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# NUMERICAL ANALYSIS OF ENTROPY GENERATION IN ARRAY OF PIN-FIN HEAT SINKS FOR SOME DIFFERENT GEOMETRIES

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## ABSTRACT

In this study, Numerical analysis has been used to investigate entropy generation for array of pin-fin heat sink. Technique is applied to study the thermodynamic losses caused by heat transfer and pressure drop in pin-fin heat sinks. A general expression for the entropy generation rate is obtained by considering the whole heat sink as a control volume and applying the conservation equations for mass and energy with the entropy balance. Analytical and empirical correlations for heat transfer coefficients and friction factors are used in the numerical modeling. Also effects of heat transfer and pressure drop in entropy generation in control volume over pin-fins have been studied. Numerical analysis has been used for three different models of pin-fin heat sinks. The models are different in cross section area. These cross section areas are circle, horizontal ellipse and vertical ellipse which mentioned in next sections. Reference velocity used in Reynolds number and pressure drop is based on the minimum free area available for the fluid flow. Also for numerical analysis in-line arrangement of fins has been investigated and their relative performance is compared. At the end, the performance of these three models has been compared.

# INTRODUCTION

In order to attain appropriate temperature for electronic device which reproduced so much heat flux, heat sinks have been highly employed for cooling electronic components. Among the cooling devices, fin pin is regarded as one of the most effective internal cooling system which highly used in electrical devices like circuit boards. Heat transfer in heat sinks depends on conduction from the body and convection to the flow over the body. Lin and Lee [1] performed a second law analysis on a pin-fin array under forced flow conditions. They evaluated optimal operational design conditions for both the in-line and staggered fin alignments. They considered the heat transfer contributions from the base wall as well as from the fin surface and found the optimal  $Re_d$  values. Jubran et al. [2] performed an experimental investigation on the effects of inter fin spacing, shroud clearance, and missing pins on the heat transfer from cylindrical pin fins arranged in staggered and inline arrays. Haq et al. [3] experimentally studied the steady state forced convective cooling of a horizontally based pin fin system. Das and Razelos [4] analyzed the heat dissipation and performance of pin fins. Yeh [5] analytically investigated the heat transfer coefficients and heat transfer from the fin tip, the optimum dimensions of rectangular fins and cylindrical pin fins. Li et al. [6] investigated the heat transfer and flow resistance characteristics in rectangular ducts with staggered arrays of short elliptic pin fins. Maveety and Jung [7] studied the heat transfer of the pin heat sink with air impingement cooling. Tahat et al. [8] experimentally and numerically simulated turbulent air impingement flow on a square pin fin heat sink. Experiments were conducted using an aluminum heat sink under uniform heat flux. Also they studied the effects of varying the geometrical configurations of the pin-fins.

Sara et al. [9] investigated steady state heat transfers from pin fin arrays for staggered and in line arrangements of the pin fins. Chen et al. [10] presented the heat transfer and friction

characteristics and the second law analysis of convective heat transfer through a rectangular channel with square cross section pin fins. Minakami and Iwasaki [11] conducted experiments to investigate the pressure loss characteristics and heat transfer performance of pin-fin heat sinks exposed to air flow in a cross-flow direction, varying the pin pitch as a parameter. They found that as the longitudinal pitch increased, the heat transfer coefficient increased and the pressure loss also increased. Jonsson and Palm [12] performed experiments to compare the thermal performance of the heat sinks with different fin designs including straight fins and pin fins with circular, quadratic and elliptical cross sections. They evaluated the thermal performance by comparing the thermal resistance of the heat sinks at equal average velocity and equal pressure drop. They recommended elliptical pin-fin heat sinks at high velocities and circular pin-fin heat sinks at mid-range velocities. Yu et al. [13] experimentally and numerically studied the thermal performances of two types of heat sinks. Kobus and Oshio [14] theoretically and experimentally studied the thermal performance of a pin fin heat sink for pure natural convection and for combined forced and natural convection. The influence of various geometrical and flow parameters on the effective thermal resistance of the heat sink were considered.

Wirtz et al. [15] reported experimental results on the thermal performance of model pin-fin fan-sink assemblies. They used cylindrical, square, and diamond shape cross section pin-fins and found that cylindrical pin-fins give the best overall fan-sink performance.

The objective of this study is to study the entropy generation based on heat transfer and pressure drop for some different pin fin configuration and compare their performances depend on the cross section area of the pin fin.

