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FINITE ELEMENT SIMULATION OF UNSTEADY FLOWS IN SECONDARY SETTLING TANKS

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ABSTRACT

We present a finite element analysis of unsteady flows in secondary settling tanks. The model takes full account of the distribution and flow features of settling solids, and incorporates the effects of turbulence. The results of numerical simulations are compared against experimental results from tests on full-scale tanks, and show good comparison, over the range of applicability of the model.

NOMENCLATURE

c	concentration
c_μ, $c_{\varepsilon 1}$, $c_{\varepsilon 2}$	constants in (7)
k	kinetic energy
p	pressure
P_1, P_2	rates of energy production
$\boldsymbol{v}$	velocity
V_s	settling velocity
ν_{eff}	effective viscosity, diffusivity
ρ , ρ_r	mass density, density of clear water
σ_s, σ_k, σ_ε	Schmidt number, Prandtl numbers

INTRODUCTION

Sedimentation by gravity is the most common and extensively applied treatment process for the removal of solids from water and wastewater in treatment processes. The secondary settling tank (SST), a crucial component of such processes, has the task of separating the activated sludge from the clarified effluent, and is also used to concentrate the settled sludge and recycle it to the biological reactor. Overall efficiency of the process depends crucially on the performance of the settling tank.

The two-phase flow in settling tanks is unsteady, and effects such as gravity sedimentation, density-driven flow, flocculation, and thickening, all play a major role in determining flow patterns and, ultimately, the tank performance. The determination of the efficiency of such tanks has therefore been the subject of numerous experimental and theoretical studies. Indeed, mathematical models for flows in SSTs of varying complexity and area of application have been developed over the last century. A comprehensive overview may be found in the work by Ekama et al. (1997).

The first numerical two-dimensional simulations of settling tanks were carried out for clear water conditions, or with solids concentrations that were so low that the influence of the solids on the flow field could be neglected (Larsen (1977)). Schamber and Larock (1981) simulated settling tank flows using the finite element method applied to the coupled governing equations for fluid flow and concentration. Density and settling velocity effects were not taken into account. DeVantier and Larock (1986, 1987) carried out a finite element analysis of a two-dimensional model with the dominant features of density-driven turbulent flow incorporated. Stamou et al. (1989) considered several particle classes, with corresponding settling velocities, but did not incorporate density effects. Lyn et al. (1992) implemented the gravity-density term in the vertical momentum equation in order to describe the sediment-induced density currents. The qualitative agreement between predictions and measurements was markedly improved in this way.

Zhou and McCorquodale (1992a) made use of the settling function of Takács et al. (1991), which led to improved modelling of the specific settling behaviour. Krebs (1991a,1991b) used the computer code PHOENICS to compute steady two-phase flow, including the influence of buoyancy flow.

The study of Vitasovic et al. (1997), based on the model proposed by Zhou and McCorquodale (1992a, 1992b), showed close agreement between the observed measurements and the computational results, and thus validated the ability of the model to describe the operation of the clarifier. SettlerCAD (Zhou and Vitasovic (1998)), a dedicated two-dimensional hydrodynamic SST simulation package, follows the model due to Vitasovic et al. (1997). It is finite difference-based and uses the idealized one-dimensional flux theory.

The aim of this work is to report in further detail on a recent two-dimensional finite element analysis of flows in secondary settling tanks (Kleine and Reddy (2005)). As is clear from that work, the problem has a number of features that provide significant computational challenges. First, it is unsteady and highly nonlinear, with nonlinearities arising from the conventional convective terms as well as the exponential dependence of the settling velocity on concentration. Gravity effects also require that great care be taken in the development of solution algorithms to ensure stability. Secondly, turbulence is a key feature of flows in settling tanks, and must be incorporated in the model. Thirdly, the boundary conditions are complex, and great care is required to ensure that they are correctly implemented in the weak formulation on which finite element approximations are based.

In the work by Kleine and Reddy (2005) full details are given of the mathematical model and of the sets of algorithms that are used to solve the resulting nonlinear problem. Here the relevant aspects of the model are reviewed, after which the aim is to focus on some of the practical aspects of the investigation and results, with a view to making recommendations for further studies that are aligned with design needs. In this way it is hoped that simulations of the kind reported here will become a widely used tool in design of settling tanks.

