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DESIGN TECHNOLOGY OF CENTRIFUGAL FAN IMPELLER BASED ON RESPONSE SURFACE METHODOLOGY

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ABSTRACT

An optimization strategy called response surface methodology (RSM) is applied to a centrifugal fan impeller optimization design in this paper. RSM is used to generate an approximated model of objective function, for which a second-order polynomial function is chosen. The Design of experiment (DOE) technique coupled with CFD analysis is then ran to generate the database. The least-squares regression method (LS) is used to determine the coefficient of the RSM function. Finally, the Genetic Algorithms (GA) is applied to the objective function in order to obtain the optimal configuration.

This paper also presents a solution to the problem of imprecise fitting of second-order RSM model by dividing the zone into several subzones which is proved to be effective in this paper. The optimization result shows that RSM is an effective and feasible optimization strategy for the centrifugal fan impeller design, and the complexity of the objective function and the overall optimization time could be significantly reduced.

NOMENCLATURE

r_{1s} Inlet shroud radius
r_{1h} Inlet hub radius
r_1 Impeller inlet radius
b_2 Impeller outlet width
r_2 Impeller tip outlet radius
N_z Blade number
n Rotation speed
ω Angular speed of shaft
T Torque of shaft
T_1 Inlet temperature
p_s Inlet static pressure
p_{sg1} Stagnation pressure at inlet
p_{sg2} Stagnation pressure at outlet
η Impeller efficiency
q_m Mass flow rate
q_v Volume flow rate
β Model coefficient of response surface
X Variable vector
$x_1, \ldots, x_k$ Independent variables of response surface
Ω The selected zone
ξ Standard variable
υ The vertex of polyhedron
m The number of the responses, the level of test
k The number of factors
p_{tF} Fan pressure

INTRODUTION

Response surface methodology(RSM) is one of the most popular optimization strategies in recent years [1-3]. Different from the traditional optimization strategies, RSM depends on a systematic methodology, and thus reduces the design cost and compensates for experience lack of designer [4].

In the traditional optimization methods of turbomachinery, the shape of objective function is usually quite complex since there is not a direct relationship between the geometrical design parameters and the aerodynamic performance [5]. This forces researcher to develop more and more complex and efficient optimization algorithms to overcome the complexity of the problem and reduces the required optimization time [5-7]. According to the new method, a simple approximation function is used to describe the correlation between the input design parameters and the performance. It not only significantly reduces the complexity of the objective function and the overall optimization time, but also simplifies the design task [4].

Although RSM method has so many benefits, there are still some drawbacks. A major drawback may be fitting all the experiment data to a second-order polynomial equation, which will cause large fitting errors. Although it is possible to establish a model equation with a higher degree, it will significantly increases the model coefficients and thus the number of data in the database is also needed to be increased, which will also increase the time cost [1].

This paper presents a solution to this problem. The first step is to divide the zone into several subzones and then a narrow range of database is established on each subzone. Finally, the trend of the response surface function is explained with the second-order equation in the subzone and the fitting error can be obviously decreased.

The purpose of this paper is to highlight the potentiality of RSM method and its capacity for optimization of turbomachinery by dividing the zone into several subzones.

METHODS OF APPROACH

RSM which combining the mathematical and the statistical method[8-10], is used to simulate and analyze the problem whose response is probably affected by multiple variables. It represents the response as a function of controllable factors and noise factors, and optimizes the response through computer simulation. In the process of optimization, it includes the following steps, establishment of approximation function, design of experiment(DOE) and the optimization for objective function.

Establishment of approximation function

The first step is to build the approximation function which correlates the performance parameters Y_j (j=1, ... , *m*) with the design factors x_i (i=1, ... , *n*):

$$Y_j = f_i(x_1, x_2, ..., x_n) \quad (1)$$

Where Y_j are the response, f_j are the unknown functions of the responses, $x_1, x_2, ..., x_n$ denote the independent variables. *m* is the number of the responses, *n* is the number of the independent variables.

The RSM approximates the objective function with polynomial (response surface), its order and shape must be chosen beforehand. In common practice, a first- or second-order polynomial function is usually used:

$$Y_j = \beta_0{}^j + \sum_{i=0}^{n} \beta_i{}^j x_i + \sum_{i=0}^{n} \beta_{ii}{}^j x_i{}^2 + \sum_{i \neq j}^{n} \beta_{ik}{}^j x_i x_k \quad (2)$$

β_0, β_i, β_{ii} and β_{ik} are the model coefficients which will be determined by LS. The database needed by LS is generated from DOE coupled with CFD analysis.

