Proceedings of the ASME 2010 3rd Joint US-European Fluids Engineering Summer Meeting and 8th International Conference on Nanochannels, Microchannels, and Minichannels
FEDSM-ICNMM2010
August 1-5, 2010, Montreal, Canada

FEDSM-ICNMM2010-30243

FLOW INSTABILITIES AND HEAT TRANSFER IN BUOYANCY DRIVEN FLOWS OF INELASTIC NON-NEWTONIAN FLUIDS IN INCLINED RECTANGULAR CAVITIES

Dennis Siginer
Mechanical Engineering & Mathematics
Petroleum Institute
Abu Dhabi, UAE
dsiginer@pi.ac.ae

Lyes Khezzar
Department of Mechanical Engineering
Petroleum Institute
Abu Dhabi, UAE
lkhezzar@pi.ac.ae

ABSTRACT

Steady two-dimensional natural convection in rectangular two dimensional cavities filled with non-Newtonian power law-Boussinesq fluids is numerically investigated. The conservation equations of mass, momentum and energy are solved using the finite volume method for varying inclination angles between 0° and 90° and two cavity height based Rayleigh numbers, $Ra=10^4$ and 10^5, a Prandtl number of $Pr = 10^2$ and two cavity aspect ratios of 1, 4. For the vertical inclination of 90°, computations were performed for two Rayleigh numbers $Ra=10^4$ and $10^5$and three Prandtl numbers of $Pr = 10^2$, 10^3 and 10^4. In all of the numerical experiments, the channel is heated from below and cooled from the top with insulated side-walls and the inclination angle is varied. A comprehensive comparison between the Newtonian and the non-Newtonian cases is presented based on the dependence of the average Nusselt number Nu on the angle of inclination together with the Rayleigh number, Prandtl number, power law index n and aspect ratio dependent flow configurations which undergo several exchange of stability as the angle of inclination $\emptyset$ is gradually increased from the horizontal resulting in a rather sudden drop in the heat transfer rate triggered by the last loss of stability and transition to a single cell configuration. Despite significant differences in the heat transfer rate and flow configurations both Newtonian and non-Newtonian fluids of the power law type exhibit qualitatively similar behavior.

INTRODUCTION

Buoyancy driven Newtonian and non-Newtonian flows in rectangular enclosures are found in a variety of engineering applications such as oil-drilling, pulp paper, slurry transport, food processing and polymer engineering. Pseudoplastic fluids are a used in compact heat exchangers or electronic modules as an enhancing cooling medium. For differentially heated two-dimensional enclosures with adiabatic side walls, the heat transfer characteristics are influenced by the inclination of the cavity with respect to the horizontal plane, Prandtl number, and the Rayleigh number based on the height of the cavity. Although the case of Newtonian liquid has received considerable attention (see Gebhart et al. [1], Ostrach [2] and Khalifa [3] for review), there is only a limited number of articles dealing with the non-Newtonian case. For the Newtonian case, flows in such configurations have been the subject of several experimental and numerical studies. Thus Catton et al. [4] and Arnold et al. [5] investigated experimentally and numerically heat transfer in inclined cavities for a range of aspect ratios, Rayleigh numbers and angles of inclination. Ozoe and Sayama [6] and Ozoe et al [7-8] experimentally investigated and numerically computed values of the Nusselt number for natural convection heat transfer in square and rectangular channels. They note the existence of several modes of two-dimensional roll cells in the flow field as the angle of inclination is gradually increased from the horizontal position. The angle of transition between modes depended upon the value of the Rayleigh number and the aspect ratio with the Nusselt number showing a discontinuous behavior. The transition of flow modes was also studied by Soong et al. [9] who noted the influence of initial conditions on the flow pattern formation. Corcione [10], , considered the effect of bi-directional differential heating in horizontal cavities of several aspect ratios for Rayleigh number between 10^3 and 10^6 and found that the increase in the number of roll cells occurring as the aspect ratio increased may be explained through the progressive breakdown of the density stratification in the fluid layers adjacent to the top and bottom walls that bring to the formation of hot and cold fluid streams moving upward and downward across the cavity with direct effect on the temperature distribution. Both Ozoe et al. and Soong et al. [8-9] and Wang and Hamed [11] have demonstrated flow mode transition and hysteresis phenomenon for a Rayleigh number greater than 3000. Wang and Hamed [11] for a range of Rayleigh numbers up to 10^4 conducted a systematic numerical study of the variation of the Nusselt number with angle of inclination and concluded that these types of flows could have dual or multiple solutions due to the effect of initial conditions. Khezzar et al [12] conducted a comprehensive numerical study of natural convection in inclined two dimensional cavities for Rayleigh numbers of 10^4, 10^5 and 10^6 of aspect ratios of 1, 3, 6, 12. They confirm the dependence of the solution on the initial conditions and clarify further the successive loss of stability with increasing angle of inclination from the horizontal, each successive loss of stability leading to flow configurations with smaller number of vortices with the final bifurcation leading to a single cell configuration. The last transition from a multiple cell to a single cell configuration is also associated with a discontinuity in the Nusselt number. The size of the jump in the value of the Nusselt number is dependent on the aspect ratio and the Rayleigh number. The transition to a single cell configuration occurs at gradually larger angle of inclinations with growing aspect ratios. The size of the discontinuity with growing Rayleigh numbers seems to be getting smaller for aspect ratios smaller than 12, however for aspect ratio 12 the size of the discontinuity seems to increase with increasing Rayleigh numbers.

When it comes to non-Newtonian liquids there are only a few references in the literature. It appears that the numerical study by Ozoe and Churchill [13] aimed at determining the threshold for the onset of Rayleigh-Bernard convection in power law fluids was one of the first in the field. The critical Rayleigh number was found to increase with the power index. However s the results showed a tendency to give exaggerated values when compared to the experimental and theoretical data reported by Tien et al. [14]. More recently, Kim et al. [15] considered transient buoyant convection in a square cavity subjected to hot and cold temperature on the vertical side walls for Newtonian and non-Newtonian power law fluids of the Ostwald-De Waele type . The study concluded that for high Rayleigh Ra= 10^5-10^7 and Prandtl numbers Pr=10^2-10^4 as the power law index n decreases, the convective activity is intensified with decreasing power law index n resulting in enhanced overall heat transfer coefficients. Ohta et al [16] studied s numerically transient heat transfer in a square cavity heated from the bottom and cooled from the top using the Sutherby model for shear thinning fluids, such as aqueous solutions of Natrosol 250H hydroxyethyl cellulose and found that shear thinning resulted in larger heat transfer rates than Newtonian fluids. Their study revealed as well that for highly peudoplastic fluids and for a large Rayleigh number equal to 10^5 complex flow patterns consisting of unstable multiple roll-cells are generated leading to an oscillating Nu with time.

