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SIMULATION OF TWO-PHASE FLOW USING CONSERVATIVE LEVEL-SET METHOD WITHOUT RE-INITIALIZATION PROCESS

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ABSTRACT

Multiphase flows are generated in several industrial domains. The numerical simulation of such flows need to have an exact tracking of the different phase interfaces. The level set method is one of the simplest methods used to study the moving front of the flow. But it is known that this method generates a non mass conservation, and do not respect the uniformity of the signed distance function. Several corrections are usually proposed to solve these problems when using the Level set method.

In this paper, a novel two steps correction method is proposed in order to guarantee the flow mass conservation and the exact shape of the flow front.

The first step concerns the correction of the mass loss. It consists to add in the transport equation, a penalty or constraint term, built to force the velocity field to satisfy the mass balance or to preserve the conservative property. This term is multiplied to an adjustable penalty factor (β). The second step consists to impose that the isocontours of the level set function (φ) always respect the same distance. With this way, the costly re-initialization procedure is eliminated.

The performance of the method is demonstrated and validated using several cases involving two-phase flow. The numerical experiments show that the accuracy and performances of our method is drastically improved compared to other methods. The approach will then applied to track an air-liquid interface in a case of an air bubble moving in a constant volume of liquid. In this case, the classical level set method reveals to be not conservative. A solution is then proposed in order to introduce a correction. To do, Navier-Stokes, continuity and energy equations are coupled to describe the flow and its thermal behavior. A finite element method is used to solve the equations. The solution is also verified by solving the dam-break problem, and bubble rising in water. Good agreements with referenced solutions are demonstrated for all tow investigated problems.

Keywords: Level-Set method, penalty, masse conservation, interface.

1. INTRODUCTION

Multiphase flows are widely encountered in many fields such as polymer engineering, bubble dynamics, sprays, wave mechanics, bioengineering, petrochemical, nuclear and combustion. Developing a method for the simulation of various types of multiphase flows is one of the most important engineering challenges. In case of large density differences across the interface, due to the significant role of the forces between the components, the simulations are very complex.

Several numerical methods have been used to simulate multiphase flows. They are divided into ''front interface tracking" and ''front interface capturing" methods. In Front Tracking Methods, the position of the interface is calculated explicitly, and a deforming mesh is used in accordance with the interface. Boundary integral methods [1] and arbitrary Lagrangian–Eulerian (ALE) methods [2] are categorized as front tracking methods. The advantage of these methods is that high resolution meshes are not mandatory. On the other hand, Front Capturing Methods are much more robust but they need high mesh resolution. In these methods, an auxiliary function is used to identify each component. Volume of fluid methods (VOF) [3], phase field methods and level set methods [4,5] are examples of Interface Capturing Methods. The volume-of-fluid

(VOF) method and a level set (LS) method are popular interface-tracking methods for computing two-phase flows with topologically complex interfaces. The VOF methods consist to track the volume fraction of each phase or component and are based on conservation laws, so they have excellent conservation properties, but encounter difficulties dealing with large topological changes and geometrical complexities. The interface is reconstructed from volume fraction values, and most of VOF interface reconstruction schemes are first-order accurate, it seems to be an inappropriate method for three-dimensional cases [6] or calculating curvature for estimating surface tension forces. The method has been improved by employing a piecewise linear interface calculation (PLIC) method [7-9], in which the advected fractional volume is evaluated geometrically from the reconstructed interface. As another Eulerian method, the Level set methods are not as strong as VOF methods in conservation properties, but they are very robust in modelling sophisticated interfaces and rapid changes in topology, such as breaking-down or coalescence [10-13], that cannot be handled by a standard front tracking method. It was developed by Sussman et al. [5] for incompressible two-phase flows with large density ratios. The Principe of the level set consists to track the interface by the level set function defined as a signed distance from the interface. Since the level set function is smooth and continuous, its spatial derivatives can be accurately discretized to compute the interface curvature and solve the transport equation. But, the discretization of the level set formulation is found not to preserve volume conservation. Subsequently, Sussman and Fatemi [14] improved the level set formulation by introducing a new constraint for the volume conservation. However, while calculating a rising gas bubble, the improved formulation did not guarantee the volume conservation, and the rise velocity could not attain a steady state. Chang et al. [15] proposed a different procedure to ensure the mass conservation.

They defined the curvature and the unit normal vector as the intrinsic interfacial physical properties, defined on the level set function zero. By changing the surface integration to a volume integral, they generalized this property to the whole volume. The correction takes more time calculation, because the procedure needs more iterations. Russo and Smereka [16] presented a new method for reconstructing (or reinitializing) the Level Set function to respect the signed distance function. Their method, based on a truly upwind discretization away from the interface, was proved to significantly reduce the movement of the interface during the reconstruction procedure of the Level Set function, but its application to two phase's flows has not yet been reported in the literature. As another approach to overcome the volume non-conservation problem of the Level Set method, a coupled level set and volume-of-fluid (CLSVOF) method is developed in [17–19]. In the CLSVOF method, the smooth Level Set function, used for evaluating the interface normal vector and curvature, is corrected (or reinitialized) for volume conservation by the piecewise linear interface, which is reconstructed from the volume-conservative VOF function. As a result, the coupled method not only can calculate an interfacial curvature more accurately than the VOF method but also achieve volume conservation well. However, compared to the Level Set method formulated algebraically, the CLSVOF method requiring geometric calculations for reinitialization of the Level Set function as well as advection of the VOF function is much more complicated to implement.