# NOMENCLATURE

Α	total base area
$A_{bp}$	area of base plate
$A_{c}$	cross section area
$A_{fin}$	surface area of fin
d	pin diameter
f	friction factor
Н	pin height
h	convective heat transfer coefficient
L	length of heat sink
Ν	total number of fins
$N_l$	number of rows in streamwise
$N_w$	number of columns in spanwise
$Nu_{fin}$	Nusselt number for fin
$Nu_L$	Nusselt number based on heat sink length
Pr	Prantel number
0	heat flux

R	resistance
$Re_L$	Reynolds number
• S gen	entropy generation rate
$S_D$	dimensionless diagonal pitch
$S_l$	dimensionless streamwise pitch
$S_w$	dimensionless spanwise pitch
Т	temperature
t	thickness
$U_{\infty}$	ambient velocity
$U_{max}$	maximum velocity

bp	base plate	
bm	bulk material	
С	contact	
fin	fin	
gen	generation	
tot	total	

Greek	symbol	S

$\Delta P$	pressure gradient
ho	fluid density

# FORMULATION OF THEORETICAL MODEL

The entropy generation associated with heat transfer and frictional effects serve as a direct measure of lost potential for work or in the case of a heat sink, the ability to transfer heat to surrounding cooling medium. According to Bejan [16] and using the law of conversation of mass and energy and entropy balance over a heat sink, the expression can be obtained for entropy generation rate:

$$\dot{S}_{gen} = \left(\frac{Q}{T_{\infty}}\right)^2 R_{tot} + \frac{m\Delta P}{\rho T_{\infty}}$$
(1)

In this expression, entropy generation rate depends on the heat sink resistance and the pressure drop over the heat sink, also the heat load, mass flow rate and ambient conditions are specified.

Total heat sink resistance is given by:

$$\boldsymbol{R}_{tot} = \boldsymbol{R}_{bm} + \boldsymbol{R}_{fin} \tag{2}$$

 $R_{bm}$  Bulk material resistance is obtained from:

$$R_{bm} = \frac{t_{bp}}{kA} \tag{3}$$

And  $R_{tot}$  the overall resistance of fins and base resistance can be written as:

$$R_{tot} = \frac{I}{\frac{N}{R_c + R_{fin}} + \frac{I}{R_{bp}}}$$
(4)

Where

$$R_c = \frac{l}{h_c A_c} \tag{5}$$

$$R_{fin} = \frac{I}{h_{fin} A_{fin} \eta_{fin}} \tag{6}$$

$$R_{bp} = \frac{1}{h_{bp}A_{bp}} \tag{7}$$

With

$$\eta_{fin} = \frac{tanh(mH)}{mH} \tag{8}$$

$$m = \sqrt{\frac{4h_{fin}}{kd}} \tag{9}$$

Dimensionless heat transfer coefficient used for cylindrical fin is developed by khan [16] which is written as follow:

$$Nu_{fin} = C_1 Re_d^{1/2} Pr^{1/3}$$
(10)

Where  $C_1$  is a constant which depends upon the arrangement of the pins and thermal boundary conditions. For isothermal boundary condition and for inline arrangement it is given by:

$$C_{I} = \frac{[0.2 + exp(-0.55S_{w})]S_{w}^{0.75}S_{I}^{0.212}}{(S_{w} - I)^{0.5}}$$
(11)

Dimensionless heat transfer coefficient used for elliptical fin is developed by

$$Nu_{fin} = 0.203 Re_{d}^{0.675} Pr^{0.3}$$
(12)

The heat transfer coefficient for the base plate  $h_b$  can be determined by considering it a finite plate. Khan [16] has developed following analytical correlation for dimensionless heat transfer coefficient for a finite plate:

$$Nu_{L} = 0.75 Re_{L}^{\frac{1}{2}} Pr^{\frac{1}{3}}$$
(13)

Where L is the length of the base plate in the streamwise direction. The mass flow rate is given by:

$$\dot{m} = \rho U_{\infty} T_{\infty} N_{w} S_{w} H d \tag{14}$$

The pressure drop associated with flow across the fins is written as:

$$\Delta P = f \, \frac{\rho U_{max}^2}{2} N_l \tag{15}$$

Where the friction factor f depends on the Reynolds number and the array geometry, and can be written as:

$$f = K_1 [0.233 + 45.78 / (S_w - 1)^{1.1} Re_d]$$
(16)

and  $K_1$  is a correction factor depending on flow geometry and arrangement of the fins. It is given by:

$$K_{1} = 1.175(S_{1} / S_{w} Re_{d}^{0.312}) + 0.5 Re_{d}^{0.0807}$$
(17)

The velocity  $U_{max}$  in Eq. (15) represents the maximum average velocity seen by the array as flow accelerates between fins, and is given by:

$$U_{max} = max \left\{ \frac{S_w}{S_w - I} U_{\infty}, \frac{S_w}{S_D - I} U_{\infty} \right\}$$
(18)

Where  $S_D = \sqrt{S_l^2 + (S_w/2)^2}$  in Eq. 18 is the dimensionless diagonal pitch.

#### NUMERICAL MODELING

Based on the control volume method, the SIMPLE algorithm is employed to deal with the problem of velocity and pressure coupling. A second order upwind scheme and structured uniform grid system are used to discretize the main governing equations. In order to obtain satisfactory solutions, a grid independence study is conducted in the analysis by adopting different grid distributions of  $7 \times 10^5$  for inline layout. At the inlet, the fluid with ambient velocity and ambient temperature enters the test section. The velocity boundary condition is applied at the inlet section, while the pressure boundary condition is used at the outlet section. The commercial program FLUENT #6.3.26 has been employed as the numerical solver. The numerical computation is ended if the residual summed over all the computational nodes satisfies the criterion  $\langle 10^{-5}$ .