Thermal convection of micro-emulsion slurry, which exhibits non-Newtonian power law characteristics, was studied numerically and experimentally by Inaba et al. [17] in rectangular cavities. They found that heat transfer rate is increased with the introduction of shear-thinning. Flow and heat transfer in a shallow rectangular enclosure filled with non-Newtonian Ostwald-De Waele liquids heated from the side under a constant heat flux assumption is studied analytically and numerically by Lamsaadi et al. [18]. They determined that if the aspect ratio and Prandtl numbers are large enough (>100) then the flow and heat transfer rate characteristics become independent of any increase in these parameters and the flow is essentially controlled by the Rayleigh number and the power-law index.

In summary it seems that no study is available on thermal convection of non-Newtonian power law fluids in two-dimensional tilted enclosures heated from below (and cooled from above) under a constant wall temperature assumption. The main goal of this article is to fill this gap and study the effect of shear-thinning and shear-thickening on heat transfer rate in such a configuration using the power-law model of Ostwald-De Waele fluids against a Newtonian fluid with a high Prandtl number. The steady state numerical solution is obtained for a range of cavity aspect ratios, Rayleigh Ra and Prandtl Pr numbers and power law indices n.

MATHEMATICAL FORMULATION

A two dimensional rectangular cavity filled with a non-Newtonian fluid is considered. The inclination angle of the cavity ϕ varies between $0^\circ \leq \phi \leq 90^\circ$. The aspect ratio is AR=L/H, the ratio of the length L of the isothermal walls to the length H of the adiabatic walls. The top (cold) and the bottom (hot) surfaces of the cavity are maintained at constant temperatures T_c and T_h, while the two side walls are kept adiabatic as shown in Figure 1.

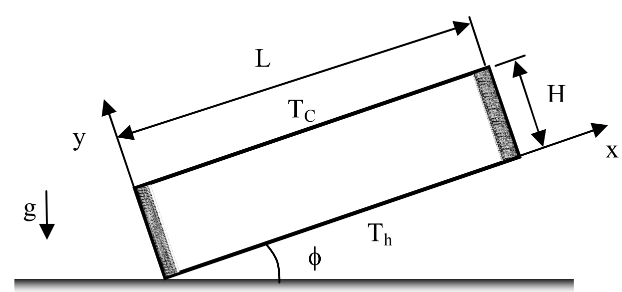


Fig. 1. Flow configuration.

Flow in the cavity is assumed laminar, steady and two-dimensional. The Boussinesq assumption is used and viscous dissipation is assumed to be negligible. The buoyancy force is caused only by the density gradient, thus:

$$\frac{\rho}{\rho_0} = 1 - \beta\left(T - T_0\right) \qquad (1)$$

where β is the coefficient of thermal expansion, ρ is the fluid density at temperature T and ρ_0, T_0 are the corresponding reference values, respectively. The field conservation equations of mass, momentum and energy are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\frac{1}{\rho_0}\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}\right)+g\beta\left(T-T_0\right)\boldsymbol{sin\,\phi}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_0}\frac{\partial p}{\partial y}+\frac{1}{\rho_0}\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}\right)+g\beta\left(T-T_0\right)\boldsymbol{cos\,\phi} \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\kappa\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Where the velocity vector is expressed in terms of its Cartesian components (u,v) along the x and y directions of the coordinate system shown in Fig. 1; p, ν, κ and g represent the pressure, the kinematic viscosity, the thermal diffusivity and the acceleration of gravity, respectively.

The fluid obeys the non-Newtonian power law given by:

$$\tau_{ij}=\mu_a D_{ij}=Ke^{T_0/T}\left(\frac{1}{2}D_{kl}D_{kl}\right)^{\frac{(n-1)}{2}}D_{ij} \tag{5}$$

where the rate of strain is given by $D_{ij}=\left(\partial u_i/\partial x_j+\partial u_j/\partial x_i\right)$ and K and n are the consistency and the power-law indices respectively. n<1 defines shear-thinning, n=1 and n>1 correspond to the Newtonian and to shear-thickening fluids, respectively. The introduction of a physical quantity with dimensions of $(\text{length})^2(\text{time})^{-1}$ to play a role analogous to the kinematic viscosity of a Newtonian fluid is required to facilitate the interpretation of results in conjunction with tools that are effective with Newtonian fluids. An effective kinematic viscosity has been adopted in previous studies on the free convection about a flat plate [19-20] and in a porous cavity [21]. In line with these efforts the following expression for the effective kinematic viscosity is introduced:

$$\nu'=\left(\frac{K}{\rho_0}\right)^{\frac{1}{(2-n)}}H^{\frac{2(1-n)}{(2-n)}} \tag{6}$$

Substituting (6) into the conventional expressions for the Prandtl and Rayleigh numbers the corresponding dimensionless numbers for non-Newtonian power-law fluids are defined as:

$$Pr=\frac{\left(\frac{K}{\rho_0}\right)^{\frac{1}{(2-n)}}H^{\frac{2(1-n)}{(2-n)}}}{\kappa} \tag{7}$$

$$Ra=\frac{g\beta\Delta TH^3}{\kappa\left(\frac{K}{\rho_0}\right)^{\frac{1}{(2-n)}}H^{\frac{2(1-n)}{(2-n)}}} \tag{8}$$

The average Nusselt number on the conducting walls is defined as follows:

$$Nu=\frac{1}{A}\int_0^A\left(\frac{\partial T}{\partial y}\right)_{wall}dx \tag{9}$$

NUMERICAL METHOD

The set of equations (2-4) is solved numerically using the finite volume technique. The Simple algorithm, the Quick scheme and PRESTO technique were used for the velocity-pressure coupling, convective terms discretization, and pressure interpolation respectively. Convergence was assumed when the normalized residuals reached a value of $5\text{x}10^{-5}$, 10^{-5} and 10^{-6} in monitoring the mass residuals, momentum and energy equations, respectively. All calculations were performed in double precision mode.