It will be seen later that, overall, the results compare well with experimental results, at least in the ranges of applicability of the model, but that many open questions remain. One of these concerns the incorporation of sludge compaction in the model, while the resolution of other questions will depend on the availability of suitable empirical data. Nevertheless, the study presented here makes a strong case for the use of computational methods as a tool in the design of settling tanks.

MODEL DESCRIPTION

The Mathematical Model

In liquid-solid flows in secondary settling tanks the activated sludge consists of a wide variety of sizes and shapes of particles, and the motion of the sludge particles and that of the fluid are highly coupled. Due to their higher density the suspended fch solids have a tendency towards sedimentation and to accumulation at the bottom of the tank, leading to a stratified flow field. The dynamics of sedimentation are influenced by the interaction between buoyancy and drag forces.

The two phases, water and suspended solids (SS), are treated as a homogeneous fluid mixture with variable density. We base our study on the mathematical model presented by Ekama et al. (1997), for which the governing equations are the following, with points in the domain referred to a cartesian coordinate system or, in the case of axisymmetric situations, to a cylindrical coordinate system.

In light of the Boussinesq approximation the continuity equation becomes the equation for incompressibility

$$\operatorname{div} \boldsymbol{v} = 0 \tag{1}$$

in which $\boldsymbol{v}$ is the velocity field. Making use again of the Boussinesq approximation, the equation of conservation of momentum is given in coordinate-free form by

$$\frac{Dv}{Dt} - \frac{1}{\rho}\operatorname{div}\sigma + \frac{1}{\rho}\nabla p = -g\frac{\rho-\rho_r}{\rho}e_2. \tag{2}$$

Here p is the pressure and $\boldsymbol{\sigma}$ is the extra stress, given by

$$\sigma = \rho\nu_{\text{eff}}(\nabla v + (\nabla v)^T). \tag{3}$$

The effective viscosity is the sum of the kinematic laminar viscosity and turbulent viscosity , which is either a constant or can be obtained from a turbulence model.

The equation for conservation of mass is given by

$$\frac{Dc}{Dt} - \operatorname{div}\left(\frac{\nu_{\text{eff}}}{\sigma_s}\nabla c\right) = \frac{\partial}{\partial y}(V_s c), \tag{4}$$

in which c is the concentration of suspended solids and V_s is the particle settling velocity, which is given as an empirical function of concentration. The eddy diffusivity is expressed here as a ratio of effective diffusivity of the solid concentration to the Schmidt number.

The settling velocity is specified as a function of concentration according to a simplified version

$$V_s = V_{s0}\exp(-nc). \tag{5}$$

of the double-exponential empirical formula due to Takacs et al. (1991). Here V_{s0} is a reference settling velocity.

The density is assumed to be related linearly to the concentration, through the equation

$$\rho = \rho_r + c\left(1 - \frac{\rho_r}{\rho_s}\right). \tag{6}$$

Here ρ_s is the density of the dry particulate solids, c is the solids concentration, and ρ_r is the reference density of clear water.

The k-ε turbulence model is used here (see, for example, Rodi (1993), Wilcox (1993)). This is characterised by the set of equations

$$\begin{aligned}\frac{Dk}{Dt} - \operatorname{div}\left(\frac{\nu_t}{\sigma_k}\nabla k\right) &= P_1 + P_2 - \varepsilon,\\ \frac{D\varepsilon}{Dt} - \operatorname{div}\left(\frac{\nu_t}{\sigma_\varepsilon}\nabla \varepsilon\right) &= c_{\varepsilon 1}\frac{\varepsilon}{k}P_1 - c_{\varepsilon 2}\frac{\varepsilon^2}{k},\\ \nu_t &= c_\mu\frac{k^2}{\varepsilon}.\end{aligned} \tag{7}$$

In addition to the above set of governing equations we need a set of boundary and initial conditions. These are discussed in detail later, in the context of the example problems.