Design of experiment (DOE) method

DOE is an efficient procedure to obtain the data for the establishment of database. The data will be used for obtain the model coefficients [11]. The number of data has a significant effect on the cost of the experiment. Meanwhile the prediction ability of the RSM is affected by the distribution of the points. Therefore DOE is very important in the whole design process. In order to reduce the fitting error, especially reduce the time cost of experiment, the design variables are selected and the distribution of the design points is decided through a proper DOE. The r_1 and b_2 are selected as design variables in this paper.

The main kinds of design method include completely randomized design, randomized block design, full factorial design, factional factorial design, and central composite design(CCD) [11]. The method used here is the full factorial design, which gives a combination of all factors, and can clearly describe the influence on responses of the input test factors. The following is an introduction for full factorial design.

$X = (x_1{}^0, ..., x_k{}^0)$ is supposed as the current value of the factors and $x_1, ..., x_k$ $x_i (i \neq 0)$ is the original unit variable. $\Omega = [x_1{}^l, x_1{}^u] \times [x_2{}^l, x_2{}^u] \times \cdots \times [x_k{}^l, x_k{}^u]$ is selected as the zone in which the relationship between responses and factors is studied. The variables are normalized as Eq.(3)

$$\xi_i = 2 \times \frac{x_i - x_i{}^0}{x_i{}^u - x_i{}^l}, 1 \leq i \leq k \quad (3)$$

Where $\xi_1, ..., \xi_k$ are standard variables and their values satisfy $\xi_i \in [-1,1], 1 \leq i \leq k$. The standard variables have no scales, but they have the same standard deviation and zero values which are convenient to compute. The current values of standard variables are located at the center of polyhedron $[-1,1]^k$. The vertexes of polyhedron $[-1,1]^k$ are denoted as $\upsilon_1, \upsilon_2, ..., \upsilon_{2^k}$.

The number of calculation of full factorial design is m^k, Where *m* is the test levels and *k* is the number of factors.

Fig. 1 shows the distribution of test points when *m*=2 and *k*=3, and Fig. 2 shows those when *m*=3 and *k*=3.

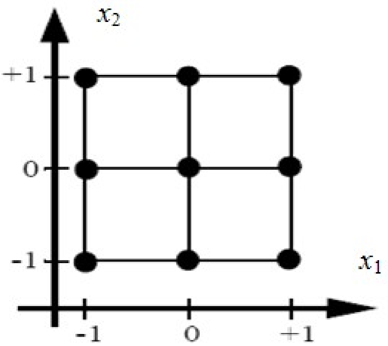

Fig. 1 The distribution of test point when *m*=2 and *k*=3

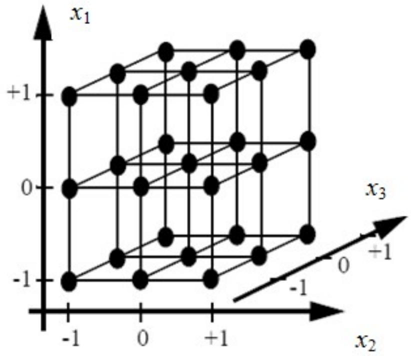

Fig. 2 The distribution of test point when m=3 and k=3

Optimization strategy for objective function

GA, as an emerging discipline, was first proposed in 1960s. GA is based on the theory of biological evolution [12]. GA is originally adopted to explain the complexity of natural system in the biological process of adaptation, simulate the biological evolution mechanism to construct artificial Genetic Algorithms, and provide a common framework for solving optimization problem of complex system. Nowadays it has been widely used in many disciplines because it does not confine itself to specific area and has robustness for varieties of problems. In this paper, GA developed by John Holland is used to obtain the optimal value of response surface function, and to perform the aerodynamic optimization of centrifugal fan impeller. **Fig. 3** shows the flowchart of GA.