Although the accuracy of the method and the dependence of the accuracy of the method on grid size were studied extensively in [22] for the Newtonian case, a set of tests was performed to investigate the effect of the power law index on grid sensitivity. It was found that when the power law index is of order $\boldsymbol{O}(1)$ the conclusions of [22] still holds while for strongly non-Newtonian cases, that is when the power law index n deviates significantly from 1, the solution is more sensitive to grid size. After some tests, it was determined that a uniformly spaced grid with N_x=100, N_y=100 provides a reasonable accuracy for the geometry with AR=1 while grids with N_x=280, N_y=70 and N_x=600, N_y=70 are suitable for AR=4 and AR=8 respectively. In addition, the results of [15] for a square cavity were used to validate the calculation method and approach. This test case consisted of calculations for a square geometry at an angle of inclination of 90˚ with $Ra=10^5$ and three Prandtl numbers Pr = 10^2, 10^3, 10^4. The dependence of the relative Nusselt number on power law index is presented in Figure 2. The results are in full agreement with those of [15] with an excellent accuracy of less than 1%. Since the problem is rather sensitive to the initial field guess [23], a procedure similar to the one used in [9] is applied. For calculations where the angle ϕ increases from 0˚to 90˚ the equations for a horizontal enclosure are first solved. For $\phi=0$ the initial field is set at zero for the flow variables and T_c for the temperature. The solution at zero inclination is then used as the initial guess at 5˚ inclination. The latter solution is subsequently used as an initial guess for 10˚ and so on until 90˚. The temperature difference in all calculations was held at 10˚.

RESULTS AND DISCUSSION

The results are discussed by considering first the vertically oriented cavity for two aspect ratios AR = 1, 4, two Rayleigh numbers $Ra = 10^4$ and 10^5 and three Prandtl numbers $Pr = 10^2$, 10^3 and 10^4. The discussion is then extended to the detailed calculations of the inclined cavities for three aspect ratios 1, 4 and 8 and Rayleigh numbers of 10^4 and 10^5, and a single Prandtl number of 100. Prandtl numbers for most non-Newtonian fluids are quite large. The Rayleigh and Prandtl number values chosen are representative of practical applications (see [23]). The power law fluids considered include shear thinning, shear thickening and Newtonian with a power index varying between $0.6 \le n \le 1.4$.

Figs. 2-3 illustrate the variation of the Nusselt number Nu_n with the power law exponent n. Nu_n for each exponent n is normalized by the corresponding value of Nu for a Newtonian fluid (n=1.0) when the angle of inclination is 90˚ for two aspect ratios AR = 1 and 4, the two Rayleigh numbers $Ra = 10^4$ and 10^5 and the effect of the Prandtl number is explored by considering Pr variation over three decades $Pr = 10^2$, 10^3 and 10^4. Compared to the case of heat transfer with Newtonian fluids shear thinning clearly enhances heat transfer when the Rayleigh number is set at $Ra=10^5$ and Prandtl numbers are in the range of $Pr=10^2$ and 10^3, whereas it reduces heat transfer for $Pr=10^4$ as was observed previously by Kim et al. [15], and the opposite is true for shear thickening. For a Rayleigh number an order of magnitude smaller $Ra=10^4$, the Nusselt number variation shows that relative to a Newtonian fluid, shear thinning reduces heat transfer with Prandtl number in the range of $Pr=10^3$ and 10^4 and increases the heat transfer rate for Prandtl number of lower orders of magnitude $Pr \le 10^2$ and the opposite is true for shear thickening fluids. The Nusselt number ratio Nu_n / $Nu_{n=1}$ is a function of the aspect ratio as well. With increasing aspect ratio the increase or the decrease in Nu_n / $Nu_{n=1}$ is enhanced regardless whether the fluid is shear-thinning or thickening all other parameters kept constant. It appears that there is a competing effect between momentum and heat transfer at high Pr and a complex relationship between the effects of Ra and Pr at this aspect ratio. Nonetheless as the Prandtl number becomes large, it tends to inhibit the convective activities that are characteristic of low Prandtl number shear thinning fluids and counteracts the effects of shear thickening by increasing the convective heat transfer in comparison to the Newtonian case.

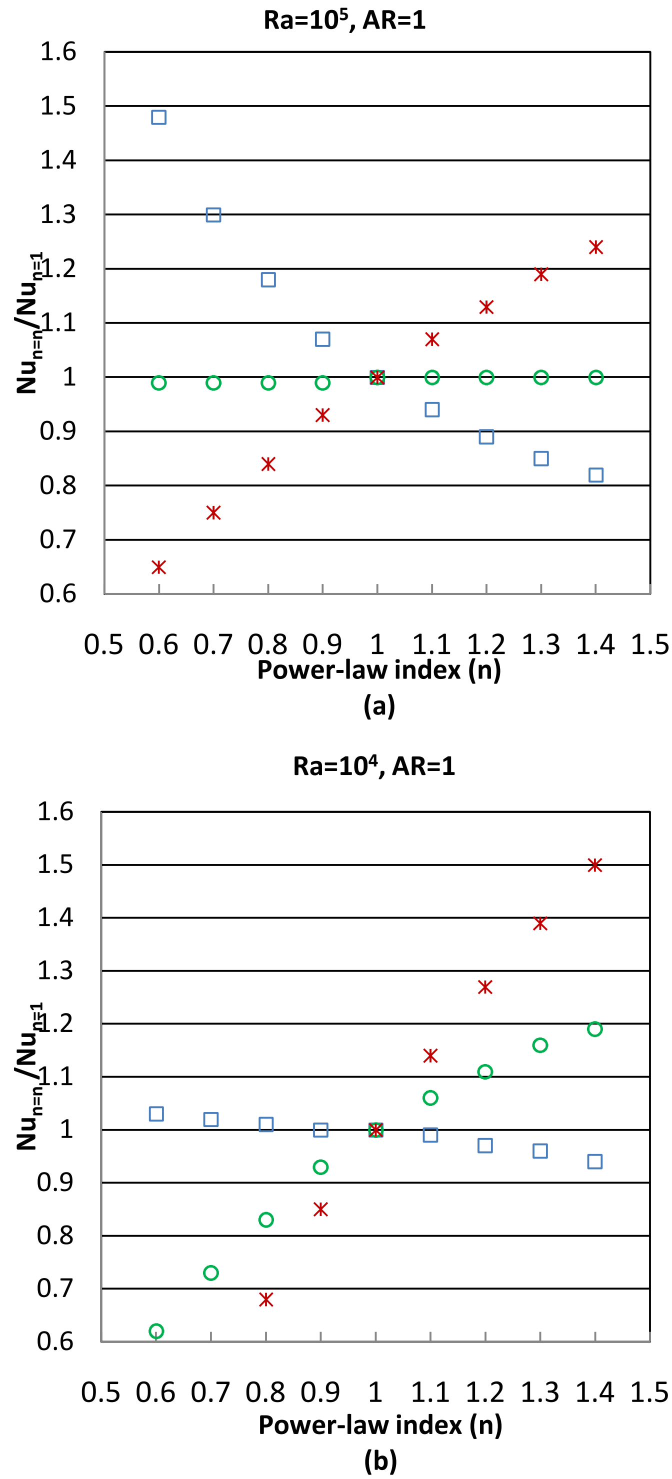


Fig.2. Dependence of the Nusselt number Nu ratio on the power-law index n for a square cavity for three Prandtl numbers □ $Pr=10^2$ ○ $Pr=10^3$ * $Pr=10^4$ (a) $Ra=10^5$, (b) $Ra=10^4$

