beta)\{b^{-1}-\alpha(1+\lambda)-\gamma\delta^{-1}\varepsilon^{-1}(1-\lambda)\}}{(2\beta+\beta^2)\sinh(\alpha)+\alpha\exp(\beta)} \tag{36}$$

Equation (35) shows that A in (33) can be neglected since both c' and $\dfrac{\alpha+\beta}{(2\beta+\beta^2)\sinh(\alpha)+\alpha\exp(\beta)}$ are small.

Thus, equation (33) can be written as

$$A_3=-\delta^{-1}(1-\lambda)A_0=\frac{\delta^{-1}(1-\lambda)}{bc'(P_r+iP_i)}A \tag{37}$$

Next step is to evaluate c' and c_i (i.e. the growth rate).

Since a_2 is small, equation (19) enables us to write equation (21) as,

$$i(r-1)\alpha\,\mathrm{Re}\,\mathrm{X}'c'+m(\mathrm{X}'''-3\alpha^2\mathrm{X}')-(\phi'''-3\alpha^2\phi')+i\alpha\,\mathrm{Re}\,a_2(r-1)\mathrm{X}$$
$$-i\alpha\,\mathrm{Re}(F^{-2}+\alpha^2S)\frac{\mathrm{X}}{c'}=0 \tag{38}$$

All functions are being evaluated at $y=0$ and all the terms involving X have been given by equations (28), (29), and (30). Finally, it remains to evaluate the term, $\phi'''-3\alpha^2\phi'$.

$$\begin{aligned}\phi'''-3\alpha^2\phi'&=-2\alpha^2\phi_1'+A_3\{\varepsilon^{-3}(d^3\phi_3/d\eta^3)-3\alpha^3\varepsilon^{-1}(d\phi_3/d\eta)\}\\&=-2\alpha^2\{b^{-1}-\alpha(1+\lambda)\}A_0+A_3\{\varepsilon^{-3}(d^3\phi_3/d\eta^3)-3\alpha^3\varepsilon^{-1}(d\phi_3/d\eta)\}\\&=-2\alpha^2\{b^{-1}-\alpha(1+\lambda)\}A_0+A_3\varepsilon^{-3}(-i\delta-3\alpha^2\varepsilon^2\gamma)\end{aligned}$$

Taking (32) and (33) into the above equation, it can be written as,

$$\phi'''-3\alpha^2\phi'=\frac{\alpha\,\mathrm{Re}\,A(Q_r+iQ_i)}{b^2c'(P_r+iP_i)} \tag{39}$$

where

$$Q_r+iQ_i=(1-\lambda)(i+3\gamma\delta^{-1}\alpha^2\varepsilon^2)-\frac{2\alpha}{\mathrm{Re}}(1-\alpha b(1+\lambda))$$

Examination of (28), (29), (30), and (39) shows that (38) is a quadratic equation in c'. This quadratic equation can be written as,

$$c'^2\{i(r-1)\alpha\,\mathrm{Re}\,\mathrm{X}'\}+mc'(\mathrm{X}'''-3\alpha^2\mathrm{X}')+c'(i\alpha\,\mathrm{Re}\,a_2(r-1)\mathrm{X})$$
$$\left\{\frac{\alpha\,\mathrm{Re}(Q_r+iQ_i)}{b^2(P_r+iP_i)}-i\alpha\,\mathrm{Re}(F^{-2}+\alpha^2S)X\right\}=0 \tag{40}$$

Equation (40) is the dispersion equation used to determine the relationship between the growth rate and the wave number.

RESULTS AND DISCUSSION

The dispersion equation is obtained by solving the Orr-Sommerfeld equations with the given boundary conditions to determine the relation between the wave number α and the growth rate c_i, which can be written as follows.

$$i(r-1)\alpha\,\mathrm{Re}\,\mathrm{X}'c'+m(\mathrm{X}'''-3\alpha^2\mathrm{X}')-(\phi'''-3\alpha^2\phi')+i\alpha\,\mathrm{Re}\,a_2(r-1)\mathrm{X}$$
$$-i\alpha\,\mathrm{Re}(F^{-2}+\alpha^2S)\frac{\mathrm{X}}{c'}=0$$

All the terms can be analytically determined except for variables α and c'. In order to validate the analysis with the experimental data [12] and the analytical solution from Yih [7], where a de-icing liquid was utilized, the same physical properties are used, as listed in Table 1.

