Design for reliability of stochastic dynamic systems by algebraically derived reduced order models

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Abstract

This paper is concerned with reduced order modeling techniques for the stochastic design optimization of complex engineering systems whose performance is dictated by their transient response. Uncertainties in design and operating parameters are considered. A novel, extended formulation of algebraically derived reduced order models is introduced and applied to the stochastic optimization of a structural component. The numerical studies show the potential of reduced order modeling techniques to enable the use of high-fidelity simulation methods for stochastic design optimization purposes.

Keywords: Reduced order models; Reliability; Stochastics; Uncertainty analysis; Optimization

1. Introduction

Two forms of reduced order models (ROMs) have been used to reduce the computational cost of analyzing engineering systems. The first are physical-based ROMs that reduce the physical interpretation of the system to a model that is easier to analyze. An example of such is modeling an airplane wing with a beam structure, or an electrostatic domain as a capacitor model. The second form of ROMs, termed algebraically derived ROMs in the following, seek to mathematically reduce a large-scale numerical model, while still capturing the essential physical phenomena. Algebraically derived ROMs, utilized in this paper to reduce the size of a finite element model, have proven to be a successful means of reducing the computational costs of a system's response in time. However, the utility of these ROMs lies only in a particular system's time integration, and any changes in the design may render the ROM highly inaccurate. The key missing component for the application of ROMs in a reliability-based design optimization (RBDO) framework, and the focus of this work, is the extension of algebraically derived ROMs (EAD-ROMs) into the space of the design and uncertainty parameters.

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2. Reduced order modeling

Reduced order models are widely used to simulate the transient response of structural systems. Assuming a linear elastic behavior of the structure, the semi-discrete form of the equations of motion for the full-order model can be written as follows:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \tag{1}$$

where $\mathbf{u}$ is the displacement vector of size n, and $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are $(n \times n)$ square matrices. Applying a proportional damping model, the damping matrix $\mathbf{C}$ is a linear combination of the mass matrix $\mathbf{M}$ and linear stiffness matrix $\mathbf{K}$.

The ROM dramatically decreases the size of the system (1) from the number of degrees of freedom n to k, where $k \ll n$. Following a Galerkin type projection approach, the system response can be approximated through k basis vectors ϕ:

$$\boldsymbol{u}(t) = \sum_{j=1}^{k} \eta_j(t)\phi_{(\mathbf{j})} = \boldsymbol{\Phi}\eta(\mathbf{t}) \tag{2}$$

where $\boldsymbol{\Phi}$ is the matrix of basis vectors, and η is the vector of generalized variables. The equations of motion (1) are reduced by substituting the approximation (2) into (1) and multiplying the resulting equation by $\boldsymbol{\Phi}^{\mathbf{T}}$, yielding the following reduced-order system:

$$\mathbf{M}_R\ddot{\boldsymbol{\eta}} + \mathbf{C}_R\dot{\boldsymbol{\eta}} + \mathbf{K}_R\boldsymbol{\eta} = \mathbf{f}_R \tag{3}$$

and

$$\mathbf{M}_R = \boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi} \tag{4}$$

$$\mathbf{C}_R = \boldsymbol{\Phi}^T\mathbf{C}\boldsymbol{\Phi} \tag{5}$$

$$\mathbf{K}_R = \boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi} \tag{6}$$

$$\mathbf{f}_R = \boldsymbol{\Phi}^T\mathbf{f} \tag{7}$$

where $\mathbf{M}_R$, $\mathbf{C}_R$ and $\mathbf{K}_R$ are $(k \times k)$ matrices and $\mathbf{f}_R$ is a vector of size (k). In comparison to the full-order model, the reduced order system (3) allows for time integration of the equations with little computational expense.

For design optimization and stochastic analysis purposes, the ROM needs to capture the changes of the system's response due to parameter variations. The system matrices **M**, **C**, **K** and the basis vectors $\boldsymbol{\Phi}$ in (4)–(7) are all functions of the design parameters p. As the evaluation of the basis $\boldsymbol{\Phi}$ is computationally expensive, the eigenvectors are approximated by a combined approximation technique [1,2,3,4,5], requiring the computation of the first-order derivatives of the eigenvectors with respect to each parameter [6,7,8]. The system matrices are rebuilt at each design change because of the small computational cost. The reader is referred to Allen et al. [9] for further details.