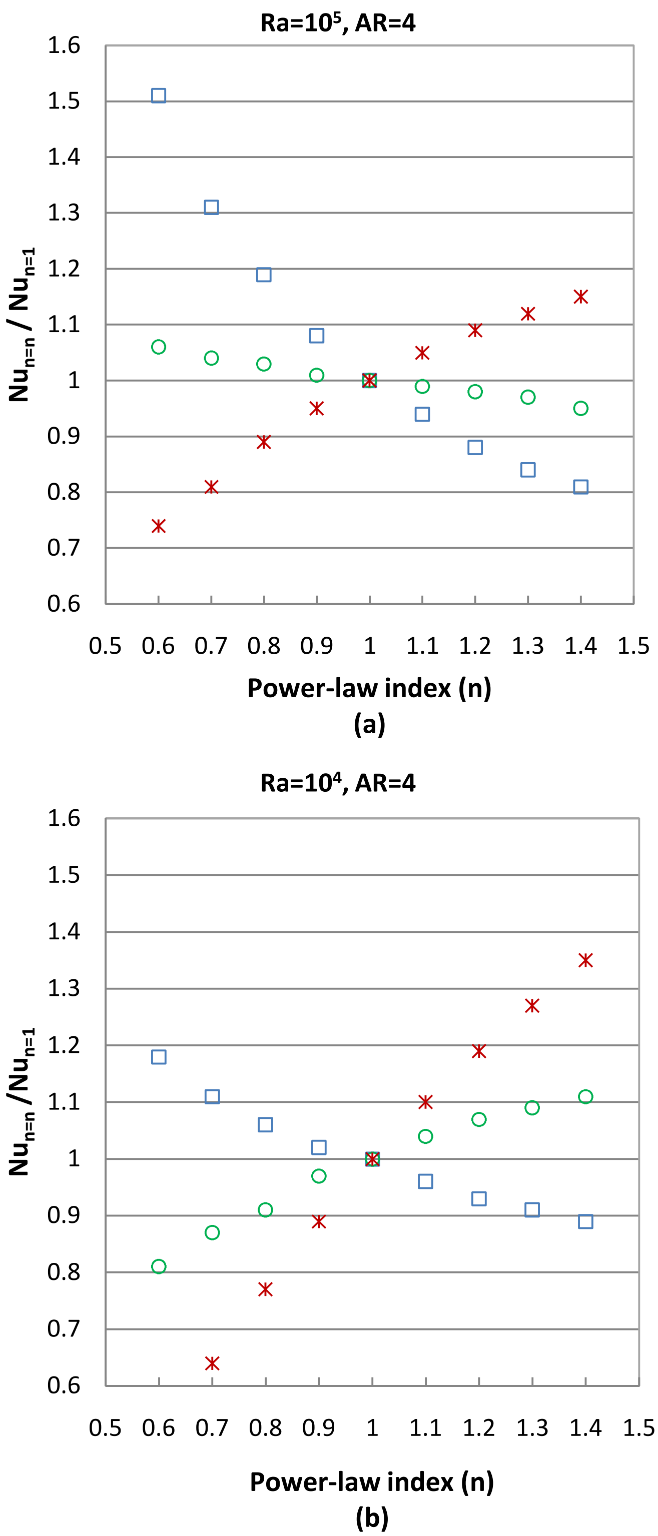
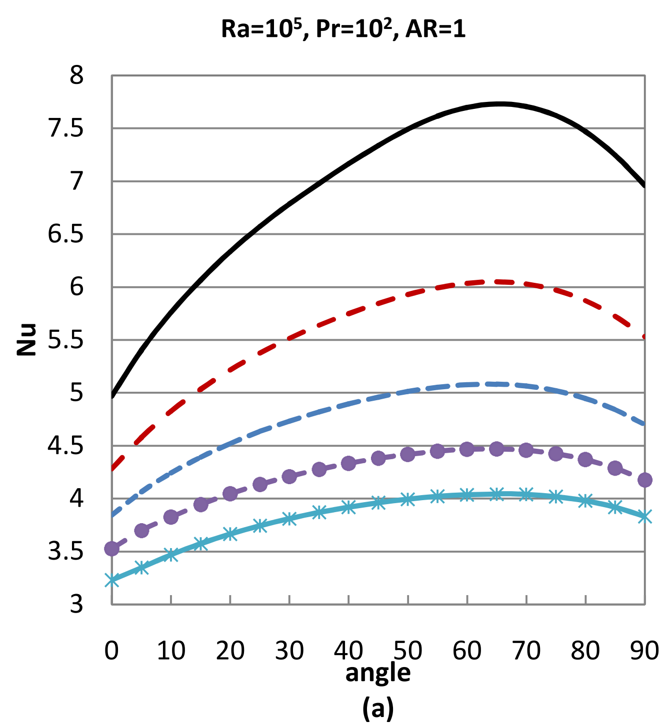


Fig.3. Dependence of the Nusselt number *Nu* ratio on the power-law index *n* for a cavity of aspect ratio four and for three Prandtl numbers □ $Pr=10^2$ ○ $Pr=10^3$ * $Pr=10^4$ (a) $Ra=10^5$, (b) $Ra=10^4$

Fig. 4 shows the variation of the average Nusselt number on the conducting walls with angle of inclination for aspect ratio AR=1 and Rayleigh numbers over two orders of magnitude $Ra=10^4$ and 10^5 with the Prandtl number set at $Pr=10^2$. The variation of *Nu* is continuous exhibiting lower values of *Nu* than the Newtonian case with shear thickening fluids and higher values with shear thinning fluids. Increasing the Rayleigh number *Ra* enhances the heat transfer rate *Nu* for all fluid types. The enhancement in *Nu* is more pronounced for shear thinning fluids. The higher the shear-thinning capability the higher the enhancement and the higher the shear-thickening characteristic the lower is the enhancement. In all cases the heat transfer rate *Nu* in the horizontal 0˚ position is substantially lower than the enhancement *Nu* in the vertical 90˚ position with the vertical walls conducting. Maximum values of *Nu* for $Ra=10^4$ occurs between 50˚-65˚ whereas for $Ra=10^5$ the maximum is located in the interval 65˚-70˚ leading to the tentative conclusion that the higher the *Ra* number the later the maximum heat transfer occurs as the angle of inclination is gradually increased from the horizontal. At the higher Rayleigh number $Ra=10^5$ all the maxima seems to occur at about the same inclination, but at an order of magnitude lower $Ra=10^4$ the maxima with shear thickening fluids is shifted to higher inclinations that the shear-thinning fluids and the higher the shear-thickening index the higher the inclination at which the maximum occurs. The heat transfer rate *Nu* variation for all cases investigated with the aspect ratio AR=1 is smooth. This is due to the single cell structure of the flow field at all angles of inclination for Newtonian as well as shear-thinning and shear-thickening fluids when the aspect ratio is one.