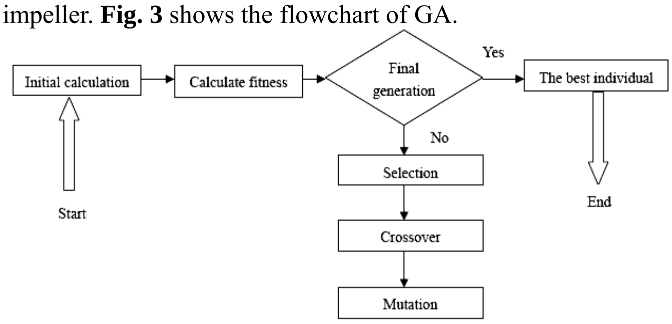

Fig. 3 Flowchart of GA

METHOD APPLICATION

Description of the problem

The internal flow of centrifugal fan impeller is very complex. The geometric parameters of impeller such as outlet width b_2, inlet width b_1, and outlet radius r_2 have significant influence on the performance of fan impeller such as efficiency η, pressure p_{tF}, mass flow rate q_m. It is difficult to obtain the best relation between the geometric parameters and the performance parameters in the process of impeller design. In order to obtain the best design, the geometric parameters are usually adjusted based on the experimental data, which not only costs a lot of time, effort and money but also largely depends on the designer's personal experience.

This paper tries to utilize RSM and GA to solve this problem. First, one simple and intuitive response surface model between design parameters and performance parameters was established. Then, we set the location of test point based on DOE. After that, the numerical results of CFD are utilized to establish a database to determine the coefficients of the response surface mode. Finally, we apply GA to the response surface function for global optimization and determine the optimal design structure which maximizes the efficiency of the impeller. The real impeller is shown in Fig. 4.The geometrical parameters and their values are reported in **Fig. 5** and **Tab. 1**.

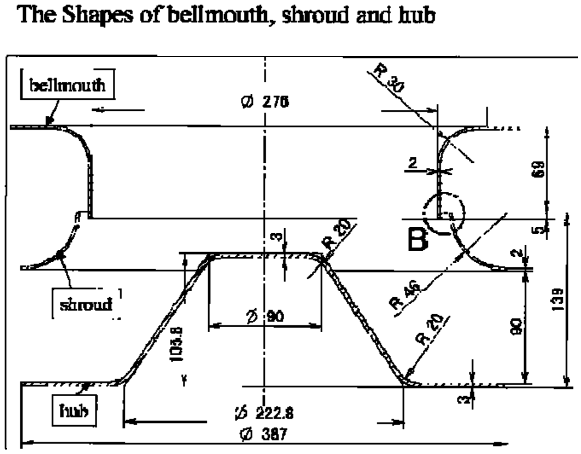

Fig. 4 The real impeller

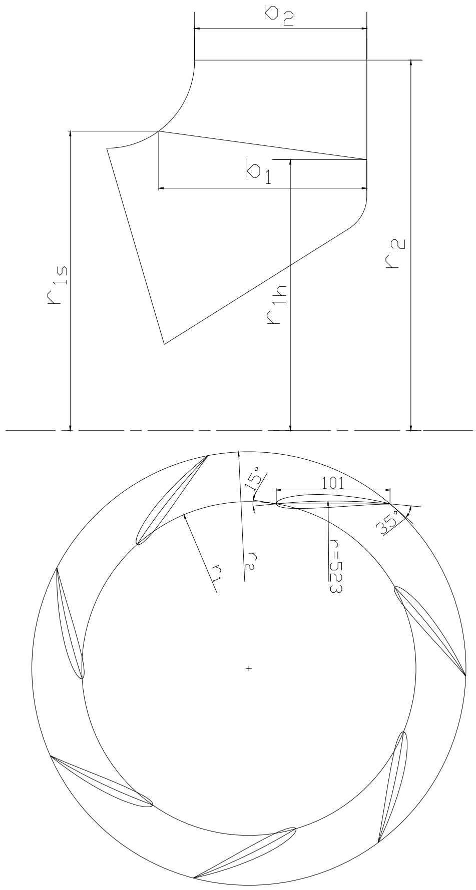

Fig. 5 The drawing of impeller

Tab. 1 Geometrical parameters and their values

Geometrical parameter	Value
r_2	198.5 mm
r_{1s}	156.45 mm
r_{1h}	141.55 mm
$r_1 = (r_{1s} + r_{1h})/2$	149 mm
b_2	90 mm
N_z	7
n	800 RPM
T_1	293 K
p_s	101325 Pa
q_m	17 $m^3/\min$

The total pressure efficiency:

$$\eta = \frac{(p_{sg2} - p_{sg1})q_v}{T\omega} \quad (4)$$

Approximation function for RSM model

In this paper, the objective function is approximated by a second-order polynomial. The input design parameters chosen here are the inlet radius r_1 and the outlet width b_2, while the flow efficiency η is chosen as the objective response. Therefore, the present RSM function is

$$\eta = \beta_0 + \beta_1 r_1 + \beta_2 b_2 + \beta_3 r_1^2 + \beta_4 b_2^2 + \beta_5 r_1 b_2 \quad (5)$$

Where β_i (i =0, 5) are the six model coefficients which will be determined by LS. The database needed for LS is generated from DOE coupled with CFD analysis.