3. Model problem: connecting rod

The EAD-ROM approach is tested on a generic structural component, shown in Fig. 1, which has been frequently studied in the context of shape optimization [10,11]. The rod is clamped at the inner circumference of the left hole, and a transient force is applied to the inner circumference of the right hole. Two geometric parameters, p_1 and p_2, control the horizontal position of the center hole, as depicted in Fig. 1.

All computations are performed within MatLab utilizing the computer aided learning of the finite element method (CALFEM) finite element toolbox [12]. The structural response is assumed to be linear elastic. All optimization problems are solved by a globally convergent version of the method of moving asymptotes [13].

3.1 Reliability-based design optimization with EAD-ROMs

The framework is tested on a RBDO of the rod in Fig. 1. The energy dissipated between $t = 0.40$ *ms* and 1.00 *ms* is used as the objective to be maximized in the design problem. However, a reliability-based constraint will be imposed on the system. The constraint will limit the standard deviation of the dissipation energy to be less than $300\mu J$, making the RBDO problem as follows:

$$\begin{array}{ll} \min\limits_{s} & (-E_{diss}) \\ subject\ to: & \sigma_E - 300 \leq 0 \\ & -4 \leq s_i \leq 4 \end{array} \tag{8}$$

where E_{diss} is the energy dissipated, and σ_E is the standard deviation of the energy dissipated. The constraint on the standard deviation forces the optimization to a more robust design, limiting the sensitivity of the system performance to uncertainties. The standard deviation is found by a Monte-Carlo simulation based on a polynomial chaos expansion. This method was chosen due to its computational efficiency and its ability to obtain the entire probability density function (PDF) of the output response. The derivatives of the standard deviation are obtained by finite differencing. The intended placement of the center hole is treated as both the design variables and the random variables. The design variables represent the mean or intended design, but a normal distribution is assigned to the actual horizontal position of both ends of the hole, each with a standard deviation of 0.20 mm.

The constraint is explored in the design and uncertainty space by sampling uniformly throughout, which is illustrated in the contour plot in Fig. 2. The plot on the left of Fig. 2 represents the contour plots for the standard deviation of the energy dissipated in μJ. The constraint boundary is highlighted in the figure, and the feasible and infeasible regions are identified. The initial design is feasible, but the deterministic design optimum is infeasible. The right half of Fig. 2 overlays the constraint boundary onto the contour plot of the objective, to give the reader the general idea of the RBDO problem.

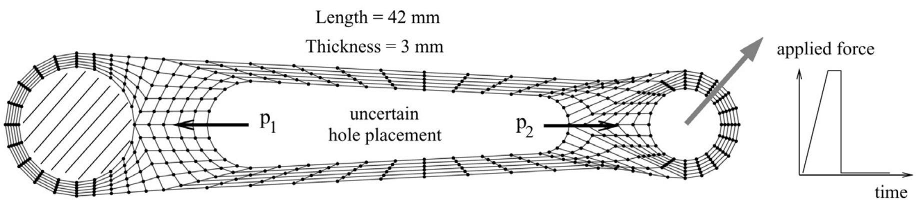

Fig. 1. Model of the connecting rod.