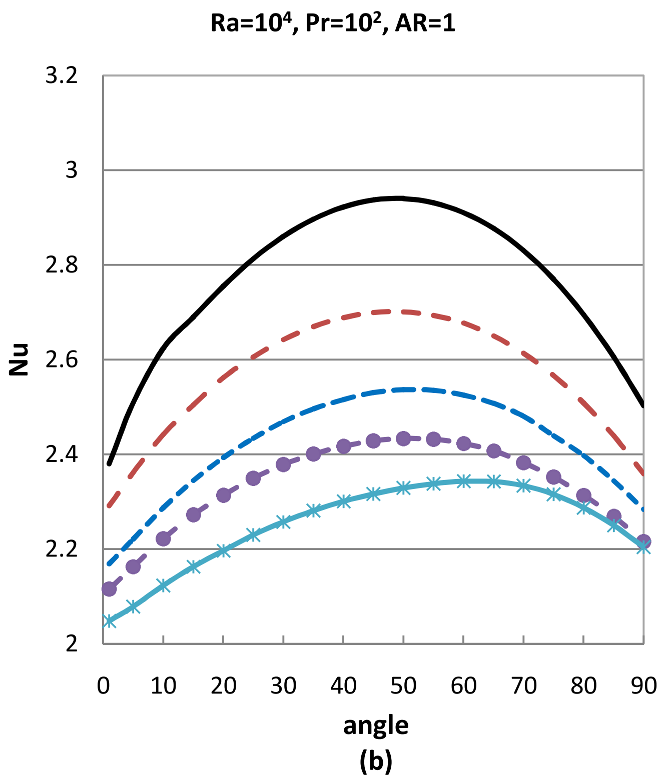
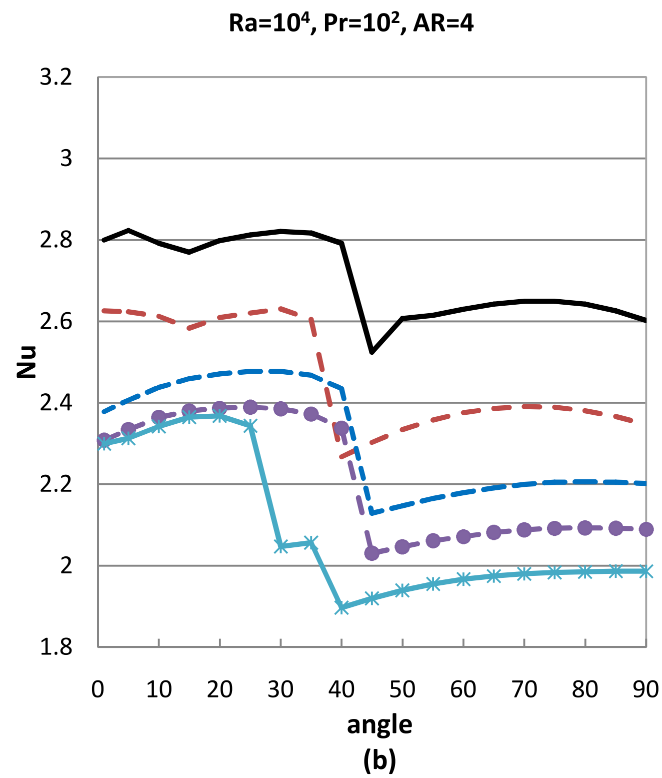


Fig. 4. Average Nusselt number *Nu* variation with the angle of inclination ∅ for a square cavity $Pr=10^2$, AR=1: (a) $Ra=10^5$, (b) $Ra=10^4$.

The variation of *Nu* with the angle of inclination for aspect ratio AR=4 is shown in Figs. 5 for two values of the Rayleigh number which differ by an order of magnitude $Ra=10^5$ and 10^4 with the Prandtl number set at a constant $Pr=10^2$. For either aspect ratio the heat transfer rate is higher for shear-thinning fluids. Similar to AR=1 the higher the power index the lower the *Nu* and the lower the power index the higher is the *Nu*. In all cases the absolute maxima for shear-thickening fluids occurs closer to the origin and is located at about 20˚ or close to 20˚ from below. At the lower magnitude Rayleigh number $Ra=10^4$ the maxima of *Nu* is closer to the horizontal position 0˚ than the shear thickening case. However with increasing Rayleigh number it moves closer to 90˚, and proves to be a function of the aspect ratio as well. At the larger Rayleigh number $Ra=10^5$ the maximum heat transfer rate *Nu* with shear-thinning fluids occurs at about 75˚ when AR=4. However figures not included here show that when the aspect ratio is doubled to AR=8 the maximum occurs much earlier at about 35˚ right before the discontinuity in *Nu*. At the lower Rayleigh $Ra=10^4$ and the higher aspect ratio AR=8 the maxima for the shear thinning fluids all move to the horizontal position of the enclosure and decrease thereafter albeit with discontinuities. For both aspect ratios and at either Rayleigh number the heat transfer rate *Nu* shows discontinuities, in fact two discontinuities, a steep decrease followed by a monotonic increase. The first



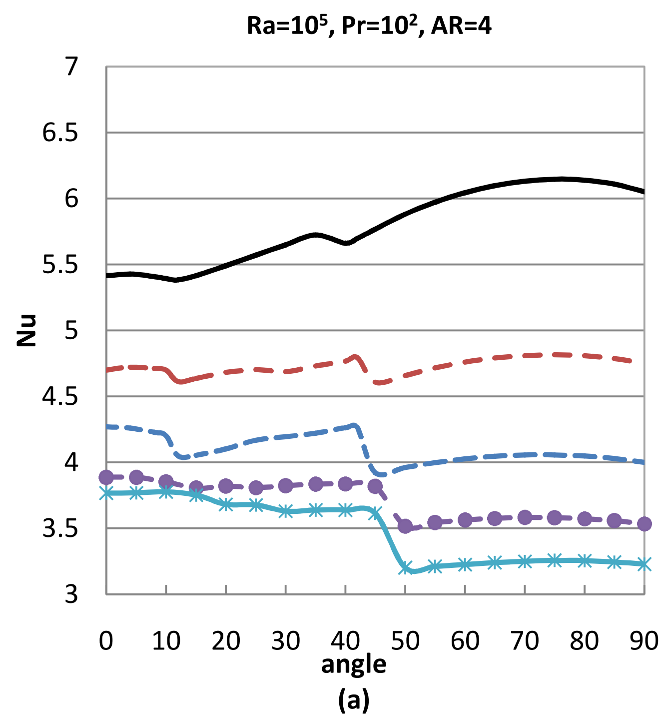


Fig. 5. Average Nusselt number *Nu* variation with the angle of inclination ∅ for a cavity with aspect ratio four $Pr=10^2$, AR=4: (a) $Ra=10^5$, (b) $Ra=10^4$.

