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CRITICAL MASS FLOW RATE THROUGH CAPILLARY TUBES

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ABSTRACT

This paper presented a numerical study that predicts critical mass flow rate, pressure, vapor quality, and void fraction along a very long tube with small diameter or capillary tub under critical condition by the drift flux model. Capillary tubes are simple expansion devices and are necessary to design and optimization of refrigeration systems. Using dimensional analysis by Buckingham's π theory, some generalized correlations are proposed for prediction of flow parameters as functions of flow properties and tube sizes under various critical conditions. This study is performed under the inlet pressure in the range of $0.8 \le p_{in} \le 1.5\, Mpa$, subcooling temperature between $0 \le \Delta T_{sub} \le 10$ °C. The tube diameter is in the range of $0.5 \le D \le 1.5\, mm$ and tube length between $1 \le L \le 2\, m$ for water, ammonia, refrigerants R-12, R-22 and R-134 as working fluids. Comparison between the results of the present work and some experimental data indicates a good agreement. Cluster of data close to the fitted curves also shows satisfactory results.

Keywords: two-phase flow, drift flux model, numerical method, capillary tube, dimensionless analysis

INTRODUCTION

Small diameter tubes or capillary tubes have an extensive implementation as a simple expansion device in the industrial refrigeration systems to reduce the pressure and temperature of a refrigerant fluid flow between a condenser and an evaporator. Because of the fact that these simple devices reduce the temperature of the flow by flashing it into vapor phase, it is complicated to analyze the flow through them. Typical diameter of a capillary tube is between 0.5 – 2 mm and its length is between 1 – 6 m. Subcooled liquid enters into the capillary tubes and flows as a single phase up to the point where the pressure reaches saturation pressure related to the flow temperature. Although it is expected that flashing into the vapor phase starts from the end of single phase region, it happens with some delay in the point where flow pressure is a bit less than its saturation pressure. This single phase region is called metastable flow region. However, this region is refused in most of numerical calculations. In two-phase region, flow starts with bubbly flow and ends with annular flow pattern if the length of the tube is long enough. By decreasing the pressure at the tube outlet, mass flow rate increases up to a critical rate in which the mass flow rate remains constant and shock happens at the end of the tube where pressure decreases with a sharp gradient and as a result the liquid flashes into vapor by a pretty high rate.

The most important models used to study the behavior of two-phase gas-liquid flow are homogeneous, separated flow, drift flux and two-fluid model. In the homogeneous model two phases are considered to flow in equilibrium from dynamic and thermodynamic point of view, but in the separated flow model an experimental correlation for the slip ratio of velocities of two phases is considered through the flow. In the drift flux model an experimental correlation is considered to relate drift velocity of the flow to the other flow characteristics. Wong and Liang [1] proposed a numerical study based on the drift flux model to find characteristics of R-134 through the capillary tubes under critical conditions. Zuber's correlation [2] for the drift velocity and Lin's two-phase multiplier [3] were used in their study. Their results compared predicted flow characteristics of R-134 with the predicted flow characteristics of R-12. Also some experimental works have been performed in order to find correlations to predict critical flow characteristics of the capillary tubes. Melo [4] experimentally studied the effect of the capillary diameter and length, inlet pressure and subcooling and type of refrigerant on critical mass flow rates under adiabatic condition. They performed the study under the inlet pressure in the range of $7-18$ bar, subcooling temperature between $2-16\ ^\circ C$ and tube diameter between $0.6-1.05$ mm for the refrigerants R-12, R-134 and R-600. It was found that the tube diameter affects critical mass flow rate more than other parameters. They presented a correlation by dimensional analysis to estimate critical mass flow rate of each refrigerant. The results of their correlations were in agreement with experimental data with 15% error for R-12 and 10% error for R-134. Kim and Choi [5] presented an empirical correlation to predict critical mass flow rate under the adiabatic condition. They explored the data affecting the critical mass flow rate and generated some dimensionless parameters by using the data of refrigerants R-12, R-22, R-134, R-152, R-407 and R-410with a deviation of 6.5%. Jabaraj et al. [6] conducted an experimental study on flow of mixture M20 (R-407c, R-600, R-290) through capillary tubes, They reported a non dimensional correlation to predict critical mass flow rate of M20 by using Buckingham's π theory. Their results revealed that approximately 95% of which has a deviation of about 10%. Recently Vins and Vacek [7] performed an experimental study on the critical flow of R-218 through capillary tubes with different tube diameters and lengths. They presented a correlation for the critical mass flow rate of refrigerant R-218 by both classical power law function and artificial neural network. The average of deviation by the power law function was -0.41% and by the artificial neural network correlation was -0.12%.