In this article, we develop a numerical method for computing gas–liquid (or two-fluid) flows with a penalty approach. The method is formulated on a variationel approach, and then applied for computation of two-fluid flows. The Level Set technique for tracking the gas–liquid interface is modified to treat the problem of non conservation masse. Also, we improve the Level Set method by introducing a simple and efficient algorithm for a better volume conservation.

For more information, the reader is referred, in particular, to the books of Sethian [4] and Osher [13] that introduce these methods.

In the present work, incompressible two component flows are considered. In section 2, we present the governing equations, and then we introduce the level set method and how it is used to solve two-phase incompressible flows. Afterwards the reinitialization method is explained. In Section 3 our new technique, based on penalty method to ensure the mass conservation, is explained. Section 4 presents the results and discussion, and section 5 propose a conclusion and suggestions for future work.

2. GOVERNING EQUATION

2.1. Transport equation

The governing equations controlling the motion of multiphase flows are level set equations, coupled with the Navier Stokes and continuity equations.

The calculated velocities, from Navier-Stokes equation solution, are the input of level set method, which determines the new position of interface in the velocity field. The level set equation is similar to the convection equation

$$\frac{\partial \phi}{\partial t} + u.\nabla \phi = 0. \tag{1}$$

The level set function is typically a smooth (Lipschitz continuous) function, denoted here as $\phi(x,t)$. It is an auxiliary scalar function, and its sign determines the phase we are dealing with. In our algorithm, the interface is the zero level set of ϕ,

$$\Gamma = \{x | \phi(x,t) = 0\}. \tag{2}$$

we assume $\phi < 0$ in low density region (gas) and $\phi > 0$ in high density region (liquid), therefore we have

$$\phi(x,t) = \begin{cases} > 0, & \text{x} \in \text{ liquid}, \\ = 0, & \text{x} \in \Gamma, \\ < 0, & \text{x} \in \text{ gas}. \end{cases} \tag{3}$$

The properties of the fluid, such as density and viscosity, are functions of $\phi(x,t)$, by means of simple mixture low:

$$\rho(\phi) = \rho_1 H(\phi) + \rho_2 \left(1 - H(\phi)\right) \tag{4}$$

and similarly,

$$\eta(\phi) = \eta_1 H(\phi) + \eta_2 \left(1 - H(\phi)\right) \tag{5}$$

where $H(\phi)$ is the Heaviside function given by

$$H(\phi) = \begin{cases} 0, & \phi<0, \\ \frac{1}{2}, & \phi = 0, \\ 1, & \phi > 0, \end{cases} \tag{6}$$

When the level set function is a smooth distance function, the interface will have a constant thickness. The unit normal vector of the interface, from gas to liquid, and its curvature can easily be expressed in terms of $\phi(x,t)$ such as

$$n = \left.\frac{\nabla\phi}{|\nabla\phi|}\right|_{\phi=0} \quad \text{and} \quad \kappa = \nabla.\left.\left(\frac{\nabla\phi}{|\nabla\phi|}\right)\right|_{\phi=0}. \tag{7}$$

Since the interface moves with the velocity of the fluid, the evolution of $\phi(x,t)$ is governed by the convection equation,

$$\frac{\partial\phi}{\partial t} + \nabla(u\phi) = 0. \tag{8}$$

2.2. Interface motion

If we use Heaviside function as described in equation (6), the thickness of the interface will be assumed zero and we will get poor results. In fact, the use of an exact Dorect delta function produces some numerical difficulties for modelling the surface tension forces. In order to overcome these problem, we give to the interface a thickness $\varepsilon = \alpha\Delta x,$ where $\alpha > 1$. We substitute a smoothed Heaviside function $H_\varepsilon(\phi)$ for the sharp Heaviside function $H(\phi)$. The smoothed Heaviside function $H_\varepsilon(\phi)$ defined as below:

$$H(\phi) = \begin{cases} 0, & \phi<-\varepsilon, \\ \frac{1}{2}\left[1 + \frac{\phi}{\varepsilon} + \frac{1}{\pi}\sin\left(\frac{\pi\phi}{\varepsilon}\right)\right], & |\phi| \le \varepsilon, \\ 1, & \phi > \varepsilon, \end{cases} \tag{9}$$

and the smoothed delta function is

$$\delta_\varepsilon(\phi) = \frac{dH_\varepsilon(\phi)}{d\phi} \tag{10}$$

and consequently, the thickness of the interface is equal to

$$\frac{2\varepsilon}{|\nabla\phi|}. \tag{11}$$

2.3. Re-initialization

In the present work, the interface must have a constant thickness so that $|\nabla\phi|$ becomes constant near the interface. Level set function must be a signed distance function and present the shortest normal distance to the interface, Γ. A signed distance function has the following properties:

$$|\nabla\psi| = 1 \text{ for } |\phi| \le \varepsilon, \ \psi=0 \text{ for x} \in \Gamma. \tag{12}$$