DOE method for selecting test points

In order to describe the performance of the impeller in one zone as large as possible, a five-level factorial design is chosen for DOE. Thus five points should be given for each variable. They are respectively assigned a value, which are shown in Tab. 2 and Tab. 3.

Tab. 2 The levels of r_1

$(r_1 + (r_2 - r_1)$*10%	153.45
$r_1 + (r_2 - r_1)$*5%	151.225
r_1	149
$r_1 - (r_2 - r_1)$*5%	146.775
$r_1 - (r_2 - r_1)$*10%)	144.55

Tab. 3 The levels of b_2

$b_2 + b_2$ *10%	153.45
$b_2 + b_2$ *5%	151.225
b_2	149
$b_2 - b_2$ *5%	146.775
$b_2 - b_2$ *10%	144.55

The test area for DOE is the combination of r_1 and b_2. The number of collected points is 25. The distribution of the points is shown in Fig. 6.

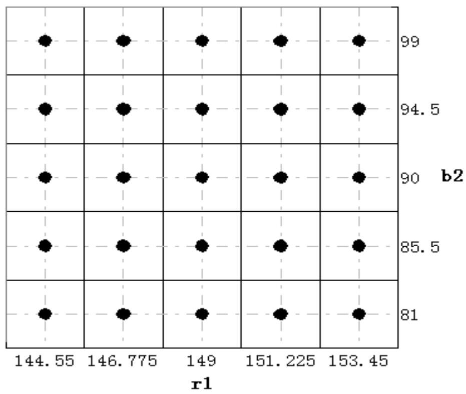

Fig. 6 the distribution of the points

CFD simulation for generating database

After the test points are determined, CFD analysis is then used to evaluate the performance of the impeller at each point to generate the database which is required to build the RSM model. At each point, CFD is carried out in single blade passage. Take the r_1 =149mm, b_2 =90mm for example, the main design and geometrical specifications of the impeller are shown in Tab. 1. Fig. 7 shows the mesh of a single passage. In order to improve the accuracy of CFD, some rules are obeyed, such as mesh's quality, the relationship of mesh and turbulent model.

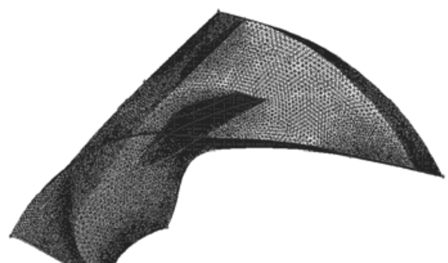

Fig. 7 Mesh of single passage to flow domain

The distribution of the test points and the CFD results are shown in Fig. 8.

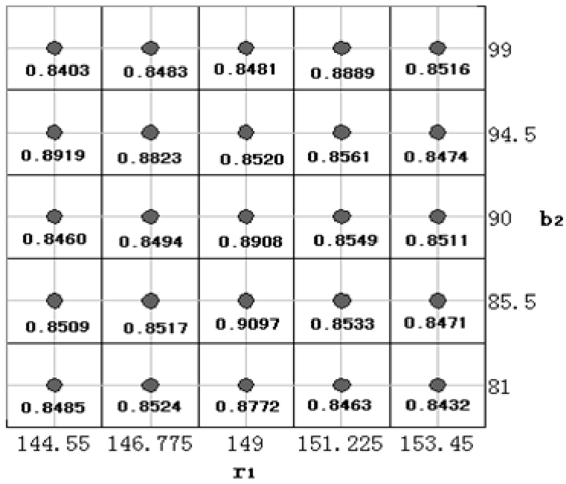

Fig. 8 The test points and the simulation results

Least-squared regression (LS) method for building the model

A. Response surface equation of the whole zone

The test points and their responses are marked in Fig. 8. The response surface equation of the whole zone is obtained by using the database of CFD. LS method is applied to all the test points and their responses. The values of the coefficients β_i are obtained and shown in Tab. 4.

