Ninth International Conference on CFD in the Minerals and Process Industries
CSIRO, Melbourne, Australia
10-12 December 2012

MULTI-COMPONENT GRANULAR SEGREGATION IN A ROTARY CLASSIFIER

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ABSTRACT

Segregation and mixing of granular mixtures is important to the minerals, food processing and pharmaceuticals industry to name just a few. It has recently been demonstrated that the rotary classifier is a suitable device for separating out binary granular mixtures, i.e. mixtures composed of only two different particle types. However, most practical granular mixtures are composed of multi-component particle types. We therefore study, using the DEM method, the capability of the rotary classifier to segregate mixtures where the particles differ either in size or density. We find that segregation is more significant for size varying systems than for density varying systems. We relate this observation to the underlying physical mechanisms.

INTRODUCTION

Granular matter represents one of the scientifically least understood yet one of the industrially most important areas of physics (Duran, 2000, Bagnold, 1941, Jaeger and Nagel, 1992, Mehta, 1994, Ristow, 2000). Segregation in granular materials occurs when moving particles differ in some fundamental property such as size, density or shape. Segregation in granular media can occur in a rotary classifier (Ottino and Khakhar, 2000, Khakhar *et al*, 1997, Meir *et al*, 2007, Pereira *et al*, 2011) which consists of a cylindrical drum roughly half-filled with the granular media. The drum is placed with its cylindrical axis perpendicular to the gravitational field and then rotated slowly about this axis (one revolution per minute is a typical speed). After a revolution, particles tend to segregate. For example, in a binary granular medium (i.e., the media consists of two different particle types) the smaller and/or the denser particles segregate to the centre, while larger and/or less dense particles segregate to the periphery. Here we extend our studies to consider the more representative case of multi-component granular mixtures (with 3 or 4 components) where the particles differ either in their size or density.

DEM MODEL & SIMULATION MEASURES

The simulation method which we use (DEM) is now a well-established and mature technique which has been extensively developed by us for a wide variety of granular flows (Cleary, 1998a and b, 2004). Here we very briefly describe the important aspects of this technique, which are important for our purposes and refer the reader to more detailed descriptions elsewhere (Walton, 1994, Cleary, 1998b, 2004).

DEM models particulate systems whose motions are dominated by collisions. It follows the motion of every particle and object in the flow and models each collision between particles and between particles and objects (i.e., inner surface of rotating drum). All forces and torques on each particle and object are summed and the equations of motion are integrated to give the resulting motion of these bodies. The collisions between particles and/or objects are modelled such that they are allowed to overlap. The amount of overlap and relative velocities between particles determine the collisional force via a contact force law. We use a linear spring and dashpot model to predict the collision dynamics. Other important parameters such as friction and coefficient of restitution are included in the model and specific values of all these can be found in a recent study (Pereira *et al*, 2011). The simulations reported in this study are carried out at angular rotation of the cylinder of one revolution every 60 seconds. The flow is on the border of the avalanching and rolling regimes (Meir *et al*, 2007). We shall use spherical particles of varying size and densities (which will be specified below).

We have previously studied binary mixtures of particles which differ only in their density in quite some detail and made comparisons with experiments (Pereira, 2011). Besides the qualitative pictures of the segregation patterns, we also give two other quantitative measures to evaluate the amount of segregation. The first one calculates the average centre-of-mass (radial centroid) for each type of particle as a function of time. The radial centroid values have the overall centre-of-mass subtracted from them and then scaled with the cylinder radius. With this measure we can not only obtain a quantitative measure of the segregation but also a temporal evolution of the segregation. The second measure is an overall segregation measure where we divide the simulation domain into small cubic boxes with an edge length of approximately 5 particle diameters. Then we count the number of each particle type in each of the boxes and calculate the deviation from a perfectly mixed (homogenous) mixture of particles (i.e., equal volume of each of the components). We use the following definition for segregation (between two components, α and β):