As the interface evolves, $\phi(x,t)$ will generally drift away from its initialized value as signed distance, thus the technique presented needs to be applied periodically in order to keep $\phi(x,t)$ approximately equal to signed distance, the process is called re-initialization. In [5], Sussman, Smereka and Osher proposed a differential equation for the process as

$$\frac{\partial\psi}{\partial\tau} = S(\phi)\left(1 - |\nabla\psi|\right), \tag{13}$$

where $S(\phi)$ is

$$S(\phi) = \begin{cases} -1, & \phi<0, \\ 0, & \phi = 0, \\ 1, & \phi > 0, \end{cases} \tag{14}$$

$\psi(x,0) = \phi(x,t)$ and τ is fictitious time. Equation (13) must be solved to reach steady-state condition. When the convergence occurs the right hand side will be zero, so $|\nabla\psi|$ will be equal to zero. Instead of sharp sign function some forms of smooth sign function may be used as

$$S_\varepsilon(\phi) = 2\left(H_\varepsilon(\phi) - \frac{1}{2}\right). \tag{15}$$

In order to analyze equation (13), we may rewrite it as

$$\frac{\partial\psi}{\partial\tau} + w.\nabla\psi = S(\phi) \tag{16}$$

where

$$w = S(\phi)\frac{\nabla\psi}{|\nabla\psi|}. \tag{17}$$

Equation (16) is a non-linear hyperbolic and its characteristic velocities point outwards from the interface in the direction of the normal vector. This means that ψ will be re-initialized to $|\nabla\psi| \equiv 1$ near the interface first. As we need a signed distance function just near the interface, we should solve Equation (16) until $|\nabla\psi| \equiv 1$ near this location, where $|\psi| \le \varepsilon$. We may use a finite number of iterations in order to reach the distance function property near the interface. For example, if the iteration step size is $\Delta\tau,$ and the total interfacial thickness is $2\varepsilon,$ then no more than $\varepsilon/\Delta\tau$ iterations are needed. In practice, only two or three iterations will be enough, because we are already close to distance function. This method has been developed by Sussman and Fatemi [14].

2.3.1. Lagrange method

In this section, we explained the importance of maintaining the level set function ϕ as a signed distance function to the interface during all the calculations process, in order to correctly capture the interface and accurately calculate the surface tension. When solving the equation of transport for the advection of the level set function ϕ, it will distort and lose its property of being a signed distance function. Consequently, during a transient simulation, the level set field ϕ needs to be reinitialised, preferentially at every time step. Ideally, the interface remains stationary during the re-initialization process, but as we solve the re-initialization equation numerically, the interface may have a small movement. Sussman and Fatemi [14] proposed an improvement to the standard re-initialization process. Since their application was multiphase incompressible flow, they focused on preserving the amount of material in each cell, i.e. preserving the area (volume) in two (three)-dimensions. We use the fact that

$$\partial_\tau \int_\Omega H(\psi) = 0 \tag{18}$$

in every cell Ω as the volume will not change due to the stationary interface. Equation (13) can be developed as

$$\begin{aligned} &\frac{\partial \psi}{\partial \tau} = S(\phi)\left(1-|\nabla\psi|\right)+\lambda f(\phi) \equiv L(\phi,d)+\lambda f(\phi), \\ &\psi(x,0)=\phi(x), \end{aligned} \tag{19}$$

where τ is a time-like variable (different from physical time, t), ϕ_0 is the initial distribution of the level set function before re-initialization, and λ is a correction coefficient ensuring mass conservation up to the first-order term in the Taylor expansion of the integral $\partial_\tau \int_\Omega H_\varepsilon(\phi) d\Omega$ constant in each cell Ω. We continue with

$$\partial_\tau \int_\Omega H'(\psi)\psi_\tau \approx \int_\Omega H'(\phi)\psi_\tau = \int_\Omega H'(\phi)\left(L(\phi,\psi)+\lambda_{ij} f(\phi)\right) = 0, \tag{20}$$

so λ may be calculated as

$$\lambda = \frac{-\int_\Omega H'(\phi) L(\phi,\psi)}{\int_\Omega H'(\phi) f(\phi)}. \tag{21}$$

It is recommended by Sussman and Fatemi [14] to set $f(\phi)$ as

$$f(\phi) \equiv H'(\phi)|\nabla\phi| \tag{22}$$

to avoid damaging the distance function property far from the interface.

In order to calculate the spatial derivatives in the finite element method, we approximate

$$H'(\phi) \approx \frac{\partial H_\varepsilon(\phi)}{\partial \phi} = \delta_\varepsilon(\phi) \tag{23}$$

And

$$\lambda_{ij} = \frac{-\int_\Omega \delta_\varepsilon(\phi)}{\int_\Omega \delta_\varepsilon^2(\phi)|\nabla\phi|}. \tag{24}$$

The interested reader is referred to [14] for more explanation on the details of the volume constrained re-initializing method. The developed constraint (equation (18)) will significantly improve the accuracy of solving equation (13). This volume constraint improves the results obtained with the ENO scheme, but it sometimes decreases the quality of results obtained with the significantly more accurate WENO scheme, so we must switch between the ENO and WENO schemes to get good results.