This study attempts to use a numerical method to predict critical flow characteristics for water, ammonia, refrigerants R-12, R-22 and R-134 through capillary tubes by drift flux model under different operating conditions. Generalized correlations have been presented for prediction of critical mass flow rate, pressure, quality and void fraction distribution through capillary tubes in a wider range of operating conditions.

NOMENCLATURE

D tube diameter, m
f friction factor
G mass velocity, kg/s.m^2
h enthalpy, J/kg
p pressure, Pa
Re Reynolds number
s entropy, j/kg.K
T temperature, $^{\circ}C$
u velocity, m/s
x vapor quality
z axial coordinate, m

Greek letters

α void fraction
ε roughness, m
φ_{fo}^2 two-phase friction multiplier
μ viscosity, kg/s.m
v kinematic viscosity, m^2/s
υ specific volum, m^3/kg
ρ density, kg/m^3
σ surface tension, N/m
ψ general variable

Subscripts

D Darcy
ex exit
f liquid
fg saturated vapor liquid difference
fo all fluid as a liquid
i index
in inlet
s single phase
sat saturation
sh superheat
t turbulent
v vapor

MATHEMATICAL FORMULATION

Flow through capillary tubes can be divided into single phase, metastable and two-phase flow. Single phase flow region continues up to the point where flow pressure reaches saturation pressure related to its temperature. In this case the length of the single phase flow region is.

$$L_s = \frac{2D\rho_f \left(p_{in} - p_{sat}\right)}{f_D G^2} \quad (1)$$

where f_D is the Darcy friction factor and for turbulent flow is calculated by Colebrook's correlation.

$$\frac{1}{\sqrt{f_D}} = 1.14 - 2\log\left[\frac{9.34}{\mathrm{Re}_{fo}\sqrt{f_D}}\right] \quad (2)$$

In the two-phase region, liquid flashes into vapor purely because of the reducing pressure. The one-dimensional, steady state governing equations of mass, momentum and energy for a two-phase flow of vapor-liquid phases through an adiabatic capillary tube are given respectively as follows.

$$\frac{d}{dz}(\rho_v u_v \alpha) + \frac{d}{dz}\left[\rho_f u_f (1-\alpha)\right] = 0 \quad (3)$$

$$\frac{d}{dz}(\alpha\rho_v u_v^2) + \frac{d}{dz}\left[(1-\alpha)\rho_f u_f^2\right] = -\frac{dp}{dz} - \frac{(f_{fo})_D G^2}{2D\rho_f}\varphi_{fo}^2 \quad (4)$$

$$\frac{d}{dz}\left[\rho_v u_v \alpha \left(h_v + \frac{u_v^2}{2}\right)\right] + \frac{d}{dz}\left[\rho_f u_f (1-\alpha)\left(h_f + \frac{u_f^2}{2}\right)\right] = 0 \quad (5)$$

where φ_{fo}^2 is two-phase friction multiplier and Lin [3] derived the following expression for that.

$$\varphi_{fo}^{2} = \left[1 + x\,(\frac{\rho_f}{\rho_v} - 1)\right]\left[\frac{A_{fo} + B_{fo}}{A_t + B_t}\right]^{\frac{1}{8}} \quad (6)$$

Where

$$A_{fo} = \left\{2.457\, Ln\left[(\frac{7}{\mathrm{Re}_{fo}})^{0.9} + \frac{0.27\varepsilon}{D}\right]\right\}^{16}$$

$$A_t = \left\{2.457\, Ln\left[(\frac{7}{\mathrm{Re}_t})^{0.9} + \frac{0.27\varepsilon}{D}\right]\right\}^{16}$$

$$B_{fo} = \left[\frac{37530}{\mathrm{Re}_{fo}}\right]^{16}, \qquad B_t = \left[\frac{37530}{\mathrm{Re}_t}\right]^{16}$$

$$\mathrm{Re}_{fo} = \frac{DG}{\mu_f}, \qquad \mathrm{Re}_t = \frac{DG}{\mu_t}$$

The vapor quality in term of void fraction is as:

$$x = \frac{\rho_v u_v \alpha}{G} \quad (7)$$

Drift-Flux Model: is essentially a separated-flow model in which attention is focused on the relative motion rather than on the motion of the individual phases. It is particularly useful when the relative motion is determined by a few key parameters instead of connection to the flow rate of each phase. Zuber [2] presented the following correlation through horizontal tubes.