Tab. 4 The model coefficients

β_0	β_1	β_2
-22.9336	0.3082	1.9571E-2
β_3	β_4	β_5
-1.0335E-3	-1.0131E-4	-7.863E-6

The response surface equation is **Eq.5**

$$\eta = -22.9336 + 0.3082 r_1 + 1.9571 e^{-2} b_2 - 1.0335 r_1^2 - 1.0131 e^{-4} b_2^2 - 7.863 e^{-6} r_1 b_2 \quad (5)$$

In order to study the accuracy and the fitting error of the response surface equation, a comparison is made between the test results and the results obtained from the response surface equation by keeping one parameter constant, which is shown in Fig. 9 to Fig. 13.

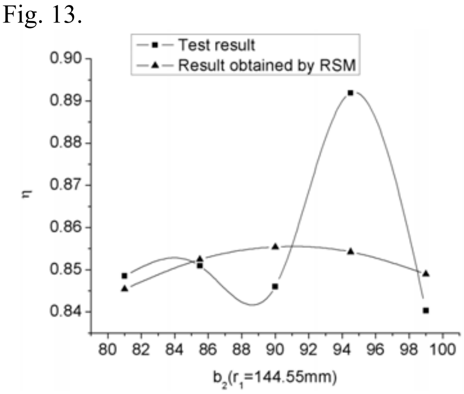

Fig. 9 Comparison of responses while r_1=153.45mm

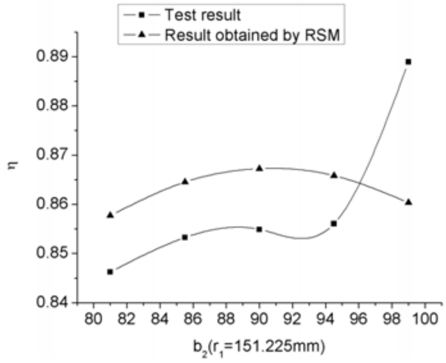

Fig. 10 Comparison of responses while r_1=151.225mm

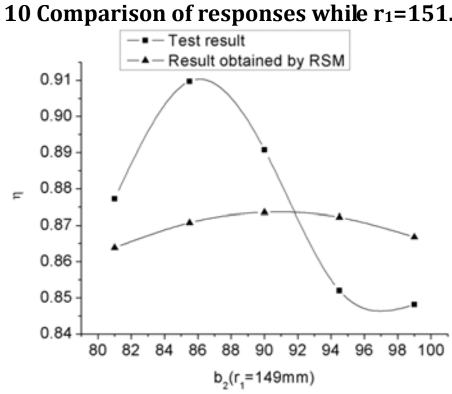

Fig. 11 Comparison of responses while r_1=149.0mm

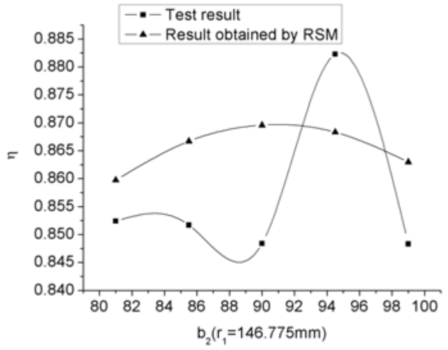

Fig. 12 Comparison of responses while r_1=146.775mm

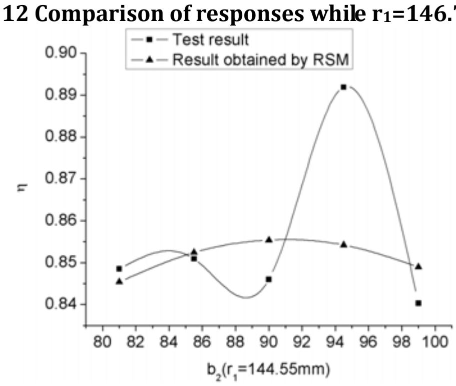

Fig. 13 Comparison of responses while r_1=144.55mm

It can be seen From **Fig. 9** to **Fig. 13** that the test results and the results obtained from the response surface equation are not consistent with each other. In order to improve the accuracy and reduce the fitting error, the whole test area is divided into four subzones in Fig. 14 and the response surface equation is established for each subzone respectively.