2.4. Surface tension modelling

In many fluid mechanic problems surface tension forces become at most importance. The surface tension force is a result of unbalanced forces exerted to the molecules near the interface by the two fluids. In the present work, the macroscopic approach suggested by Brackbill et al. [20] has been used and surface tension is modelled as a body force in the vicinity of interface. Similar methods have been proposed by Unverdi and Tryggvason [21], and Chang et al. [22] that also represent surface tension as a body force.

The magnitude of the force is proportional to the curvature of the interface $\kappa(\phi)$ and is calculated by solving the following relation:

$$F_{surface\ tension} = -\sigma\kappa(\phi)\nabla H_\varepsilon(\phi) = -\sigma\kappa(\phi)\delta_\varepsilon(\phi)\nabla\phi. \tag{25}$$

When the level set function is a signed distance function, the curvature of the interface $\kappa(\phi)$ can be found by means of the following relation:

$$\kappa = \nabla.\left(\frac{\nabla\phi}{|\nabla\phi|}\right) \tag{26}$$

As mentioned above this force is localized near the interface and is equal to zero far from it.

2.5. Flow equation

The interface separating the two-fluid is tracked by the level set function, ϕ, which is defined as a signed distance from the interface. The negative sign is chosen for the gas phase and the positive sign for the liquid phase. In the current implementation of level set method, as mentioned above, the interface is captured implicitly, so the method is expected to be robust in handling sophisticated topological and geometrical changes. Sussman and Fatemi [14] used incompressible Navier–Stokes equations for calculating the velocities as below: the surface tension acts only at the interface and we can add this force to the Navier–Stokes equations as a singular interface term by using a δ-function.

$$\begin{aligned} &\rho(\phi)\frac{\partial u}{\partial t}+\rho(\phi)u.\nabla u+\nabla p-\nabla.\left(2\eta(\phi)D\right) = -\sigma\kappa(\phi)\delta(\phi)\nabla\phi+\rho(\phi)g, \\ &\nabla.u = 0, \end{aligned} \tag{27}$$

Here, κ is the curvature for two-dimensional flows, and **n** is a properly oriented unit vector normal to the front. *n* is a normal coordinate to the interface, with *n* = 0 at the interface.

where ρ and η are density and viscosity, respectively, δ is the Dirac delta function, **u** is the velocity, p is the pressure and D is the rate of deformation tensor.

3. Correction with penalty method

3.1. Correction of mass conservation

The principle of this method is to introduce the kinematic constraint condition "absolute" type:

$$\frac{1}{2}\beta\left(\int_{\Omega} H(\phi)\,d\Omega - Vol^{*}\right)^{2} \tag{28}$$

where β is the penalty factor and $Vol^{*} = \int_{\Omega} H_{\varepsilon}(\phi_0)\,d\Omega$ is the initial volume of the dispersed phase. The effect of this constraint has to minimizing volume loss by imposing an updating after each iteration.

To minimize the volume change, we must minimize the following functional:

$$\text{Min}: \quad J(\phi) = \frac{1}{2}\beta\left(\int_{\Omega} H(\phi)\,d\Omega - Vol^{*}\right)^{2} \tag{29}$$

The minimum part in brackets (standard deviation of volumes) is much smaller than the parameter β is large.

So we resolve the derivative of the following modified functional:

$$\frac{\partial J}{\partial \phi} = \frac{\partial}{\partial \phi}\left(\frac{1}{2}\beta\left(\int_{\Omega} H(\phi)\,d\Omega - Vol^{*}\right)^{2}\right) \tag{30}$$

This functional expresses the derivative with the Level-Set function, this gives after development of the derivative the follows expression

$$\frac{\partial J}{\partial \phi} = \beta\left(\int_{\Omega} \frac{\partial H(\phi)}{\partial \phi}\,d\Omega\right)\left(\int_{\Omega} H(\phi)\,d\Omega - Vol^{*}\right) \tag{31}$$

whith

$$\frac{\partial H(\phi)}{\partial \phi} = \delta(\phi) \tag{32}$$

Where $\delta(\phi)$ is a regularized Diract function.

Hence,

$$\frac{\partial J}{\partial \phi} = \beta\left(\int_{\Omega} \delta(\phi)\,d\Omega\right)\left(\int_{\Omega} H(\phi)\,d\Omega - Vol^{*}\right) \tag{33}$$

where β is prescribed as a penalty parameter. The solution is governed by the equation

$$\frac{\partial \phi}{\partial t} + u.\nabla\phi + \beta\delta(\phi)\left(\int_{\Omega} H(\phi)\,d\Omega - Vol^{*}\right) = 0 \tag{34}$$

The penalty method is easy to implement. The equation with the penalty method is an approximation of the problem of Level-Set, when the latter model is obtained in the limit $\beta \to \infty$. When β is large, the computing time is larger because it increases the number of iterations, but the accuracy is even finer. When β is too small, the obtained solution leads to a loss of the mass..

Another interesting approach is implemented in this paper. This approach consists in the correction of the interface without reinitialization. In the case of two-phase flow, the advantage of this approach is a direct application in the transport equation, without adding another equation to reset the increasing number of degrees of freedom.