$$u_{vj} = 1.48\left[\frac{(\rho_f - \rho_v)\sigma g}{\rho_f^{\,2}}\right]^{0.25} \quad (8)$$

The drift velocity of vapor relative to the center of mass, u_{vm}, is given by:

$$u_{vm} = u_v - u_m = \frac{\rho_f}{\rho_m} u_{vj} \quad (9)$$

where the average density and mass velocity are defined as:

$$\rho_m = \alpha\rho_v + (1-\alpha)\rho_f \quad (10)$$

$$u_m = \frac{\alpha\rho_v u_v + (1-\alpha)\rho_f u_f}{\rho_m} \quad (11)$$

During flashing, we assume the equilibrium prevails between the inside and outside of the bubble before growth starts and the initial liquid superheat can be evaluated with Clausius-Clapeyron equation as follows.

$$T_{sh} = T_f - T_{sat} = \frac{4\sigma v_{fg}}{d s_{fg}} \quad (12)$$

where σ is the surface tension and $s_{fg} = h_{fg}/T$. From the available data, an initial bubble diameter may be assumed as $d = 2.5\times10^{-5}\ m$, [8].

Combining Eqs. (8-11), you can obtain easily u_f as a function of u_v

After inserting Eq. (7) into Eq. (6) and $u_f = f\,(u_v)$ into Eqs. (3-5) and differentiating the results, the general form of the differential equation for three variables u_v , α and p is given by:

$$a_i \frac{du_v}{dz} + b_i \frac{d\alpha}{dz} + c_i \frac{dp}{dz} = d_i \quad (13)$$

where, the coefficients a_i, b_i, c_i and d_i are functions of the physical properties of the liquid-vapor phases and are given in Appendix A.

SOLUTION METHOD

In the above analysis, if a subcooled single-phase liquid inters a capillary tube, Eq. (1) is first applied to the subcooled liquid region with the known initial conditions at the orifice inlet, for instance, pressure and mass flow rate. Once the saturated condition is reached, the two-phase flow calculations are started until the mass flow rate reaches the critical condition. When this case happens, the critical flow rate is almost independent of the downstream pressure. In this case Eqs. (13) for $i = 1, 2, \text{and } 3$ are solved simultaneously by the fourth order Rung-Kutta method for unknown variables u_v , α, and p under their specified initial conditions. The existing derivatives of $d\phi/dp$ for $\phi = \rho_f , \rho_v , h_f , h_v$ in Eq. (13) can be obtained by fitting the fluid properties data versus pressure from the thermodynamic table of the matter.

RESULTS AND DISCUSSION

To predict the critical mass flow rate and the critical pressure distribution through the capillary tubes, the present model is examined for a range of inlet pressure of $0.8 \le p_{in} \le 1.5 Mpa$, subcooling temperature between $0 \le \Delta T_{sub} \le 10$ °C. The tube diameter is in the range of $0.5 \le D \le 1.5 mm$ and tube length between $1 \le L \le 2 m$ for water, ammonia, refrigerants R-12, R-22 and R-134 as working fluids. Dimensions of the capillary tubes include length of $1 \le L \le 2$ m, diameter of $0.5 \le D \le 1.5$ mm with surface roughness of $0.5 \le \varepsilon \le 2\ \mu$m .

Critical Correlations: In order to obtain the generalized correlations for the critical flow parameters, the set of equations (13) for $i = 1, 2 \text{ and } 3$ is first solved numerically for the unknown variables u_v , α and p, then the critical flow parameters are assumed to be as functions of physical properties and flow geometry as follows.

$$\psi_{cr} = f\left(L, D, \varepsilon, p_{in}, \Delta T_{sub}, \rho_f, \rho_v, \mu_f, \mu_v\right) \quad (14)$$

where ψ_{cr} refers to the dependent variables and includes:

$$\psi_{cr} = m_{cr}, P_{cr}, x_{cr}, L_{cr} \quad (15)$$

Using Buckingham π theory, the following dimensionless parameters are obtained.