$$\Phi_{\alpha\beta} \equiv \frac{1}{V}\sum_{i=1}^{N_{cell}}\left(V_\alpha(i) - V_\beta(i)\right)^2 V(i) \,, \qquad (1)$$

where the total number of small cubes is N_{cell}, V is the total volume of particles in the domain, while volumes in the *i*th cell are indicated with (*i*). In a perfectly mixed sample, the volumes of each of the components should be the same in all cells, so that the difference in (1) is zero. Hence Φ is zero. For a segregated sample, the difference in (1) is non-zero and so Φ will be non-zero. The more segregated the sample is, the larger the numerical value of Φ. The second factor in the sum in equation (1) gives a weighting to cells with more particles in them. For samples with more than two components we give a value for Φ with each of the other components and also with the sum of the other components.

RADIAL SEGREGATION IN THE CLASSIFIER

Here we study the radial segregation that occurs in the rotary classifier. In most practical cases the length of the classifier is larger than the diameter. As has previously been shown (Pereira *et al*, 2011), both axial and radial segregation occur in the classifier. However, the axial segregation is limited to a region of about 10-15 particle diameters from each wall. In the middle region the axial segregation does not vary. To limit the numerical size of the simulations we consider a thin periodic slice (in the axial direction). The periodic length is about 10 particle diameters (2 cm) and diameter of the cylinder is 10 cm (so R = 5 cm). Particles exiting on one side of the slice then re-enter on the opposite side. We firstly consider segregation in a mixture where particles differ in density and then segregation where particles differ in size.

Density segregation

It has been shown that a binary mixture of spherical particles in a slowly rotating drum forms a "core" or "sun" pattern with the denser particles in the centre and lighter ones on the outside (near the cylinder wall). See Pereira *et al* (2011) for typical segregation patterns which can evolve.

Here we use spherical particles with a diameter of 2 mm. The lightest particles have a density of 2595 kg/m^3 (glass density). Densities from here on will be quoted as a multiplicative factor of this glass density. In Table 1 we give the overall segregation values as well as the maximum difference between the equilibrium (radial) centroid values for a range of binary mixtures. In these simulations (Table 1) the value of Φ varies up to about 7×10^{-12} (for large density ratio, i.e. greater than 6). The maximum (equilibrium) normalised difference in centroids is also given in Table 1.

Particle densities (based on glass)	Φ $(\times10^{-12})$	ΔR
1 , 2	4.4572	0.13
1 , 2.97	5.4325	0.13
1 , 4.38	6.6117	0.11
1 , 6	7.1482	0.14
1 , 8.5	7.0397	0.13

Table 1: Overall segregation values and maximum normalised difference in radial centroids, ΔR, for binary density particle mixtures.

We now consider the segregation for a ternary granular mixture, where we have equal volumes of each particle type (1/3 each). We consider four combinations of particle density (see Table 2). Figure 1a displays the equilibrium particle distribution (after 300 secs of rotation) and also centroids of the three component mixture consisting of particle densities of 1, 2 and 4.38. The blue particles are the most dense and segregate predominantly to the innermost region, while the yellow particles are least dense and segregate to the outermost region (near cylinder wall). The red particles are mostly in between the other two components. However, their segregation is not complete with scattering of yellow into the red region and vice versa and scatter of blue particles into the red region and vice versa. Note that the segregation is sufficiently strong that there are no yellow particles in the core or blue particles in the outer yellow region. The densest particles (blue) have a centroid value which is negative and indicates they are closer in to the centre of the cylinder than average. The other two particle types have positive centroid values and so are further from the cylinder centre. The red and yellow curves are comparatively close together which indicates these two particles type are much more de-segregated than the blue particles, whose centroid curve is far apart from the other two curves.