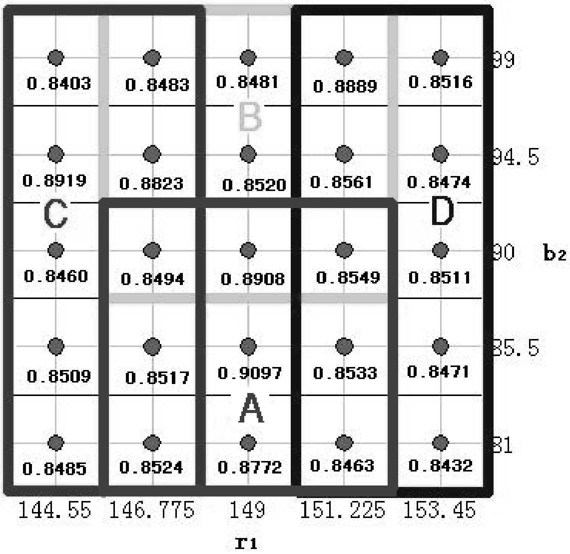

Fig. 14 The distribution of subzones in test area

B. Response surface equation for each subzone

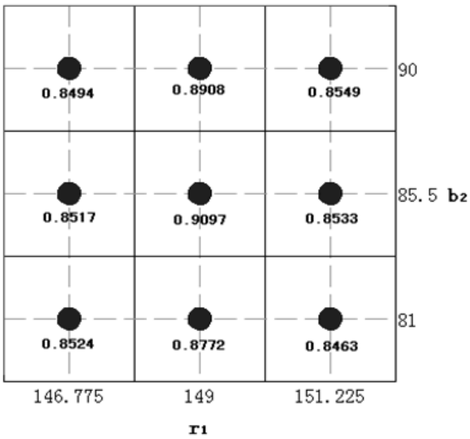

Fig. 15 Test result of subzone A

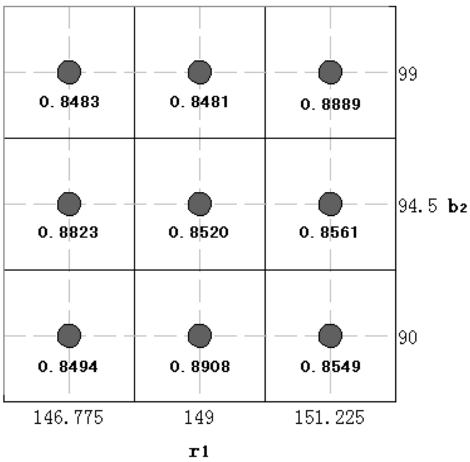

Fig. 16 Test result of subzone B

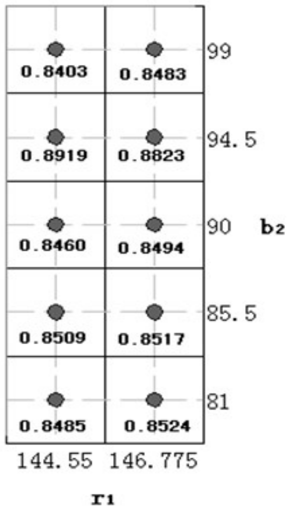

Fig. 17 Test result of subzone C

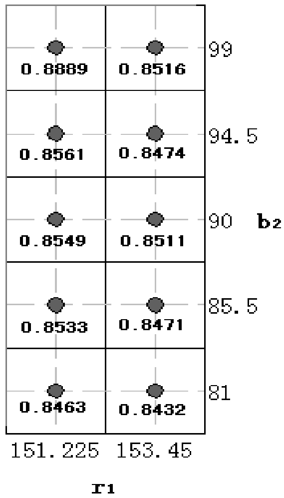

Fig. 18 Test result of subzone D

In each subzone, LS method is applied to all test points and their responses to obtain the value of the coefficients β_i which are shown in Tab. 5.

Tab. 5 Coefficients of the model

Serial number	β_0	β_1	β_2
A	192.332	-2.649	0.136
B	-9.942	0.239	-0.150
C	0.126	-0.104E-1	0.318E-1
D	-0.83	-0.298E-1	0.955E-1
Serial number	β_3	β_4	β_5
A	0.853E-2	-0.187E-2	0.124E-2
B	-0.111E-2	0.143E-4	0.989E-3
C	0.444E-4	-0.156E-3	-0.214E-4
D	0.291E-3	0.781E-4	-0.700E-3