Finite Element Approximations

To formulate the set of spatially discrete equations we proceed in the normal way, by defining finite-dimensional spaces of velocities, pressures, concentrations, turbulent kinetic energy, and dissipation. The equations are rendered into weak form in the usual way (Kleine and Reddy (2005)), and by choosing the polynomial spaces appropriately we arrive at the set of five simultaneous equations

$$
\begin{aligned}
GU &= 0, \\
M_v \frac{dU}{dt} + H_v(U) + S_v U + G^T P &= F_v, \\
M_c \frac{dC}{dt} + H_c(U,C) + S_c C &= F_c, \\
M_k \frac{dK}{dt} + H_k(U,K,E) &= 0, \\
M_\varepsilon \frac{dE}{dt} + H_\varepsilon(U,K,E) &= 0.
\end{aligned} \qquad (8)
$$

Here ***U, P, C, K*** and ***E*** are respectively vectors of nodal velocities, pressures, concentrations, and the two turbulence variables, while the ***H*** matrices are nonlinear functions of the unknown vectors.

The solvability of this set depends crucially on the choice of elements, particularly those for the velocity and pressure (Brezzi and Fortin (1991)). Since in the present context it is important to obtain continuous pressure approximations directly from the solution procedure, an alternative approach, the characteristic-based split stress method (Zienkiewicz and Wu (1991)) is used. This method exploits the existence of the time-discretisation scheme to render stable an element choice in which velocities and pressures are both continuous and of equal order (this would not be possible in a steady problem). We choose the same order of polynomial approximation for all five unknown variables, viz. bilinear approximations on quadrilateral elements.

Finally, it is mentioned that the system (8) of ordinary differential equations is solved by making use of the highly stable, second-order accurate fractional step scheme due to Glowinski and Periaux (1987). In conjunction with this, the projection scheme developed by Chorin (1968) and Van Kan (1986) is used to solve iteratively for the velocity and pressure.

RESULTS

Numerical results for the model presented are illustrated using as a benchmark the tests carried out by de Haas (1998) on SSTs at the Darvill wastewater treatment plant, Pietermaritzburg, South Africa. Results for a third tank geometry and flow conditions may be found in Kleine and Reddy (2005). The numerical results here are compared with the simulation results of Ekama and Marais 2000), who made use of the two-dimensional finite difference SST simulation package SettlerCAD (Zhou and Vitasovic (1998)), which is based on the one-dimensional flux theory.

A constant turbulent viscosity was chosen for the Darvill old and new tanks, with constants given as (Ekama et al. (1997))

$$c_\mu = 0.09, c_{\varepsilon 1} = 1.44, c_{\varepsilon 2} = 1.92, \sigma_k = 1.0, \sigma_\varepsilon = 1.3.$$

The geometry of the simplified circular Darvill old and new tanks are shown in Figure 1. The old tank has a horizontal floor and six suction lifts for the settled sludge, while the new tank has a 10 % sloping floor, and the sludge is scraped to a central hopper. A 6.0 m diameter skirt baffle acts as flocculator at the centre well to water depths of 1.8 m and 2.7 m, respectively. A peripheral Stamford baffle at the effluent outlet extends 1.2 m and 1.7 m from the side wall, respectively.