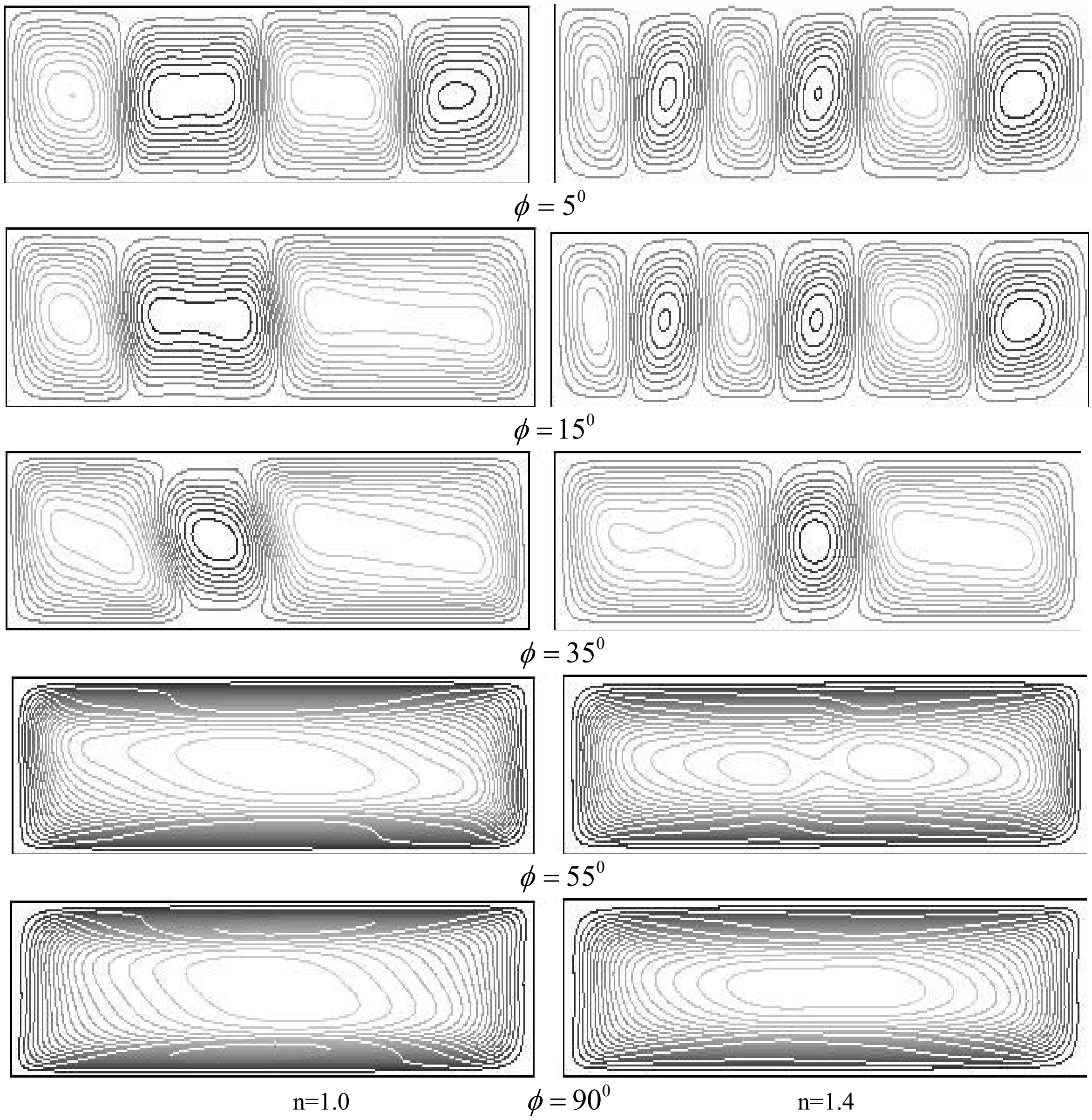


Figure 6. Comparison of stream function contours, *Ra*=10^5, *Pr*=100 and AR=4, *n*=1 and n=1.4.

discontinuity, which occurs at about 10˚, is more pronounced for the Newtonian and shear-thinning cases in comparison with the shear-thickening case, in fact it is completely smoothed over when the fluid is shear-thickening, and it is smaller in size than the second discontinuity in all cases. The second discontinuity occurs much later as the angle of inclination is increased and is located in the interval 35˚< ∅ <50˚ in all cases. The steep decreases in *Nu* stem from flow mode changes. Each loss of stability resulting in a decreased number of cells as the angle of inclination is increased also generates a steep decrease in *Nu*. The smaller the number of roll cells before the loss of stability the larger the decrease in *Nu*. Thus the second discontinuity occurring at a larger angle of inclination is larger in size compared to the first because it is associated with the transition to a single cell flow configuration at any aspect ratio except the square enclosure. In general shear thickening tends to reduce the heat transfer in comparison with the Newtonian fluid and the opposite is true for shear thinning, whereas heat transfer is augmented with increasing *Ra* in all cases.

Flow configuration consists of one cell structure throughout the inclination angle variation from 0˚ to 90˚ when the aspect ratio is one. There is no loss and exchange of stability and therefore no discontinuities exist in the heat transfer rate *Nu* variation at this aspect ratio. The Nusselt number is strongly