So far, the response surface equation for each subzone has been fully established as follows,

$$\eta_A = 192.332\text{-}2.649r_1 + 0.136b_2 + 0.00853r_1^2 \quad (6)$$
$$\text{-}0.00187b_2^2 + 0.00124r_1 b_2$$
$$\eta_B = \text{-}9.942\text{+}0.239r_1 \text{ -}0.150b_2 \text{-}0.00111r_1^2 \quad (7)$$
$$+0.0000143b_2^2 + 0.0009887r_1 b_2$$
$$\eta_C = 0.1258\text{-}0.0104r_1 + 0.0318b_2 + 0.0000444r_1^2 \quad (8)$$
$$-0.000156b_2^2 \text{-}0.0000214r_1 b_2$$
$$\eta_D = \text{-}0.83\text{-}0.0298r_1 + 0.0955b_2 + 0.000291r_1^2 \quad (9)$$
$$+0.0000781b_2^2 \text{-}0.0007r_1 b_2$$

In order to study the fitting accuracy of the response surface equation for subzones, the test results and the results obtained from the response surface equation of subzone A are compared in **Fig. 19**, **Fig. 20**, and **Fig. 21**. As show in these figures, a good agreement is obtained. Though only the two kind results of subzone A is compared in this paper, the same result can be concluded for the other subzones. In **Fig. 19**, **Fig. 20**, and **Fig. 21**, the results obtained from the response surface equation of whole area is also compared with the test results and the results obtained from the response surface equation of subzone A, it can be obtained that the fitting accuracy of the results obtained from the response surface equation of subzone A increasing distinctly.

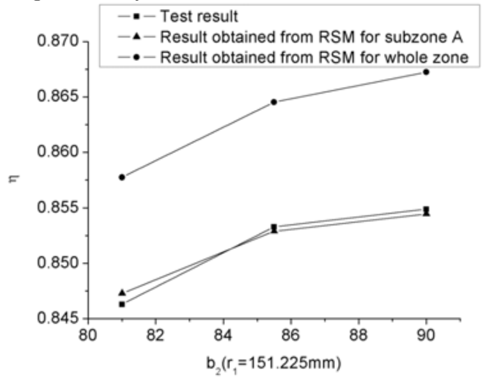

Fig. 19 Comparison of responses while r_1=151.225 in zone A

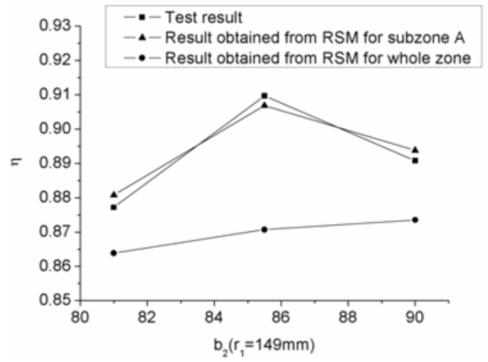

Fig. 20 Comparison of responses while r_1=149 in zone A

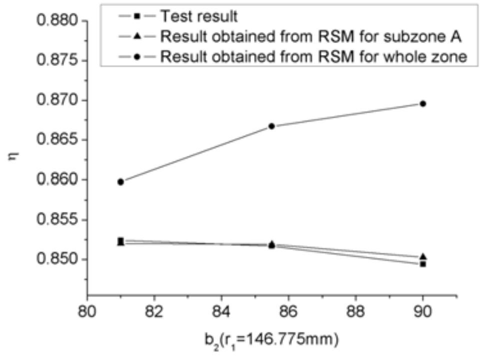

Fig. 21 Comparison of responses while r_1=146.775 in zone A

GA application for determining the optimum configuration

In this section, GA is applied to the four RSM functions of each subzone to determine the optimum configuration [12]. The optimum results of each RSM function are compared with each other to decide the best one.

Tab. 6 shows the optimum results of each RSM function obtained by GA. It can be concluded that the best design is r_1 =151.2, b_2 =86.8 and the efficiency η=0.906.

The real impeller shown in Fig. 4 has been used for a long time in Daikin, Ltd., and its parameters are r_1 =149, b_2 =90. It is the prototype of optimization. The optimum result of subzone B is very close to the real parameter. But according the optimum result of subzone A, the impeller's r_1 and b_2 ought to be changed.

Tab. 6 GA calculation results

Serial number	r_1 (mm)	b_2 (mm)	η
A	151.2	86.8	0.906
B	148.7	90.8	0.874
C	146.77	91.4	0.864
D	151.26	98.9	0.88

Convergence process of GA in seeking optima in subzone

In subzone, the convergence of GA is good. Such as, in subzone B, the convergence process is shown in the **Fig. 22**. When GA obtain the max fitness, the generation is 132, the variables r_1=148.7436mm, b_2=90.83099mm, and the chromosome is 011110001010000000001000101110100011.