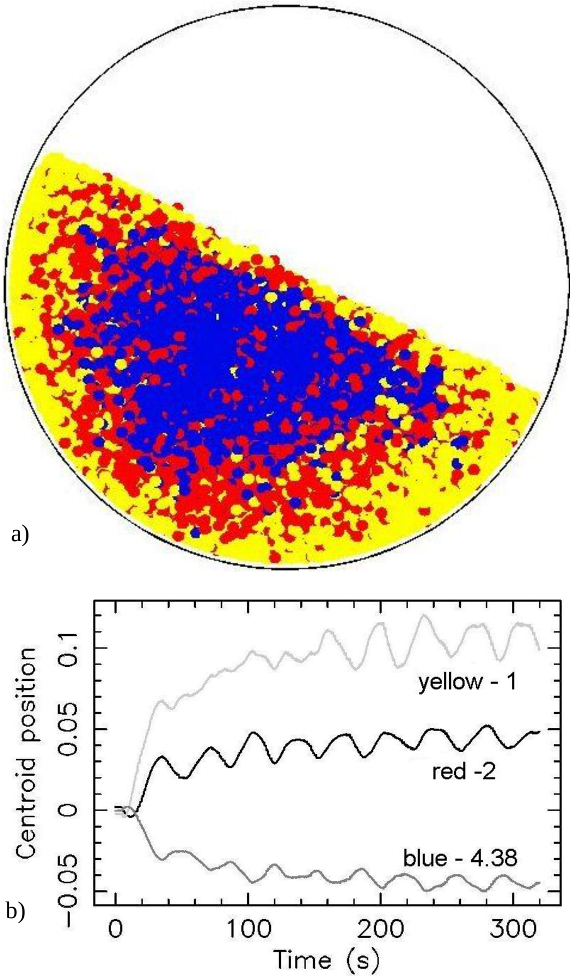

Figure 1: (a) Particle distribution at 305 secs and (b) radial centroid values, as a function of time for a 3 component density mixture. Density are 1.0 (yellow particles), 2.0 (red) and 4.38 (blue).

The time dependence of these curves is also interesting. For the first 100-150 secs, the particles are migrating towards their equilibrium positions. By about 200 secs they have reached their equilibrium positions after which the only variation in average position is due to the periodical rotation of the cylinder (which shows as oscillations in the centroid curves). A possible explanation for these oscillations is as follows: There is a slight shift in the bed (as a whole) during rotation. That is, the bed moves slightly clockwise (with cylinder rotation) before it slips back (anti-clockwise). The maximum difference between centroid values (b/w yellow and blue curves) is 0.17.

The segregation values, Φ for this case are given in Table 2. Since there are now more than two components, we give a segregation value, Φ, of each component with each other component (so for a ternary mixture there will be three such values) and also with the sum of other components (again for a ternary mixture there are three such values). In this way we can determine whether a particular component is segregating from one or all of the other components. The second column in Table 2 gives three segregation values (for particle density 1.0 and 2.0, 1.0 and 4.38, 2.0 and 4.38), while the third column gives a further three segregation values of one component with the other two components. The first entry is particles with density 1.0 segregating from 2.0 and 4.38, second is for density 2.0 segregating from 1.0 and 4.38 and the third is density 4.38 segregating from 1.0 and 2.0. One can see particle densities 1.0 and 2.0 have a low segregation value and so a small amount of segregation. On the other hand, the densest particles have the highest segregation value, both with each of the other components separately and as a whole. All these asymptotic values are consistent with both the particle distributions and centroid curves (Fig. 1).

Particle densities (based on glass)	$\Phi\,(\times 10^{-12})$ (with other component)	$\Phi\ \times 10^{-12})$ (with sum of other components)	ΔR
1, 2, 4.38	0.29,1.79,2.23	2.08,2.52,4.02	0.17
1, 2.97, 4.38	0.88,2.75,2.23	3.63,3.12,4.98	0.14
1, 2.97, 6	0.39,2.82,1.37	3.21,1.76,4.19	0.165
1, 4.38, 8.5	0.44,3.05,1.68	3.48,2.12,4.73	0.13
1, 2, 2.97, 4.38	0.09,0.38,1.09, 0.12,0.68,0.29	1.57,0.89,0.78, 2.1	0.18

Table 2: Values of overall segregation and maximum difference in centroids, ΔR, for 3 (first four rows) and 4 (last row) component density particle mixtures.