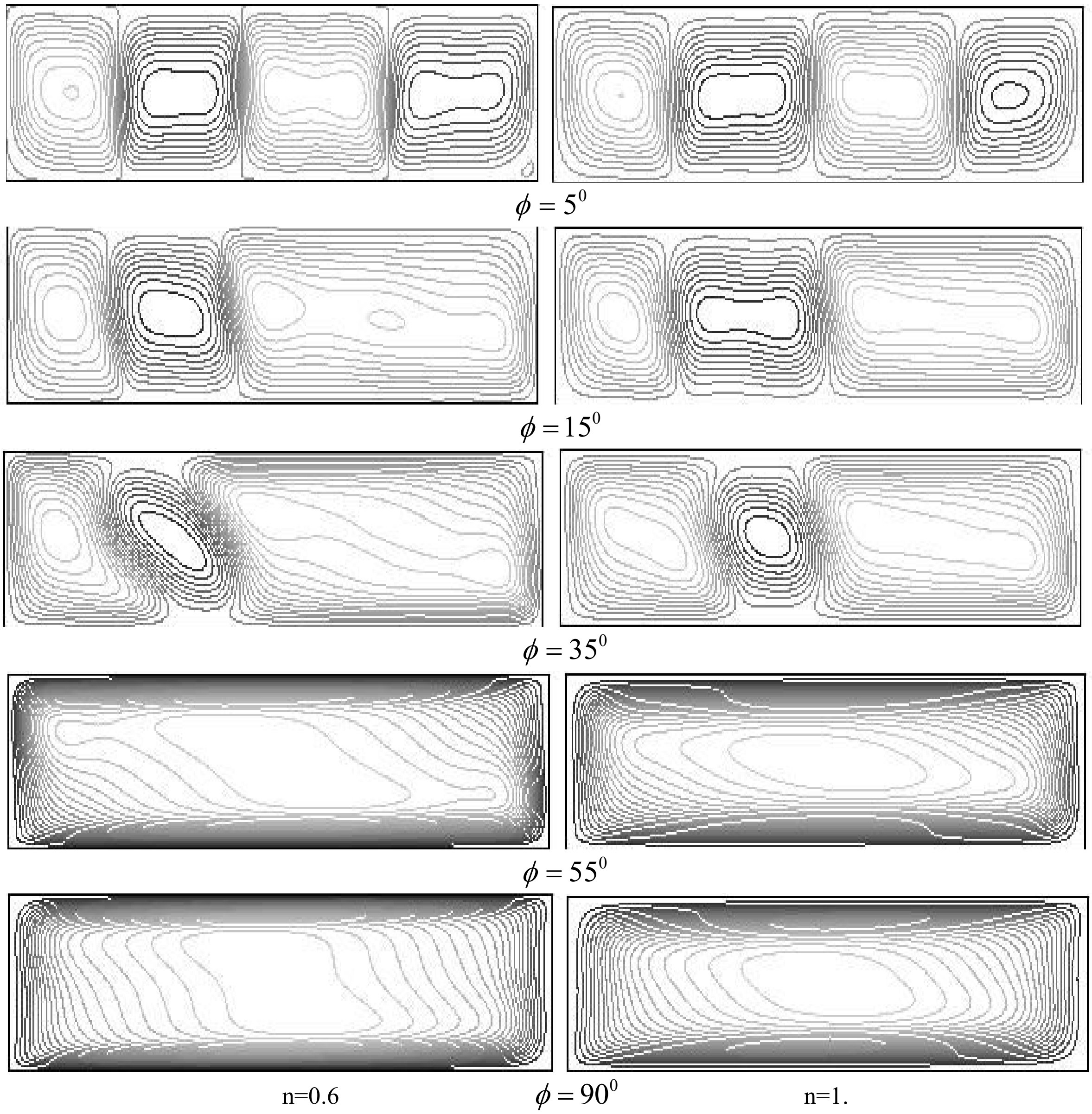


Figure 7. Comparison of stream function contours, $Ra=10^5$, $Pr=100$ and AR=4, $n=0.6$ and n=1.

dependent on the power-law index at a fixed angle.

Figs. 6-7 illustrate contours of the stream function for the aspect ratio AR=4 as the angle of inclination is gradually increased from the horizontal in the first quadrant. The number of cells in the multiple cell flow structure in the vicinity of the horizontal depends on the power-law index. For indices $n=1-\delta < 1$, $0< \delta <1$ that is for shear-thinning fluids the number of roll cells in the horizontal position of the enclosure is smaller than the number of cells for the shear-thickening fluid with index $n=1+ \delta >1$ with $0< \delta <1$. Fig. 6 and 7 depict the roll cells for δ = 0.4. The shear thinning fluid with n = 0.6 and the shear thickening fluid with n = 1.4 display four and six cells, respectively, at 0˚. The shear thinning fluid behaves more like a Newtonian fluid concerning the locations of the loss and exchange of stability. The first loss of stability for the Newtonian fluid occurs at an angle less than 5˚ probably around 3˚~4˚. The shear-thinning fluid with n = 0.6 loses stability in the same neighborhood. However for the shear-thickening fluid the transition takes place much later for an angle of inclination between 5˚ and 15˚, and there seems to be several exchanges of stability within a rather short range of angle of inclination. At $\emptyset$=5˚ the shear thickening fluid with index n = 1.4 has 6 cells and at $\emptyset = 15^\circ$the number of cells are reduced to 3. Although a thorough search has not been conducted it is quite likely that several additional cascading loss and exchange of stability occurred for $5^\circ < \emptyset < 15^\circ$ as the angle is increased reducing the number of roll cells gradually from 6 to 3 within a short interval. Nevertheless this exchange of stability and change in the number of roll cells has almost no effect at all on the heat transfer rate Nu for fluids with $n >$ 1and minimal effect on fluids with $n < 1$, Fig. 5. Shear-thickening appears to have a stabilizing effect as the flow pattern changes take place at relatively low angles of inclination. The final exchange of stability occurs in the transition to a single roll cell configuration. This exchange of stability triggers a sudden drop in the heat transfer rate which is more pronounced at the lower Rayleigh number $Ra = 10^4$. The drop is a function of the aspect ratio as well and is more pronounced at larger aspect ratios at the same Rayleigh number, for instance for AR=4 it is less pronounced than for AR=8 at say $Ra = 10^4$. The more shear-thinning the fluid is the earlier and the more shear thickening the fluid is the later the transition to a single cell configuration occurs as the angle of inclination is increased although the difference in angles is not much. All transitions to a single cell configuration are completed before $\emptyset = 35^\circ$ is reached when n is confined to the interval $0.6 < n < 1.4$. The results also show the stabilizing influence of the Prandtl number Pr on the flow by comparison with the Newtonian fluid of a Prandtl number an order of magnitude lower at Rayleigh numbers of comparable magnitude and similar aspect ratios, [10, 18].