The figure of the response surface equation is shown in **Fig. 23**. It can be obtained that the max fitness locates at the neighborhood of r_1=149mm, b_2=90mm.

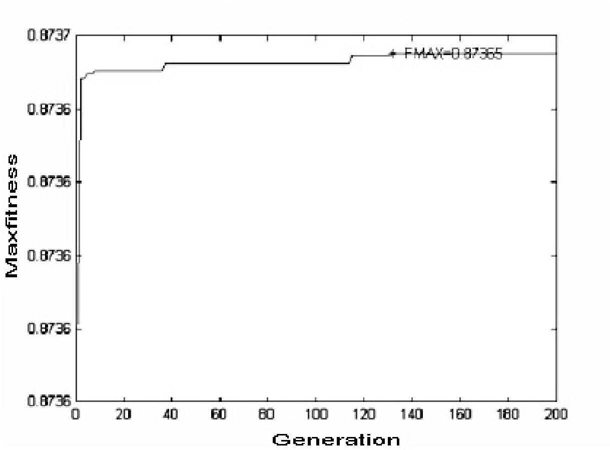

Fig. 22 convergence process of GA in subzone B

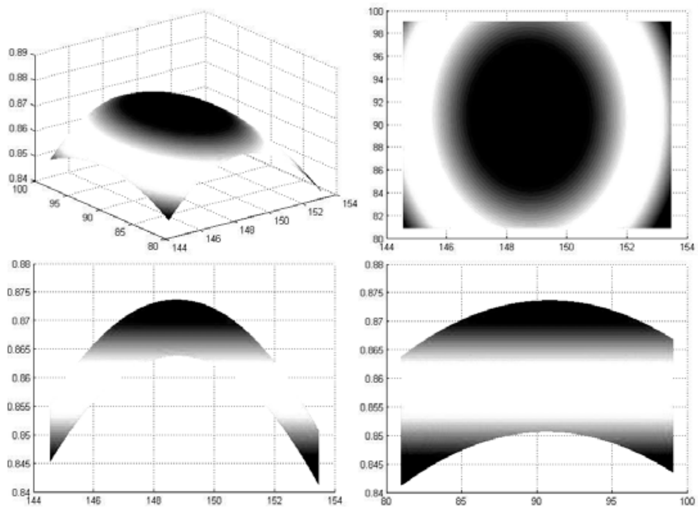
Fig. 23 The response surface of subzone B

In Tab. 7, different crossover probability and mutation probability are used for GA in subzone B, their convergence point almost at the same point and the max fitness is the same one.

Tab. 7 Max fitness of different crossover probability and mutation probability in subzone B

N	P_m	P_c	Precision	r1	b2	η
1000	0.7	0.01	0.00001	148.7404	90.86695	0.873654
1000	0.8	0.01	0.00001	148.7424	90.7913	0.873654
1000	0.8	0.005	0.00001	148.7436	90.79447	0.873654
1000	0.7	0.005	0.00001	148.7373	90.82652	0.873654
1000	0.8	0.01	0.0001	148.7425	90.84461	0.873654
1000	0.8	0.01	0.00001	148.743	90.83358	0.873654
1000	0.8	0.01	0.00001	148.7473	90.83473	0.873654

CONCLUSION

In this paper, the RSM combined with GA is applied to the optimization design of centrifugal fan impeller. The results show that the optimal design structure of centrifugal fan impeller could be determined through the selection of response surface model, the design of experimental point, the simulation with CFD software for establishing database, Least-squares fitting for coefficients and GA.

For the whole area, from **Fig. 9** to **Fig. 13**, it can be obtained that the second order model is not accurate and the fitting accuracy ought to be improved. In order to cut this drawback, the area is divided into several subzones and sub-piece fitting is used for each subzone. Two kinds of results of the subzone A is compared and it can be obtained that the fitting accuracy is improved. So it can be concluded that the drawback of the second order model can be cut by dividing the zone into several subzones.

If higher order model is used, the fitting accuracy may be improved, but more data are needed because of the increase in the number of coefficients, which results in a more computational work.

The optimization result proves that the RSM method by dividing the whole area into several subzones is feasible for the optimization design of the centrifugal fan impeller. Compared with the traditional optimization design strategy, the RSM method can not only reduce the design time and the cost but also compensate for designer’s lack of experience.

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