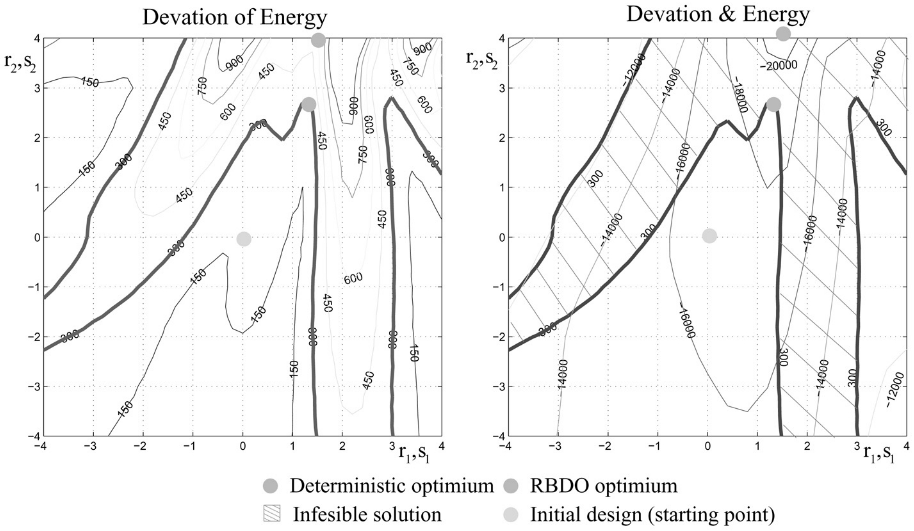

Fig. 2. Stochastic constraint in RBDO problem.

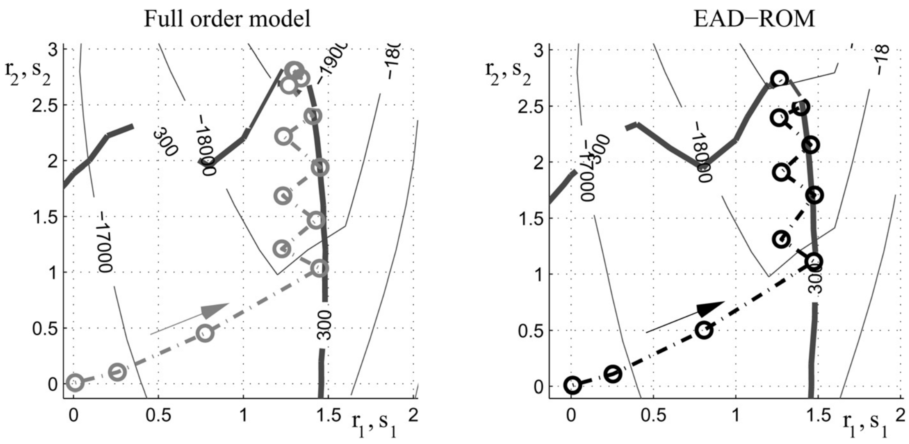

Fig. 3. Convergence of the RBDO problem.

3.2. Full order model versus EAD-ROM

The convergence of the RBDO problem using the full order system analysis is illustrated on the left of Fig. 3, and the convergence utilizing the EAD-ROM analysis is illustrated on the right of Fig. 3. These plots are a zoomed-in portion of the design and uncertainty space illustrated in Fig. 2. The reader may note the similarities between the two optimization procedures in terms of search directions, step sizes, and converged solutions. The EAD-ROM approach sufficiently models a system within a certain region around the design point for which the EAD-ROM was calibrated. Therefore, a trust region framework needs to be established to incorporate the EAD-ROM into optimization problems with large bounds. Recalibration of the design is done at least when a predefined outer bound is reached within the optimization problem. For this paper the EAD-ROM is recalibrated once for each iteration of the global optimization problem. Within each iteration the stochastic analyses and function evaluations for the line search are computed using the EAD-ROM. The EAD-ROM framework converges quickly to the general solution of the RBDO problem, and the results are summarized in Table 1.

The overall computational savings of the EAD-ROM in an optimization framework are dependent upon the costs of recalibration, and the frequency with which

Table 1
Full and EAD-ROM model RBDO results

Analysis method	s_1	s_2	iterations	time (hrs)
Full model	1.31	2.80	11	6.27
EAD-ROM	1.27	2.88	8	3.36

recalibration is required. Again, the EAD-ROM is most beneficial in RBDO frameworks, which require many analyses about a mean design that can be used as the recalibration point. Table 1 demonstrates the effectiveness of the EAD-ROM to save running time of the RBDO problem. The EAD-ROM saves approximately 46% of the time it takes the full order model to run. When moving to larger models, the time saved running an EAD-ROM will be significant.

4. Conclusions

This study has illustrated the potential of utilizing reduced order models for designing dynamic structures under uncertainty. The extended formulation of the algebraically derived reduced order models provides a formal approach to use high-fidelity numerical simulation models for stochastic analysis and design optimization purposes at significantly lower numerical costs. The feasibility and potential of the approach will be studied for larger numerical models undergoing nonlinear, transient, coupled multi-physics phenomena.

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