V. CONCLUSIONS

Natural convection of Newtonian and non-Newtonian power law type fluids in two dimensional rectangular tilted enclosures were investigated numerically for angles of inclination in the first quadrant $0^\circ \leq \emptyset \leq 90^\circ$. Flow configurations and heat transfer rates in enclosures of aspect ratios 1, 4 for $Ra=10^4$ and $Ra=10^5$ and $Pr=10^2$ have been examined for the range of the power law index $0.6 \leq n \leq 1.4$ representative of a substantial spectrum of shear-thinning and shear-thickening fluids. The effect of the Pr is further highlighted by investigating the heat transfer rates as compared to the Newtonian case for three Prandtl numbers that differ by an order of magnitude $Pr = 10^2, 10^3$ and 10^4 and for the aspect ratios 1, 4 in the vertical position $\emptyset = 90^\circ$. Shear-thinning and thickening result in significant increases and decreases, respectively, in the heat transfer rate Nu in comparison to the heat transfer rate of a Newtonian fluid. The increase and decrease in Nu for shear-thinning $n < 1$ and shear-thickening $n > 1$ fluids is Rayleigh number Ra, Prandtl number Pr, aspect ratio and power law index n dependent. The more shear-thinning or the more shear-thickening the fluid the higher and the lower, respectively, is the heat transfer rate Nu at the same Ra and Pr and aspect ratio. Heat transfer rates go up as the Rayleigh number increases with all other parameters kept constant at any angle of inclination. The flow configuration starts out with multiple roll cells at the horizontal position 0° and goes through a number of exchanges of stability each one of which reduces the number of roll cells further ending up with a single cell configuration as the angle of inclination is gradually increased. Shear-thinning fluids start out with the same number of roll cells as the Newtonian fluid at 0° and the shear-thickening fluids with a larger number of cells than the Newtonian fluid in the range of the power law indices studied $0.6 \leq n \leq 1.4$ regardless of the values of the remaining parameters. The transition to a single cell flow structure is delayed as the power index n grows with $n > 1$ and occurs earlier as the power index n gets smaller with $n < 1$, and triggers a steep decrease in the heat transfer rate Nu more pronounced for shear-thickening fluids in particular at the Rayleigh number of the lower magnitude studied $Ra = 10^4$. In general higher Prandtl numbers as well shear-thickening have a stabilizing effect on the flow.

REFERENCES

[1] Gebhart, Y. Jaluria, R.L. Mahajan, and B. Sammakia, Buoyancy Induced Flows and Transport, Hemisphere, Washington, D.C., 1988

[2] S. Ostrach, Natural convection in enclosures, in Harnett, J.P. and Irvine, T.F. (Eds), Advances in Heat Transfer, 8, Academic Press, London, 1972.

[3] A.J.N. Khalifa, Natural convective heat transfer coefficient – a review, II. Surfaces in two- and three-dimensional enclosures, Energy Convers. Manage., Vol. 42, pp. 505-517, 2001.

[4] I. Catton, P.S., Ayyaswamy, and R.M. Clever, Natural convection flow in a finite, rectangular slot arbitrarily oriented with respect to the gravity vector, Int. J. Heat Mass Transfer, vol. 17, pp. 173-184, 1974.

[5] J. N. Arnold, I. Catton and D. K. Edwards, Experimental Investigation of Natural Convection in inclined rectangular regions of differing aspect ratios, J. Heat Transfer, pp. 67-71, 1976.

[6] H. Ozoe, H. Sayama, and S. Churchill, Natural convection in an inclined square channel, Int. J. Heat and Mass Transfer, vol. 17, p. 401-406, 1974.

[7] H. Ozoe, and H. Sayama, 1976, Natural convection in an inclined rectangular channel at various aspect ratios and angles-experimental measurements, Int. J. Heat Mass Transfer, vol. 18, pp. 1425-1431, 1976.

[8] H. Ozoe, H. Sayama, and S. Churchill, Natural circulation in an inclined rectangular channel heated on one side and cooled on the opposite side, Int. J. Heat Mass Transfer, vol. 17, pp. 1201-1217, 1974.

[9] C.Y. Soong, P.Y. Tzeng, D.C. Chiang, and T.S. Sheu, Numerical study on mode-transition of natural convection in differentially heated inclined enclosures, Int. J. Heat Mass Transfer, vol. 39, pp. 2869-2882, 1996.

[10] M. Corcione, Effects of the thermal boundary conditions at the sidewalls upon natural convection in rectangular enclosures heated from below and cooled from above, Int. J. Thermal Sciences, 42, pp. 199-208, 2003.

[11] H. Wang, and M. Hamed, Flow mode-transition of natural convection in inclined rectangular enclosures subjected to bidirectional temperature gradients, Int. J. Thermal Sciences, vol. 45, pp. 782-795, 2006.
[12] L.Khezzar, D. A. Siginer and I. Vinogradov, Natural convection in inclined two dimensional rectangular cavities, Int. J. Heat Mass Transfer, forthcoming.
[13] H. Ozoe and S. Churchill, Hydrodynamic stability and natural convection in Ostwald-De Waele and Ellis fluids: the development of a numerical solution, AIChE J., vol. 18, pp. 1196-1207, 1972.
[14] C. Tien, H. Tsuei, and Z. Sun, Thermal instability of a horizontal layer of non-Newtonian fluid heated from below, Int. J. Heat Mass Transfer, Shorter Commun., vol. 12, pp. 1173-1178, 1969.
[15] G.B. Kim, J.M. Hyun, H.S. Kwak, Transient buoyant convection of a power-law non-Newtonian fluid in an enclosure, Int. J. Heat Mass Transfer, vol. 46, pp. 3605-3617, 2003.
[16] M. Ohta, M. Akiyoshi, and E. Obata, A numerical study on natural convective heat transfer of pseudo-plastic fluids in a square cavity, Numer. Heat Transfer A, vol. 41, pp. 357-372, 2002.
[17] H. Inaba, C. Daib, and A. Horibe, Natural convection heat transfer of micro-emulsion phase-change-material slurry in rectangular cavities heated from below and cooled from above, Int. J. Heat Mass Transfer, vol. 46, pp. 4427-4438, 2003.
[18] M. Lamsaadi, M. Naimi, M. Hasnaoui, and M. Mamou, Natural convection in a tilted rectangular slot containing non-Newtonian power-law fluids and subject to a longitudinal thermal gradient, Numer. Heat Transfer A, vol. 50, pp. 561-583, 2006.
[19] A. Emeri, H. Chi, and J. Dale, Free convection through vertical plane layers of non-Newtonian power law fluids, ASME J. Heat Transfer, vol. 93, pp. 164-171, 1971.
[20] J. Dale, A. Emery, The free convection of heat from a vertical plate to several non-Newtonian pseudoplastic fluids, ASME J. Heat Transfer, vol. 94, pp. 64-72, 1972.
[21] G. Getachew, W. Mincowycz, D. Poulikakos, Natural convection in a porous cavity saturated with a non-Newtonian fluid, J. Thermophys. Heat Transfer, vol. 10, pp. 640-651, 1996.
[22] L. Khezzar, D. Siginer, Natural convection in inclined two-dimensional rectangular cavities, Proceedings of the fluids engineering division summer meeting, FEDSM2009-78566, August 2-5, Vail, Colorado, USA, 2009.
[23] F.F.T. Pittman, J.F. Richardson, and C.P. Sherrard, An experimental study of heat transfer by laminar natural convection between an electrically-heated vertical plate and both Newtonian and non-Newtonian fluids, vol. 42, pp. 657-671, 1999.