Three other three-component mixtures were also considered (see Table 2). The particle distributions and centroid curves are all similar to those shown in Fig. 1. The densest particles consistently segregate the best of all (based on their Φ value) out of the three components. In all cases a fairly uniform layer of dense particles forms near the cylinder walls. There appears to be no systematic trend in Φ values or centroid values with increasing particle densities (i.e. from one simulation to the next), but in future we will investigate this in greater detail.

Next consider a mixture of four different particle densities. The lightest particle type is glass and the other densities are 2.0, 2.97 and 4.38 times that of glass. The mixture has equal fractions of each particle type (i.e. 1/4 by volume each). Figure 2 shows the equilibrium particle distribution and radial centroid evolution curves. The particle distribution (Fig. 2a) shows much less segregation than was observed for the three component mixtures. The same general segregation features are demonstrated with the densest particles (blue) migrating to the core and least dense (yellow) moving to be adjacent to the cylinder wall. However, now there is much less segregation between the various components. The centroid curve (Fig. 2b) shows a general segregation between the various components. The top and bottom curves have similar values to the three component cases (range is now 0.18 compared 0.17 previously), which indicates the core and outermost layer are similarly segregated as in the ternary mixtures. Any differences are now apparent in the two intermediate particle density types, which are forced to be close to each other. So in this case, by virtue of the fact that there are more components, which must necessarily fill the same region (c.f. Fig. 1), different components are forced to be nearer to each other.

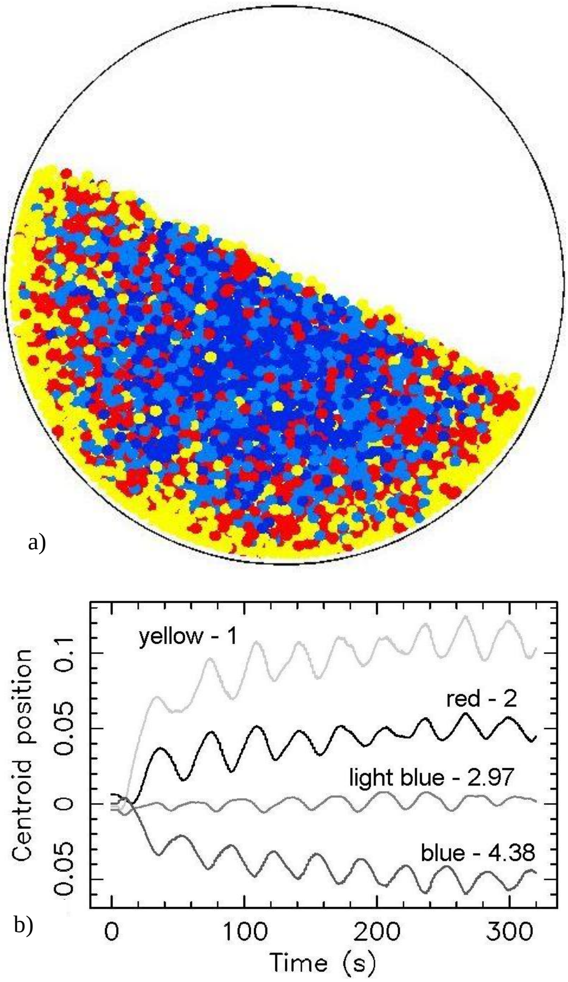

Figure 2: (a) Particle distribution at 305 secs and (b) radial centroid values, as a function of time for a 4 component density mixture. Densities are 1 (yellow particles), 2 (red) 2.97 (light blue) and 4.38 (blue).