3.2. Interface correction method without Reinitialization

This section presents another technique to keep the characteristic of the distance function. This new variational formulation forces the Level-Set function to meet the definition of the signed distance function, and it eliminates the return to the reinitialization procedure.

This variational formulation is based on a method of penalty. Thus the functional to minimize is:

$$\mathrm{E}(\phi) = \int_{\Omega} \frac{1}{2}\chi\left(|\nabla\phi| - 1\right)^{2} d\Omega \tag{35}$$

This function is a metric characteristic that allows redefining the signed distance function. With χ is a parameter greater than zero, which controls the effect of the deviation of the penalization of the Level Set function in the signed distance function.

Minimizing the functional E will follow the following procedures:

$$\frac{\partial \mathrm{E}(\phi)}{\partial \phi} = \frac{\partial}{\partial \phi}\left(\int_{\Omega} \frac{1}{2}\chi\left(|\nabla\phi| - 1\right)^{2} d\Omega\right) \tag{36}$$

This gives after development of the derivative:

$$\frac{\partial \mathrm{E}(\phi)}{\partial \phi} = \left(\int_{\Omega} \chi\left[\Delta\phi - div\left(\frac{\nabla\phi}{|\nabla\phi|}\right)\right] d\Omega\right) \tag{37}$$

Where:

$$\Delta\phi - div\left(\frac{\nabla\phi}{|\nabla\phi|}\right) = div\left[\left(1 - \frac{1}{|\nabla\phi|}\right)\nabla\phi\right] \tag{38}$$

The factor $\left(1 - \frac{1}{|\nabla\phi|}\right)$ is the rate of diffusion. If $|\nabla\phi| > 1$ the rate of diffusion is positive and the effect of this term is a simple diffusion, which keeps ϕ higher and reduce the gradient $|\nabla\phi|$. If $|\nabla\phi| < 1$ this term will affect the inverse scattering, this promotes the increase of the gradient. This approach has been used by Li et al. [23], in the field of image processing by the method of Level-Set.

Finally the Level Set equation to solving in two-phase flow is as follows:

$$\frac{\partial \phi}{\partial t} + u.\nabla\phi + \beta\delta(\phi)\left(\int_{\Omega} H(\phi)\, d\Omega - Vol^{*}\right) + div\left[\chi\left(1 - \frac{1}{|\nabla\phi|}\right)\nabla\phi\right] = 0 \quad (39)$$

4. Application of the method to monitor the dynamics of bubble: test of conservation of mass

As noted above, the Level-Set method has limitations. Related to numerical errors, the first one on the non-conservation of the exact mass of the fluid, and the second one the gradual erosion of the stiffness of the interfaces during simulation.

The following case offers as proof the two limitations of the numerical method. Consider the simulation of the rise of a gas bubble in a liquid initially at rest. To ensure the conservation of mass of the two phases, the correction method by penalty method has been tested by using several factors of penalty for this method. It was developed in the context of this work.

We assume that the bubble is gas which viscosity and density is a data, and is immersed in a fluid which the viscosity and density are higher.

The motion of single bubble is relatively well understood as extensive experimental data on the shape and terminal velocity in the form of empirical correlations are available in the literature and they are summarized in the book by Clift et al. [24]. In spite of a significant amount of experimental work, the understanding of bubble motion is still rather rudimentary in describing the flow dynamics.

Finally, the scale-up of bubble column demands a comprehensive study of the effect of column with on bubble shapes and their rise trajectories.

The possibility to simulate complex motions of bubbles directly is still a delicate case to solve by numerical methods.

Therefore, it is clear that so far most of the attention has been to evaluate the effect of bubble diameter on the fluctuating bubble motion. In addition to bubble motion in a stagnant liquid, it is necessary to clarify the effect of column diameter on the bubble shape and its rise trajectory. However, very few attempts have been made to numerically investigate these effects. Another aspect which has not received enough attention is the coalescence and break-up of multiple bubbles. More work is thus required for better understanding of multi-bubble behaviour where a number of interacting bubbles generate a high level of turbulence compared to that with a single bubble.

In this study, we have analysed fluctuating motions of single bubble in a liquid column. The new Level set approach has been used for numerical simulations. Bubbles ranging from 5-100mm are simulated using water and air as the continuous and dispersed phases respectively.

Different characteristics of single bubble trajectories have been numerically evaluated to model the hydrodynamics of gas–liquid bubble columns. The simulations were performed for several diameters of bubble columns. Initially, a simple case of a single bubble rising in water was studied. The effect of bubble size on bubble rise trajectory and fluctuation in bubble shape were analysed in detail. Bubble shapes were compared with the Grace-Diagram (Figure 1). Selected results were compared with the experimental work in the literature.

4.1. Physical properties

The properties of air and water were used in the transport equations when the computational cell was in the liquid or the gas phase, respectively. Detail of physical properties of air and water used in simulations are given in Table 1.