$$\pi_1 = \frac{\dot{m}}{D\mu_f},\ \pi_2 = \frac{D^2 \rho_f p_{in}}{\mu_f^2},\ \pi_3 = \frac{D^2 C_p \Delta T_{sub}}{{v_f}^2},\ \pi_4 = \frac{\varepsilon}{D}$$

$$\pi_5 = \frac{\rho_f}{\rho_v},\ \pi_6 = \frac{D^2 h_{fg}}{\upsilon_f^2 \mu_f^2},\ \pi_7 = \frac{L}{D},\ \pi_8 = \frac{p_{in} - p_{ex}}{p_{in}}$$

Figure 1 illustrates the mass flow rate of water, ammonia, refrigerants R-12, R-22 and R-134 through the capillary tubes under the critical conditions. The ordinate axis has been dimensionless by the capillary tube diameter and the liquid viscosity. The abscissa axis is indicated by some dimensionless parameters. Comparison between the present results with some experimental data presented by Melo [4], Mikol [9] and Li [10] shows a good agreement. All the results including the numerical results of the present model and the experimental data are also fitted by a line with the following dimensionless expression.

$$\frac{\dot{m}_{cr}}{D\mu_f} = 0.72\left[\frac{D^2 \rho_f p_{in}}{{\mu_f}^2}\right]^{0.689}\left[\frac{D^2 c_p \Delta T_{sub}}{{v_f}^2}\right]^{0.116}\left[\frac{\rho_f}{\rho_v}\right]^{0.1} \times \left[\frac{D^2 h_{fg}}{\upsilon_f^2 \mu_f^2}\right]^{-0.229}\left[\frac{L}{D}\right]^{-0.465} \quad (16)$$

Figure 2 predicts the pressure distribution under the critical conditions along the capillary tubes for water, ammonia, R12, R-22 and R-134 as working fluids. The y- and x-axes have been dimensionless by the inlet pressure and some dimensionless parameters respectively. The figure also includes a curve fitted by all numerical results of the present. The equation of the curve is as follows.

$$\frac{P_{in} - P}{P_{in}} = 0.39\eta - 0.6\eta^2 + 0.37\eta^3 \quad (17)$$

$$\eta = \left[\frac{L}{D}\right]^{0.078}\left[\frac{\rho_v}{\rho_f}\right]^{0.036}\left[\frac{D^2 c_p \Delta T_{sub}}{{v_f}^2}\right]^{-0.016}\left[\frac{D^2 h_{fg}}{\upsilon_f^2 \mu_f^2}\right]^{0.016}\frac{Z}{L}$$

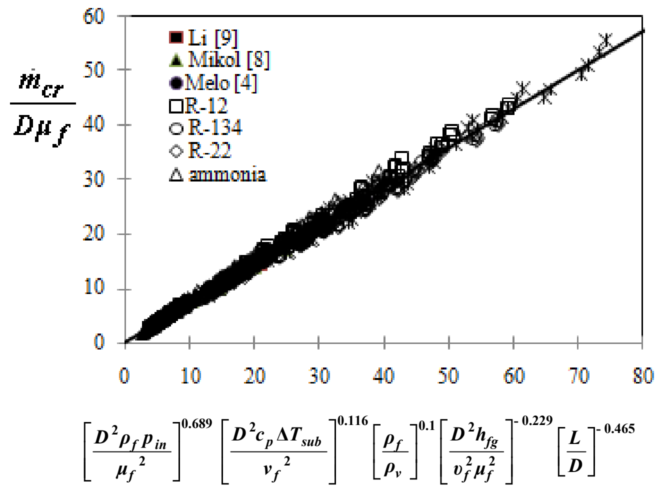

Fig. 1 Comparison between predicted critical mass flow rate and the experimental results through capillary tubes

Figures 3 and 4 present the vapor quality and void fraction along the capillary tube under the critical flow conditions, respectively. Each figure also includes a curve fitted by all the