The segregation values (see last row in Table 2) for this case bears out the fact that the amount of segregation is much less significant in this case than for three-component mixtures. Even though the densest component has the same density as the densest component for the first two cases for the three component mixtures (viz, 4.38) it has a significantly smaller Φ value (at 2.07 c.f. to 4.02 and 4.98 for three components). All other components also have smaller Φ values than for the three component mixtures. Thus the extent of density segregation in the rotary classifier for four components is significantly diminished, compared to binary and ternary mixtures. We shall discuss physical mechanisms for this later.

Size segregation

We now consider particles all of the same density (that of glass) but with different sizes. The smallest spherical particle used had a diameter of 1 mm. We consider segregation in binary, 3-component and 4-component granular mixtures for a variety of size combinations, (see Tables 3 and 4). Firstly we focus on binary mixtures. Figure 3 shows the particle distribution and centroids variation for this case. The smaller particles segregate towards the centre of the cylinder, while larger particles move towards the cylinder walls. What is clearly noticeable from Fig. 3a is the degree of segregation between the two particle types. The regions of red are almost completely devoid of yellow and vice-versa. The centroids also show a clear and rapid segregation for the two components. The maximum difference between the two curves is 0.2 and the segregation occurs very rapidly - within 60-80 secs or one rotation of the cylinder.

Particle diameter (mm)	Φ $(\times 10^{-12})$	ΔR
1, 2	8.1654	0.20
1, 3	8.6212	0.21
1, 4	7.4413	0.19
2,3	6.1124	0.15
3,4	4.5475	0.12

Table 3: Values of overall segregation and maximum difference in centroids, ΔR, for binary size particle mixtures, for various particle diameters.

The segregation measures for this case (see Table 3) are large and are another indication of the strength of the segregation. We also considered four other binary size combinations, shown in Table 3. They show similar behaviour with the smaller particles forming the (inner) core and larger particles mainly around the periphery. The segregation measures for the three cases with 1 mm particles are roughly the same at around 7.5 to 8.5. For 2 mm/3 mm and 3 mm/4 mm mixtures the amount of segregation decreased significantly. This is borne out by both the overall segregation and maximum centroid differences.

Next we consider ternary mixtures. The first has equal volumes of 1, 2 and 3 mm particles. The equilibrium particle distribution (after about 300 secs) and centroid curves are given in Fig. 4. There is good segregation between all three components with the smallest particles (blue) forming the innermost core, underneath this is a kidney-bean shaped region of red (2 mm) particles and around the outside (nearest to cylinder walls) largest particles (yellow). Regions of blue are almost devoid of other particles and the kidney-bean shaped region of red particles consists almost entirely of 2 mm particles. The centroid curves (Fig. 4b) show that segregation is rapid (within 100 secs) and the difference between the three asymptotic radii is significant. The maximum difference is around 0.21 which is comparable to the binary sized-particle mixtures. The smallest particle centroid curve has the largest separation from the other curves and indicates the smallest particles segregate best from other particle types. The physical mechanism underlying this will be discussed later.