The parameters chosen for this simulation are presented in the following table:

TABLE 1 PHYSICAL PROPERTIES OF LIQUID AND GAS

Surface tension between the two fluids	0.0728 Kg/s2
Gravity g	9.81 m/s2
Initial diameter of the bubble d	1 m
Volume density of water $\rho_{liquide}$	1000 Kg/m3
Volume density of gaz ρ_{gaz} (air)	1.226 Kg/m3
Viscosity of water $\eta_{liquide}$	1.137 10-3 Kg/ms
Viscosity of air η_{gaz}	1.78 10-5 Kg/ms

4.2. Study of the Effect of Bubble Size on Bubble shape evolution

4.3.3. Problem position

Initially, a spherical air bubble was positioned stationary in the computation domain with its centre located bubble diameter above the bottom. At the walls, no-slip boundary condition was imposed. The column was modelled as an open system.

In this example, the spherical bubble has a zero initial velocity is fixed and the reports of density ρ_l / ρ_g and viscosity η_l / η_g is equal to 813 and to 64 (the index l denote the liquid, and the index g denote the gas), respectively. Under of these conditions, we can consider the density ρ_g and viscosity η_g gas in the bubble effects negligible in those of the surrounding fluid. Under the influence of gravity the bubble rises and becomes distorted.

Regarding the evolution of the shape of the bubble we can compare our model with respect to a generalized graphical representation given by Clift et al. [24] and [25] (see **Figure 1**), which specifies the form of bubbles of varying diameters and different physical properties, characterized by the dimensionless numbers: the number of Morton, $Mo = \frac{g\eta_L^4(\rho_L - \rho_g)}{\rho_L^2 \sigma^3}$, the number of EÖTVÖS, $Eo = \frac{\Delta\rho g d^2}{\sigma}$, and is used to characterize the shape of bubbles or drops moving in a fluid. It is the ratio of buoyancy and surface tension, and the Reynolds number, $\mathrm{Re} = \frac{\rho_L U_\infty d}{\eta}$.

ρ_L and η_L are the density and viscosity of the fluid and U_∞ the terminal velocity of the bubble. The numbers of Mo and Eo are connected by the Reynolds number and Weber ($\mathrm{Re} = \left(\frac{\ddot{E}o^3}{Mo}\right)^{1/4}$, $\mathrm{We}=\mathrm{Eo}$) if we choose the reference speed equal to $U_R = U_\infty = \sqrt{gd}$, the reference length is the diameter of the bubble L_R = d and $\Delta\rho = \rho_l$ (in the case of an air bubble in water).

The changing shape of the bubble depends directly on the size of the bubble. So for the purposes the air-water ($Mo=2.63*10^{-11}$, and Log (Mo)=-10.6), also for our case we find Eo = 136.9. This corresponds to the form developed experimentally in [29] which shows the shape of Skirted (or skirt) of the bubble see (figure2)

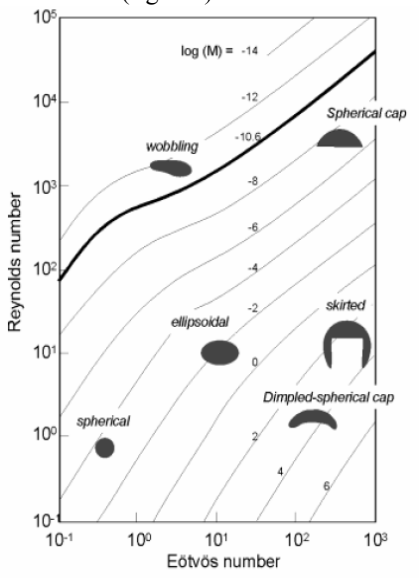

FIGURE 1- GRACE1 DIAGRAM OF FOR THE SHAPE AND TERMINAL RISE VELOCITY OF A GAS BUBBLE IN QUIESCENT NEWTONIAN LIQUID. [24] ET [25]

Figure 2 shows a number of snapshots depicting the rise of a single bubble. It is found that the stiffness of the interface is maintained throughout the simulation. As and when the climb, the bubble has a spreading, leading to a region closer to the interface where the stretching and bending are great.

Figure 3 summarizes the evolution of the shape of various size bubbles at several times. It can be observed that for every bubble, the shape of the bubble is influenced by two global parameters, the first parameter is the force of gravity, this parameter is very important when the diameter of the bubble is large. Forces of surface tension drive the second parameter; this force becomes important when the diameter of the bubble is small. However, when the diameter of the bubble is of average size, an interaction between these two forces is proved, such as competition between the gravitational force and the force of surface tension will generate oscillations of the bubble. As it has mounted along the water column.