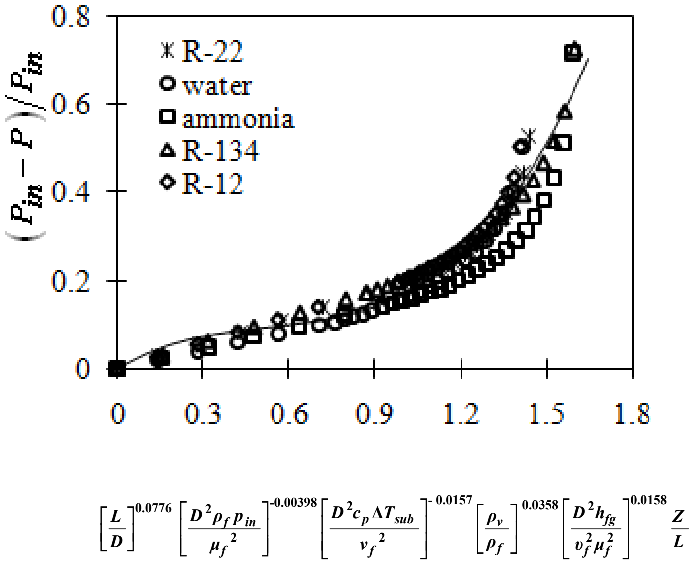

Fig. 2 Prediction of pressure distribution under critical condition through capillary tubes

predicted data by the present model and they are expressed respectively by:

$$x_{cr} = 194\eta^3 \quad (18)$$

Where,

$$\eta = \left[\frac{L}{D}\right]^{-1.033} \left[\frac{D^2 \rho_f \, p_{in}}{\mu_f^{\,2}}\right]^{0.161} \left[\frac{D^2 c_p \Delta T_{sub}}{v_f^{\,2}}\right]^{0.444} \left[\frac{\rho_f}{\rho_v}\right]^{0.397} \times \left[\frac{D^2 h_{fg}}{\upsilon_f^2 \mu_f^2}\right]^{-0.618} \frac{Z}{D}$$

$$\alpha_{cr} = 5.4\eta^{0.9} \quad (19)$$

Where,

$$\eta = \left[\frac{L}{D}\right]^{-1.145} \left[\frac{D^2 \rho_f \, p_{in}}{\mu_f^{\,2}}\right]^{0.326} \left[\frac{D^2 c_p \Delta T_{sub}}{v_f^{\,2}}\right]^{0.468} \left[\frac{\rho_f}{\rho_v}\right]^{0.779} \times \left[\frac{D^2 h_{fg}}{\upsilon_f^2 \mu_f^2}\right]^{-0.771} \frac{Z}{D}$$

In these two figures, all data are clustered close to the fitted curves and they indicate satisfactory results. The average amount of void fraction at the tube length is about $\alpha_{ext} \simeq 0.93$.

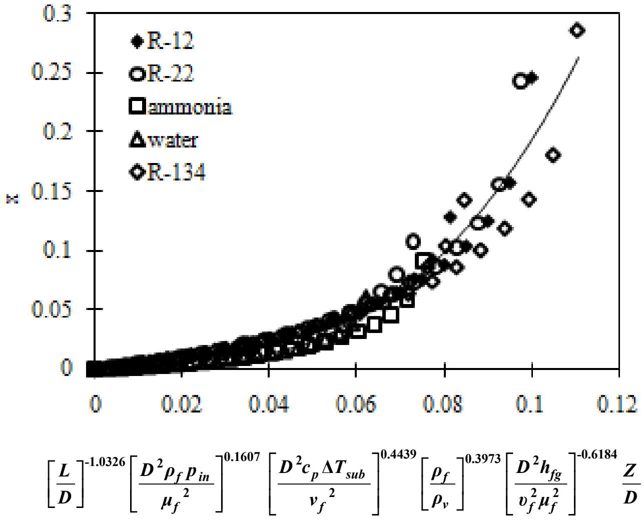

Fig. 3 Prediction of quality distribution under critical condition through capillary tubes