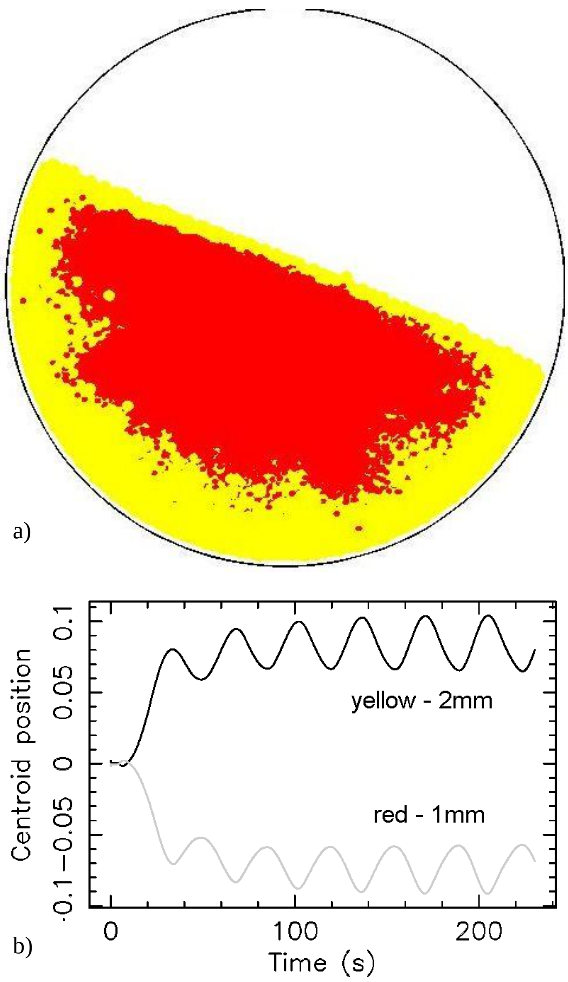

Figure 3: (a) Particle distribution at 235 secs and (b) time variation of the radial centroid location for a binary particle size mixture. Particles are 1 mm (red) and 2 mm (yellow).

Particle diameter (mm)	Φ $(\times 10^{-12})$ (with other component)	Φ $(\times 10^{-12})$ (with sum of other components)	ΔR
1, 2, 3	3.78,4.81,0.43	8.60,4.21,5.24	0.21
1, 2, 4	3.66,4,66.0.43	8.32,5.09,4.09	0.22
1, 2, 3, 4	1.86,2.56,2.73, 0.25,0.38,0.04	7.15,2.49, 2.86,3.16	0.21

Table 4: Values of overall segregation and maximum difference in centroids, ΔR, for 3 (first two rows) and 4 component (last row) size particle mixtures, for different diameter combinations.

The segregation measures (see first row of Table 4) are relatively large between 1 and 2 mm particles and 1 and 3 mm particles. However, it is much smaller between 2 and 3 mm particles. There is a region on the left-hand side of the cylinder in Fig. 4a, where red and yellow particles are have not segregated fully. This is the reason for the low segregation value between these two components. Overall segregation between the three components is also significant (i.e., large) especially for the 1 mm particles.

The other ternary mixture (1, 2 and 4 mm particle diameters) showed very similar results to what we have just discussed. The centroid values and segregation values were almost identical which indicates segregation is also very good for this mixture.