Fig 1. 3D modeling of geometry

Figures of geometries for three model has been showed in fig. 1 to 4 as follow







Fig 3. Vertical elliptic fin





## **RESULTS AND DISCUSSION**

In this section, Diagrams of pressure drop and entropy generation vs. velocity inlet for inline arrangement of pin fin heat sink are plotted in fig. 6 and 7. For the pin fins, the major dimensions are shown in table 1. The objective is set to select the heat sink dimension to fit the  $50 \times 55$  mm base plate and overall height of 20 mm. It is also assumed that a total heat dissipation of 10 W is uniformly applied over the entire base plate which has a uniform thickness of 2 mm. The ambient temperature is fixed at 300 K and the problem is solved for the two thermal conductivities 170 and 400 (enhanced aluminum and copper).

Geometry	$S_w$	$S_{l}$	а	b	d
Circle	10	11	-	-	2
Hor. elliptic	10	11	2	3	-
Ver. elliptic	10	11	2	3	-

Table 1. Major dimension of heat sink

As expected, the pressure drop and entropy generation increase with the rise of frontal velocity. Clearly the vertical elliptic fin cross section is the worst choice from points of views due to the highest pressure drop and the lowest heat transfer rate which led to the highest entropy generation.

The drag force for the horizontal elliptic geometry decreases monotonically from the circular geometry to the flat plate. The heat transfer rate increases from circular geometry to horizontal elliptic geometry. So the horizontal elliptic geometry could be the best choice from both points of views of pressure drop and heat transfer.

Figure 5 shows the effects of approach velocity on the pressure drop for the selected geometries. It is clear that, for high approach velocities, flat plate is superior to other geometries considered due to lowest drag force. The horizontal elliptic geometry is the next favorable geometry as far as drag force is concerned. It should be noted that, there is no optimum approach velocity to provide a minimum pressure drop for any geometry, since it increases monotonically with the approach velocity. It is interesting to note that, for very low approach velocities, the importance of geometry disappears and all the curves approach to the same value as  $U_{\infty}$  approaches zero.



Fig 5. Diagram of pressure drop vs. velocity inlet

Figure 6 shows the variation of entropy generation rate  $\dot{s}_{gen}$ with the approach velocity,  $U_{\infty}$  for the selected geometries. The conductivity coefficient has been set to 400 for copper pin fins. The ambient temperature  $T_{\infty}$  is kept constant. As the approach velocity increases, the best choice it moves from elliptical geometry to the flat plate. The vertical elliptic geometry gives the highest entropy generation rate for the entire range of the approach velocities. The total entropy generation rate  $\dot{s}_{gen}$ includes the contributions due to heat transfer and viscous friction. As the approach velocity is increased, the contribution due to heat transfer decreases and that of viscous friction increases for each of the geometry considered. Also for low approach velocity the effect of heat transfer for entropy generation minimization is higher than the effects of viscous friction.



Fig 6. Entropy generation vs. velocity inlet

Figure 7 shows the variation of entropy generation rate  $S_{gen}$  with the approach velocity for aluminum pin fins with

conductivity coefficient set to 170. It is clear that although a high thermal conductivity heat sink is superior to all other cases, but a low thermal conductivity heat sink with high pin density still results in acceptable performance in terms of entropy generation rate.

Variation of total resistance based on velocity inlet for conductivity coefficient set to 170, has been shown in figure 8. As can be seen, total resistance decreased when the velocity inlet increased. It can be observed from the diagram that circular fin and horizontal elliptic fin are the same for thermal resistance but the vertical fin has different trend as the other geometries. The vertical elliptic fin has the highest resistance among the other fins and it's more efficient to use horizontal elliptic or circular fin.

Figure 9 shows total resistance based on velocity inlet for conductivity coefficient set to 400. As can be seen, the order of resistance decreases by increasing of conductivity coefficient. For velocity more than 2 m/s the circular and horizontal elliptic are the same in total resistance.



Fig 7. Entropy generation vs. velocity inlet



Fig 8. Total Resistance vs. velocity inlet



Fig 9. Total Resistance vs. velocity inlet

## CONCLUSION

This scientific study is presented for determining optimum heat sink conditions given the consideration of both heat transfer and viscous dissipation. The effects of approach velocity and heat sink thermal conductivity are examined with respect to its role in influencing entropy generation, pressure drop and optimum conditions among these three models and the overall performance of the heat sink has been compared. Different fin geometries are compared from the point of views of heat transfer and total entropy generation rate. As expected, the pressure drop and entropy generation increase with the rise of frontal velocity. The elliptic pin fin shows the lowest pressure drops. Whereas, the circular geometry appears as the best from the point of view of the total entropy generation rate for low approach velocities and the elliptical geometry is the next favorable geometry from the point of view of total entropy generation rate for higher approach velocities. Eventually vertical elliptic fins shows the highest pressure drop and entropy generation among these different geometries.

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