Table 1 Properties used in analytical calculations

Properties	Units
liquid viscosity near 0^oC	10 Pa· s
air viscosity at -10 ^oC	1.67×10^{-5} Pa· s
surface tension at -10 ^oC	31.3 mN/m
liquid-air density ratio	972
free-stream speed	27.28 m/s
the chord length	0.279 m
mean liquid film depth d	1.1mm

The analytical results are shown in Table 2, together with the experimental data and the analysis from Yih [7]. It is observed that the results agree well with the experimental data in terms of the wave number. Compared to the analysis of Yih [7], it seems that our results are more accurate respect to the wave number with the same magnitude phase speed and maximum growth rate.

Table 2 Comparisons of the theoretical results with experimental data

Characteristic	wave number	Phase speed	Maximum growth rate
Experimental data [12]	0.5		
Theoretical results of this work	0.5	1.657e-05	2.88e-05
Theoretical results(Yih [7])	0.33	1.52e-05	1.46e-05

Studies have also done on the effect of liquid film depth on the stability of a thin liquid film flowing along an inclined surface. The analysis is based on a film velocity of 15 *m/s*. The results are presented on Figure 2, where the range of *Re* number represents a liquid film depth *d* from 0.5 to 2.0 *mm*. Figure 2 shows the effect of *Re* on the maximum growth rate and its corresponding wave number of a thin liquid film flow. It reveals that when the *Re* number increases (e.g. from 560 to 2240), the maximum growth rate shifts to a larger wave number (from 0.5 to 0.6). Therefore, it can be concluded that an increase in the depth of a thin liquid film can increase the maximum growth rate of the flow and also shifts the corresponding wave number to a larger value.

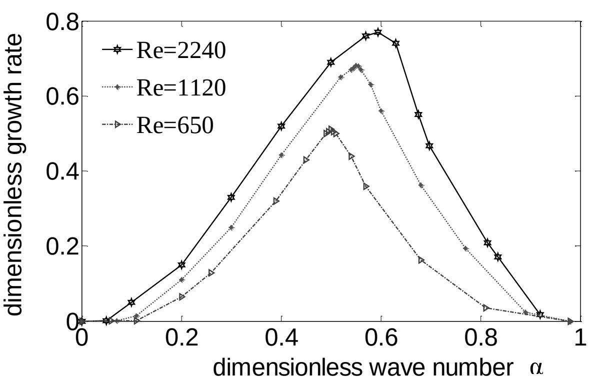


Figure 2 Effect of liquid film depth on stability of a thin liquid film flowing along an inclined surface

CONCLUSION

The purpose of this study is to investigate the effect of shear mode instability on the wave formation for a thin liquid film flowing down an inclined substrate ($\leq 8^o$). The Orr-Sommerfeld equations for both liquid and gas phases are derived by linearizing governing equations and introducing perturbations for both phases. Then the analytical solutions are obtained by solving the Orr-Sommerfeld equations with the corresponding boundary conditions. The results show good agreement with previous experimental data and Yih's study in the maximum growth rate and its corresponding wave number for air flowing along a layer coating with de-icer. The good agreement between the analytical solution and the experimental data suggest that for a thin liquid film flowing down a substrate with a small inclining angle, the shear mode instability is paramount in importance compared to the gravity mode instability. The studies on the effect of liquid film depth on the wave formation of a thin liquid film flow show that the maximum growth rate keep increasing with an increase in liquid film depth and the corresponding wave number also shifts to a larger value.

AKNOWLEGEMENT

The authors would like to thank the Natural Sciences and Engineering Research Council Canada (NSERC) for the financial support.

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