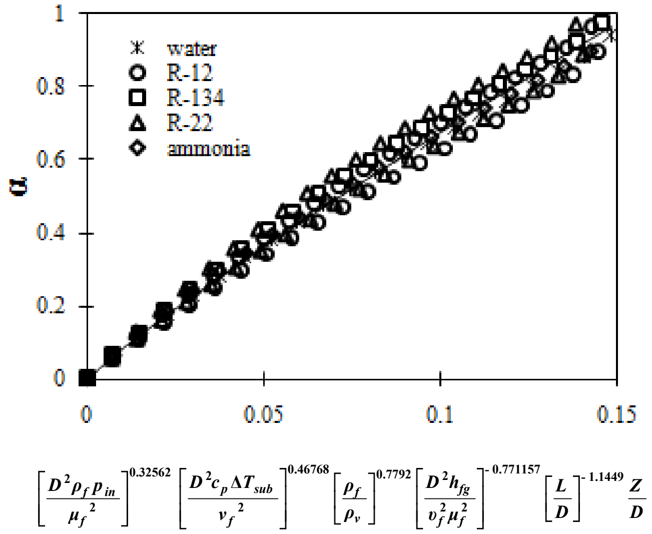

Fig. 4 Prediction of void fraction distribution under critical condition through capillary tubes

Finally, the critical tube length is also obtained from the following expression.

$$\frac{L_{cr}}{D} = 0.14 \left[\frac{D^2 \rho_f \, p_{in}}{\mu_f^2}\right]^{-0.33} \left[\frac{D^2 C_p \Delta T_{sub}}{v_f^{\,2}}\right]^{-0.47} \left[\frac{\rho_f}{\rho_v}\right]^{-0.78} \times \left[\frac{D^2 h_{fg}}{\upsilon_f^2 \mu_f^2}\right]^{0.77} \left[\frac{L}{D}\right]^{1.15} \quad (20)$$

CONCLUSIONS

In this work a numerical method based on the drift flux model was presented to predict the flow variables through an adiabatic capillary tube under the critical conditions for water, ammonia, R12, R-22 and R-134 as working fluids. Using dimensional analysis by Buckingham's π theory, some generalized correlations were proposed for the mass flow rate, pressure, quality and void fraction as functions of flow properties and tube sizes. Comparison between the results of the present work and some experimental data show a good agreement.

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APPENDIX A

$$a_1 = \rho_v \alpha + \rho_f (1-\alpha)$$

$$b_1 = \rho_v u_v - \rho_f u_f - \frac{2.61\ \rho_f^{0.5}}{1-\alpha}\left[\sigma(\rho_f - \rho_v)\right]^{0.25}$$

$$c_1 = u_v \alpha \frac{d\rho_v}{dp} + 0.6525\ \frac{\sigma^{0.25}\ \rho_f^{0.5}}{(\rho_f - \rho_v)^{0.75}} \times \frac{d\rho_v}{dp}$$

$$d_1 = 0 \qquad \text{(A1)}$$

$$a_2 = 2\alpha\rho_v u_v + 2(1-\alpha)\rho_f u_f$$

$$b_2 = \rho_v u_v^2 - \rho_f u_f^2 - \frac{5.22\ u_f}{(1-\alpha)\rho_f^{0.5}}\left[\sigma(\rho_f - \rho_v)\right]^{0.25}$$

$$c_2 = 1 + \alpha u_v^2 \frac{d\rho_v}{dp} + 1.305 \frac{u_f \rho_f^{0.5}\sigma^{0.25}}{(\rho_f - \rho_v)^{0.75}} \times \frac{d\rho_v}{dp}$$

$$d_2 = F_w \qquad \text{(A2)}$$

$$a_3 = \alpha\rho_v h_v + \frac{3}{2}\alpha\rho_v u_v^2 + (1-\alpha)\rho_f h_f + \frac{3}{2}(1-\alpha)\rho_f u_f^2$$

$$b_3 = \rho_v u_v \left[h_v + \frac{u_v^2}{2}\right] - \rho_f u_f \left[h_f - \rho_f \frac{u_f^2}{2}\right]$$

$$-2.61\frac{\rho_f^{0.25}}{1-\alpha}\left[\sigma(\rho_f - \rho_v)\right]^{0.25}\left(h_f + \frac{3}{2}u_f^2\right)$$

$$c_3 = \alpha u_v h_v \frac{d\rho_v}{dp} + \alpha u_v \rho_v \frac{dh_v}{dp} + \alpha \frac{u_v^3}{2}\frac{d\rho_v}{dp} + (1-\alpha)\rho_f u_f \frac{dh_f}{dp}$$

$$+0.6525\frac{\rho_f^{0.25}\sigma^{0.25}\left(h_f + \frac{3}{2}u_f^2\right)}{(\rho_f - \rho_v)^{0.75}} \times \frac{d\rho_v}{dP}$$

$$d_3 = 0 \qquad \text{(A3)}$$