The barycentric velocity of the bubble is defined by the relationship:

$$U = \frac{\int_\Omega \left(1 - H(\phi)\right)\vec{v}.\vec{n}d\Omega}{\int_\Omega \left(1 - H(\phi)\right)d\Omega} \qquad (44)$$

with $\int_\Omega \left(1 - H(\phi)\right)d\Omega$ the volume of gas, $\vec{v}$ the local velocity

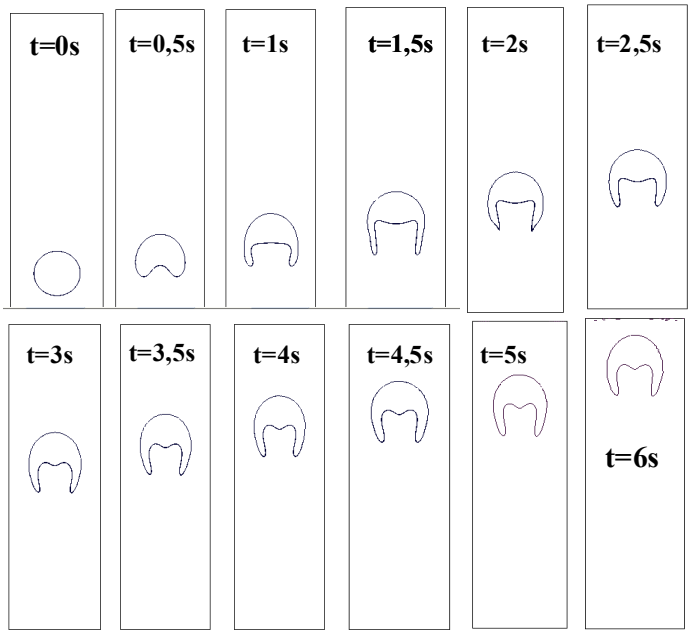

FIGURE 2 - EVOLUTION OF THE BUBBLE IN THE WATER COLUMN AT DIFFERENT TIMES FOR A RATIO OF 500 OF PENALTY FACTOR AND FOR A MESH OF 11,194 TRIANGULAR ELEMENTS (40 * 125)

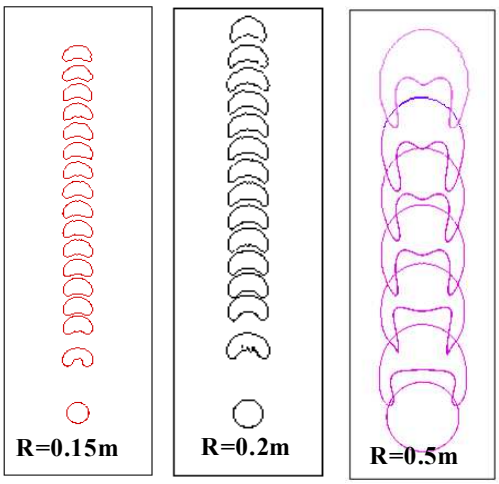

FIGURE 3 -THE SHAPE OF THE BUBBLE FOR DIFFERENT RAYS

of gas et $\vec{n}$ the unit vector parallel to **g.**

When the diameter of the bubble is large, the terminal velocity of the bubble measured experimentally by Davies and Taylor [26] is given by the following equation:

$$U_B = 0.707\sqrt{gd_{bulle}} \quad (45)$$

With **g** the gravitational force and d the diameter of the bubble.
Clift et al. [24] introduced a model that takes into account the surface tension at the rise of the bubble, the model is written as follows:

$$U_T = \sqrt{2.14\sigma / \rho d_{bulle} + 0.505 g d_{bulle}} \quad (46)$$

where ρ is the density of the liquid, σ surface tension, d the diameter of the bubble, and g is the gravitational force.
The comparison of changes in the speed of the bubble calculated from this work with the terminal velocity given by Davies and Taylor [26], and with that given by the model of Clift et al. [24] is shown in Figure 4.

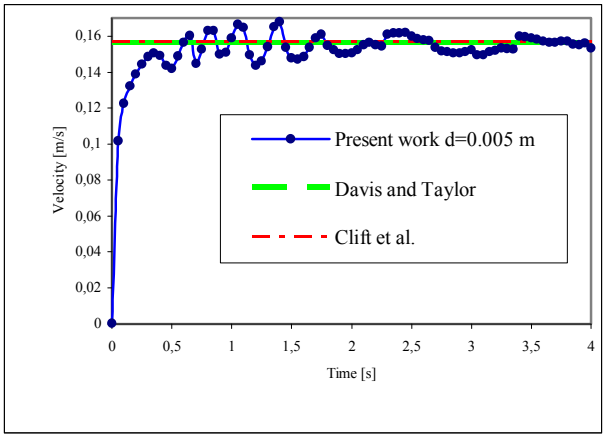

FIGURE 4 - COMPARISON OF EXPERIMENTAL AND NUMERICAL RESULTS OF THE VELOCITY OF A BUBBLE DIAMETER D = 0.005 M

In the time interval [0, 1s], the velocity of the bubble increases rapidly due to the effects of gravity and then stabilizes around the terminal velocity given experimentally by Davies and Taylor [26] U_b = 0.156 m / s, and Clift et al. [24] U_T = 0.157 m / s.
The terminal velocity averaged from t = 1s to t = 4s (the time for which the bubble reaches a maximum speed) equal to V = 0, 157 m / s. The speed obtained by calculation is consistent with the terminal velocity given by Clift et al. [24], validating our model against data from the literature. Nevertheless, we want to ensure that this model is valid over a wide range of diameters of the bubble, for this we have implemented a series of calculations of the rise of air bubble in a water column and we compared these velocities from the two models given above and compared to experimental data of Talia et al. [27] see Figure 5.
This figure confirms the accuracy of our method in the case of two-phase flows; we find that the values of terminal velocity calculated remain near the terminal velocity given by the model of Clift et al. [24] for different diameters of the bubble.