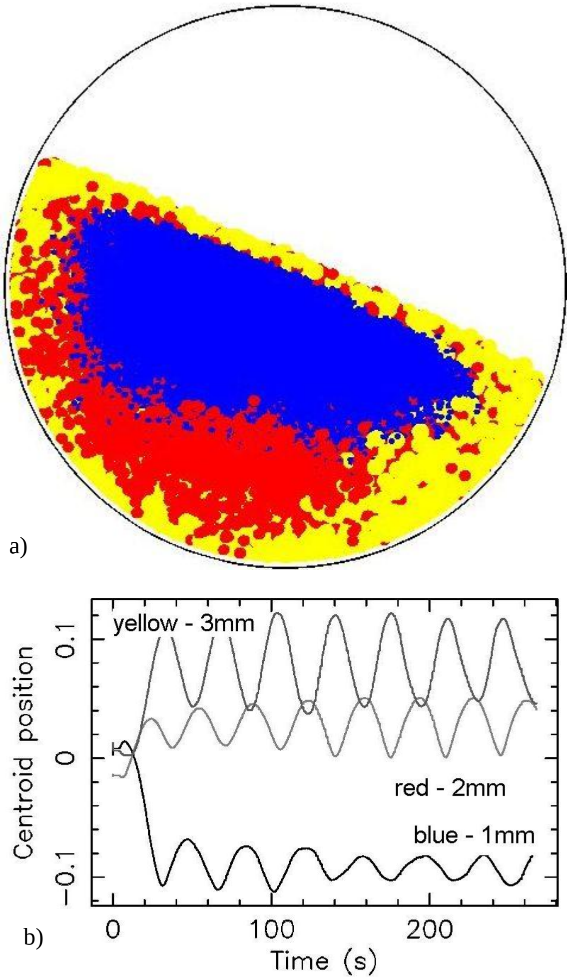

Figure 4: (a) Particle distribution at 300 secs and (b) radial centroid values, as a function of time for a 3-particle-sized mixture. Particles sizes are 1 mm (blue), 2 mm (red) and 3 mm (yellow) particles.

Finally we consider segregation for a 4-component mixture consisting of 1, 2, 3 and 4 mm diameter, spherical particles. The equilibrium particle distribution (Fig. 5a) shows strong segregation for the smallest particles (blue) in the innermost core region and fairly good segregation for the 2 mm particles (light blue), which are predominantly located in a thick band directly below the 1 mm particle region and in a thin layer on top. The segregation for the 3 and 4 mm particles is quite weak. Both coarser particle sizes occupy the region closest to the cylinder walls. The centroid curves (Fig. 5b) for the two largest particle sizes have a significant overlap while the other two particle sizes are well separated. The segregation measures for this case (bottom row of Table 4) are consistent with this description. The first three entries in the second column of the bottom row are large and result from segregation between 1 and 2 mm, 1 and 3 mm and finally 1 and 4 mm particles. This indicates the 1mm particles segregate well from all other components (so the overall value is also large). The segregation between 2 and 3 mm particles is much smaller at 0.25×10^{-12} while the value between 2 and 4 mm particles is similar. Finally, the segregation value between 3 and 4 mm particles is very low at 0.04×10^{-12} indicating these two components remain quite de-segregated.

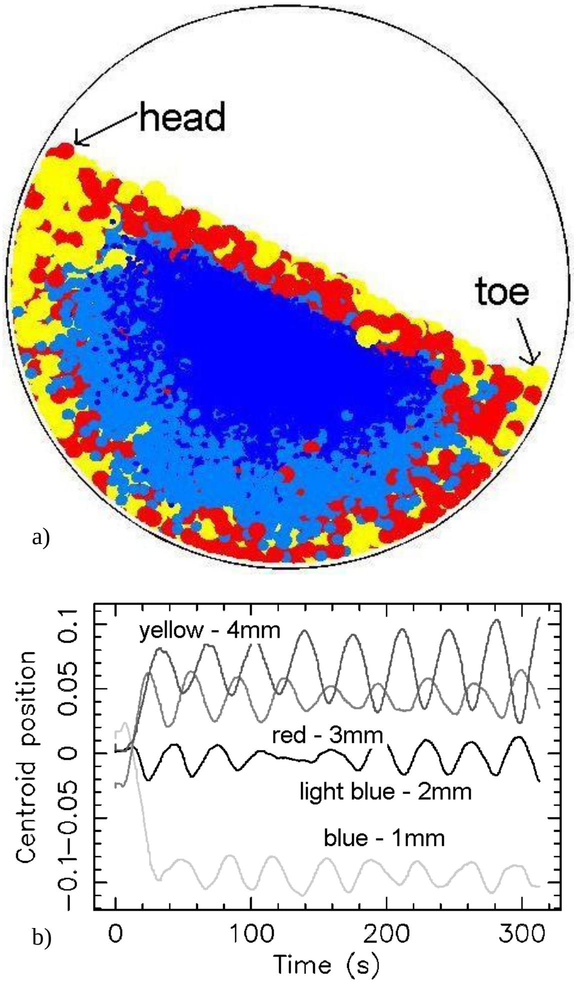

Figure 5: a) Stable segregation pattern in a rotary classifier for a 4-component mixture with 4 mm (blue), 3 mm (blue), 2 mm (red) and 1 mm (yellow) particles. b) Radial centroid variation for each size as a function of time. In (a) we also point out the head and toe of the bed, for later discussion.