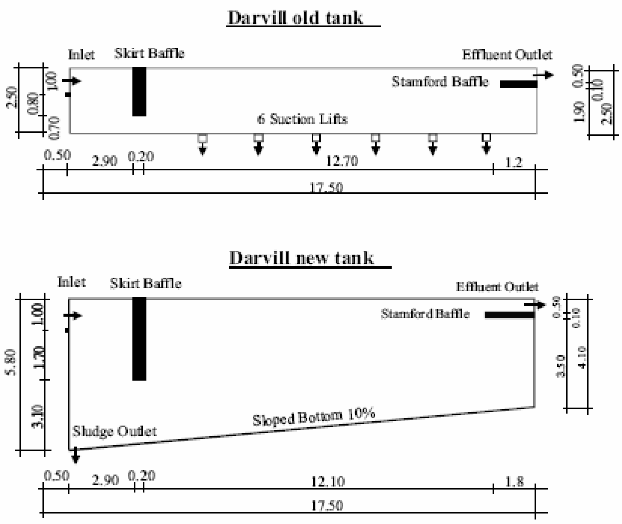

Figure 1. Geometry of the old and new Darvill tanks

Boundary conditions

The boundary conditions are specified as follows: (a) the *inlet conditions* are idealized, in that all values of the unknowns are assumed to be constant and uniformly distributed across the inflow boundary; (b) the gradients or fluxes of velocity and concentration are set equal to zero at the *effluent outlet boundary;* (c) the baffles and the effluent weir are treated as a *reflecting boundary*, so that the normal derivative of the sludge concentration is set equal to zero there; (d) the normal gradients of k and of ε are set equal to zero; (e) the velocity normal to the *bottom* is set equal to the return activated flow divided by the removal outlet area affected; (f) it is assumed that there is negligible change in water surface elevation over the tank, and that the vertical velocity and the horizontal surface traction are set equal to zero at the free surface. The turbulence decreases at the free surface, which leads to a reduced turbulent viscosity. In the numerical model this fact is taken into account by allowing the dissipation to vary inversely with depth, as proposed by Ekama et al. (1997).

Boundary conditions for the concentration are set similarly. The flux at the bottom is set so as to assure a balance of mass, while walls are treated as reflecting boundaries.

Finally, wall functions (Hill and Baskharone (1987)) are used to mitigate the poor behaviour of the k-ε model in the near-wall region.

Presentation and discussion of results

Extensive numerical experimentation, using the full transient model, was carried out to determine the appropriate degree of mesh refinement that would provide results of acceptable accuracy and resolution of the key features of the flow. For the actual simulation quadrilateral meshes of 85 elements in the horizontal and 25 in the vertical direction were used for the old tank, and 58 elements in the vertical direction for the new tank. The average element dimensions of approximately 150mm square were thus sufficient to resolve flow structures which were of the order of 1 – 2 m.

(a) Darvill old tank

In all cases the comparison is made with test 3 in de Haas (1998). In addition to a base flow of 948m^3/h, flows of 13.3% higher are considered, as a means of testing functionality and susceptibility to failure.

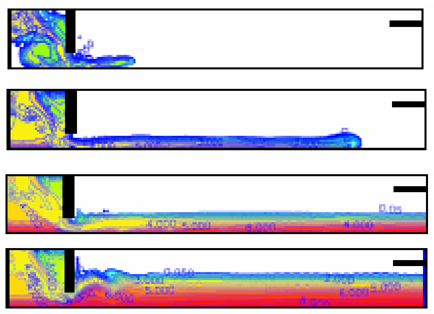

Figure 2. Sludge concentration profiles at (from top to bottom) 100s; 10min; 60min; 6h

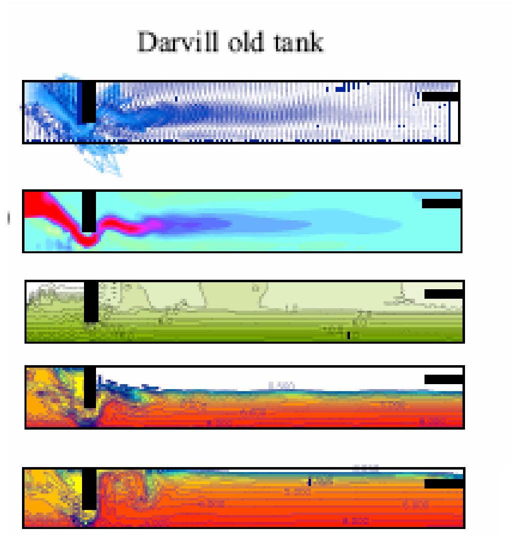

Figure 3. Steady state situation in the Darvill old tank, corresponding to base flow (from top to bottom): velocity field, horizontal velocities, pressure field, and sludge distribution; and bottom-most, corresponding to base flow + 13.3%

The concentration profiles corresponding to unsteady flows are illustrated in Figure 2. Mass conservation and viscous forces influence the flow pattern in the settling section, resulting in complete flow circulation with a strong bottom forward current and a reverse current at the water surface. After 15 minutes the sludge current reaches the end of the tank, the region of the upward current to the effluent outlet. The sludge level rises continuously, as does the sludge concentration.