After validating the speed of the bubble, we must verify that the method complies with the conservation of mass. In the remainder of this section, we proceed to use different penalty coefficient to ensure the conservation of mass. For this, we will detail the procedure for calculating the volume of the bubble. The instantaneous volume of the bubble is given by,

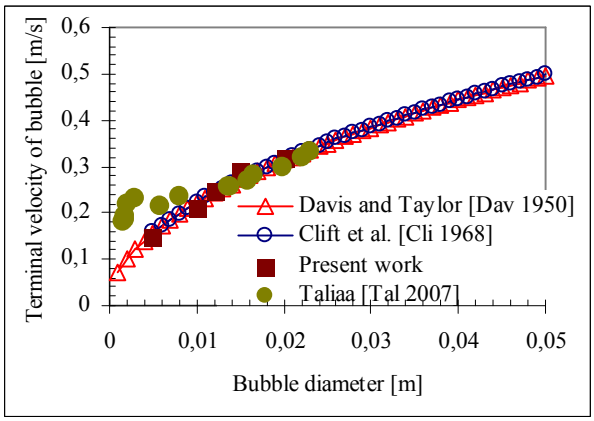

FIGURE 5 - COMPARISON OF TERMINAL VELOCITY BETWEEN EXPERIMENTAL DATA AND OUR NUMERICAL MODEL

$$V(t) = \int_\Omega (1 - H(\phi)) d\Omega \quad (47)$$

$(\phi \le 0)$ delimits the part of the volume of gas defined by the Level-Set, and the volume of computational domain. There V0 the initial volume of the bubble.
To quantify the relative change in percent of the volume of the bubble, we define the following relationship.
is ΔV (t) the rate of change in the volume of fluid 1 associated with the Level-Set function in the computational domain:

$$\Delta V(t) = 100 \frac{\int_\Omega \left((1 - H(\phi)) - \phi_0\right) d\Omega}{\int_\Omega \phi_0 d\Omega} \% = 100 \frac{V_g(t) - V_g(t=0)}{V_g(t=0)} \% \quad (48)$$

The comparison of successive volumes of the bubble will be done each time to monitor the relative volume of gas and ensure its conservation throughout the rise of the bubble. The correction method developed earlier by penalty is applied in order to overcome this possible loss of mass that generates the application of the method of Level-Set. Several penalty coefficients are simulated in order to better understand the effectiveness of this method.

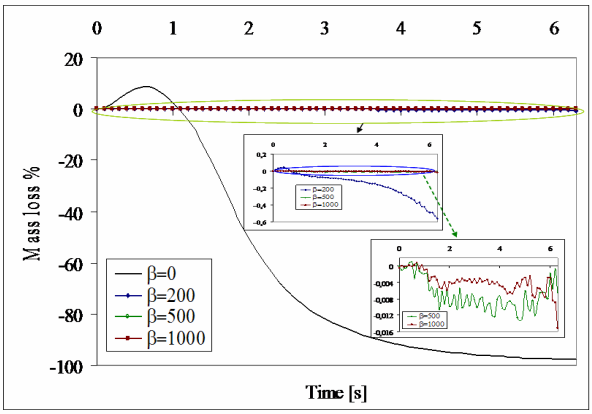

FIGURE 6 - CORRECTION OF MASS WITH DIFFERENT PENALTY FACTORS

Figure 6 illustrates the correction of mass for different penalty coefficients. We note that when this factor increases, the mass conservation is much better. Only point we can criticize this method is the computing time becomes increasingly expensive as this factor increases. Looking in detail in this figure, where the Level-Set function is applied with a zero correction factor, we note that from a number of iterations, the mass change of the bubble changing dramatically, to the point where it may disappear altogether if it lasts longer: 11.1% mass loss in 0.4 seconds. $\nabla . U = 0$) are not preserved. As illustrated in Figure 20, the volume of the bubble and hence a liquid (since This is harmful, when should use this method in simulations of real cases, as in filling molds for example.

The correction proposed in this work is to check at each iteration that the condition of signed distance is well respected. A penalty factor is introduced to limit the spread of the interface defined by the Level-set function. This factor is more effective than its value is important. Unfortunately, the stronger it is more time it requires expensive computation. A comparison of the degree of confidence based on the penalty factor is represented in the graph in Figure 6. When the penalty coefficient is high, the rate of relative change of the mass becomes almost zero over time, reaching 0.001% for a coefficient $\beta = 1000$. The computation time in this case is multiplied by 10 compared to the computation time with $\beta = 10$. So, apart from the time of calculation, this method allows us to free ourselves, rigorously, the problem of non conservation of mass, inherent in the use of the Level-Set method.

5. CONCLUSION

In this work, a new level set approach is developed and used to simulate the trajectories of single bubble rising in column of liquid. Bubbles of different sizes were simulated. The computed bubble shapes strongly resembled the shapes expected on basis of the Grace diagram. The new level set model is applied to study the case of the dam break. These results present an excellent agreement with experimental observations of Martin and Moyce. Finally, best conservation of mass is ensured. In conclusion, it is demonstrated that the new level set method is able to provide an accurate description of the interface shape in two fluids flows.

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