Comparison between size and density segregation

Generally we have seen the segregation in mixtures composed of different sized particles is much stronger than for mixtures of different densities. This is demonstrated by all the measures we have considered. The quantitative measures (maximum centroid values and segregation values) are both significantly larger for the mixtures which differed by size rather than density. This indicates that the driving mechanism for segregation is not only different for these two scenarios but it is stronger for particles which differ in size compared to those which differ in density.

The other noticeable difference in the segregation pattern between these two scenarios is that for particle size segregation (2, 3 or 4 components mixtures) the extent of the segregation is larger. A relatively pure (innermost) core and a fairly pure surrounding region of the next smallest component are consistently formed. In contrast, for mixtures whose particles differ in their density the innermost core tend to remain moderately de-segregated (see Fig. 1 and especially Fig. 2) while the region near the cylinder wall tends to be relatively well segregated. This is an important difference and may be useful in any applications of these devices.

For both scenarios the segregation in the rotary classifier results in an onion-like pattern of layers. Firstly, an innermost core of the densest or smallest particles moving out to a layer, furthest from the centre (and closest to the cylinder wall), corresponding to the least dense or largest particles.

PHYSICAL MECHANISMS FOR SEGREGATION

Simulations of multi-component granular mixtures, which can differ either in size or density of particles, have demonstrated that segregation occurs differently for the two material properties. It is well known in binary-sized particle mixtures that segregation is driven by percolation (small particles flow through the gaps created by larger particles). For mixtures whose particles differ in their density, segregation is driven by a buoyancy effect (denser particles sink deeper into a bed of particles than lighter particles). One would expect these are the same driving mechanisms for multi-component mixtures. However, we need to modify the models of these mechanisms somewhat for multi-component mixtures.

We first consider the buoyancy mechanism. It has been shown (Khakhar *et al*, 1997) that the important region where segregation occurs is a layer of about 10-15 particles deep along the top surface of the particle bed – called the *active layer* (see Fig. 18 of Pereira *et al*, 2011). At the head (see Fig. 5a) of the particle bed, particles which have been transported around the bed (via rotation with the cylinder) become free to move (relative to each other). They avalanche down the free surface of the bed. It is now that the buoyancy mechanism becomes important. Denser particles tend to *sink* into the bed, while the less dense flow quickly along the top of the bed, down to the toe (see Fig. 5a). The magnitude of segregation is proportional to $(1-\rho/\langle\rho\rangle)$, where ρ is the density of the particle which has just reached the head of the bed and $\langle\rho\rangle$ is the average density of particles in the active layer. What this implies for a multi-component case is that the average density of the core increases. For example, for a sample with particle densities of 1, 2, 2.97 and 4.38, we should expect the segregation of the densest particle to be equivalent to a binary mixture with a density ratio of about 2. This would imply a much larger amount of mixing than a 1 and 4.38 or 2 and 4.38 binary mixtures.

For mixtures whose particles differ in size, segregation again occurs in the active layer, but now it is dominated by the void volume (between particles). The active layer, itself, is far more dilated than the rest of the particle bed, so has more voids and so more mobility of fine particles. This region has a constant influx of circulating particles passing through it, so there is sufficient free volume for the smaller particles to percolate through and join the central core. Hence the segregation of the smallest particles will tend to be similar to that in a binary mixture.

CONCLUSIONS

We have investigated the segregation of multi-component granular mixtures in a rotary classifier. We have found size segregation tends to be faster and more complete than density segregation. A relatively pure core formed in the case of size segregation. For density segregation, the particles near the cylinder wall are most strongly segregated. We have related this to the underlying physical mechanisms for segregation. Future work will investigate the mechanisms for segregation in greater detail, extend our simulations to include axial segregation and also consider different fractions of the components in the mixtures rather than uniform volume weightings of components.

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