The steady state case, illustrated in Figure 3, is reached when the potential and kinetic energies have reached equilibrium. The numerical result shows stable behaviour, as in the stress test in de Haas (1998). The sludge blanket reaches equilibrium and does not rise to the outlet for the clarified water.

The bottom-most figure in Figure 3 shows also the steady state result with a 13.3 % higher influent flow. Here the sludge blanket reaches the effluent outlet and the current coming from the inlet chamber is raised to the water surface.

(b) The Darvill new tank

In Figure 4 the computational results at steady state are shown for the new tank, with influent flow 13.3 % higher than the experimental stress test. The incoming fluid cascades down, in the process drawing in fluid from the inlet chamber. During the transient phase the sludge current moves along the bottom to the end of the tank, also causing complete flow circulation.

The new tank exhibits safe behaviour; which can be ascribed to its much greater storage zone at inlet when compared with the old tank. This leads to stable and constant behaviour over a wide range of loading conditions.

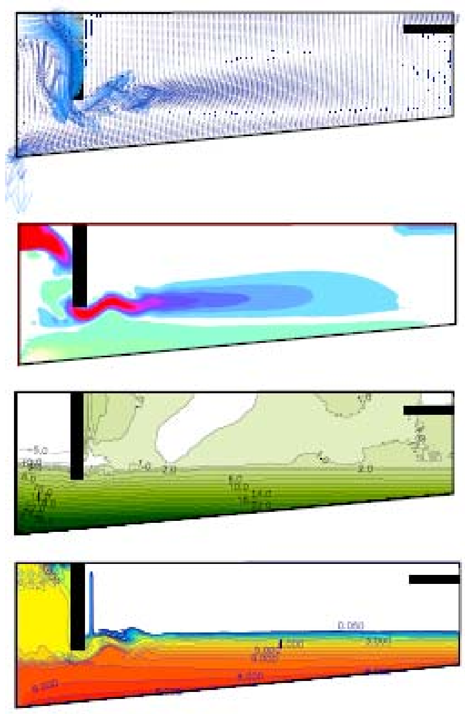

Figure 4. Steady state situation in the Darvill new tank, corresponding to base flow + 13.3% (from top to bottom): flow pattern, horizontal velocity, pressure field, and sludge distribution.

Table 1 gives a summary and comparison with the experimental results, as well as with the computational results obtained using SettlerCAD.

CONCLUSION

It is clear from this work that finite element analyses are able to capture many of the complexities inherent to settling tank operations. These include the highly nonlinear nature of the material model, the necessity to model turbulence, and to model in a realistic way the set of boundary conditions. In particular, it has been possible to show qualitatively the flow patterns and sludge distributions in these two tanks, which in turn has given a better understanding of their distinct behaviour in general, and has confirmed the superior efficiency of the Darvill new tank.

	Darvill old	Darvill old + 13.3%	Darvill new + 13.3%
Experiment	Safe	Not tested	Not tested
Present work	Safe	Fail	Safe
SettlerCAD	Safe	Fail	Safe

Table 1. Summary of tank behaviour corresponding to different flows

While the model and simulations discussed here are in many ways satisfactory there remain a number of areas that require further investigation.

Certain three-dimensional features such as rotating suction lifts cannot be incorporated in a two-dimensional model, and a full three-dimensional simulation is a goal of high priority. A further limitation of the mathematical model used here is that it does not incorporate the effects of sludge compaction at the bottom of the tank. It is therefore not surprising that numerical results of situations in which the sludge compaction is significant, lead to poor predictions of sludge distribution in compaction zones.

The Darvill tanks have been modelled using a constant turbulent viscosity and diffusivity, and in order to achieve the results obtained from measurements the Schmidt number had to be adjusted to a value greater than one, which is in contrast to the recommended values found in the literature.

In general, flow and sludge distribution react very sensitively to turbulent viscosity and diffusivity. The assumption that the turbulent diffusivity, which governs the solids mixing in the tank, is proportional to the turbulent viscosity, cannot be confirmed.

The limitations of the present study are therefore due to a combination of assumptions built into the mathematical model, the absence of key empirical data, and the restriction of the present study to two space dimensions.

Nevertheless, the study presented here provides a range of new insights, as well as a solid platform from which to pursue the range of extensions and improvements alluded to.

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