Active control and drag optimization for flow past a cylinder at high Reynolds number using genetic algorithms

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Abstract

We propose here a new approach to the optimal active control of incompressible viscous flow past a circular cylinder for drag minimization by rotary oscillation. The flow at $Re = 15000$ is simulated by solving 2D Navier-Stokes equations in stream function–vorticity formulation, using high-accuracy compact schemes for space discretization and a fourth-order Runge-Kutta scheme for time integration. The objective function – time-averaged drag – is optimized using a real coded genetic algorithm with respect to the decision variables, the maximum rotation rate and the forcing frequency. The optimal design variables obtained match very well with experimentally obtained values.

Keywords: Flow control; Drag optimization; Genetic algorithms; DNS

1. Introduction

Bluff-body wake control and drag reduction is an active area of research. The efficacy of rotary oscillations in reducing the drag of a cylinder placed in a viscous incompressible flow was demonstrated experimentally for $Re = 15000$ in [1], where drag reduction by a factor of six was reported as compared to the case of the stationary cylinder. Two important non-dimensional parameters needed for the flow description are maximum rotation rate (Ω_1) and the forcing frequency (S_f). The rotation rate of the cylinder at any instant is given by

$$\Omega = \Omega_1 \sin(2\pi S_f t) \tag{1}$$

If A_{max} and f are the physical peak rotation rate and the forcing frequency respectively, then $\Omega_1 = A_{max}d/2U_\infty$ and $Sf = fd / U_\infty$. A number of researchers have attempted numerical studies of this flow, references for which are given in [2]. However, the cases studied are for $Re = 40$ to 3000 – small compared to the case in [1]. Shiels et al. [3] investigated the flow phenomenon for $Re = 15000$ using the viscous vortex method.

The fact that there has been only one numerical study, other than the present paper, for $Re = 15000$, reveals numerical difficulty in computing such flows at high Re. To accurately simulate such a flow, a high resolution method is essential to resolve all spatial and temporal scales. In [4] and [5], details of the requirements of such a scheme are discussed. The numerical method uses the OUCS3 scheme of [4] for spatial discretization and fourth-order Runge-Kutta scheme for time marching. There have been a couple of studies of *active* control of flow past a rotationally oscillating cylinder. Protas et al. [6] demonstrate a rigorous feedback algorithm that finds an optimal rotation rate (Ω) using a conjugate gradient method over a time interval, called the optimization horizon. The study was for $Re = 40$ to 150. A similar study in [7] at $Re = 1000$ reports optimization using a different numerical technique. The optimization functional used in [6] and [7] – energy spent in a time interval – was turned off, furthermore, after a while in [7].

The objective of the present work is the cost-effective application of genetic algorithms (GA) to actively control the unsteady flow at high Re for a circular cylinder. Usually, stochastic methods (as in [8]) are very costly for flow control problems because of high computational costs. To circumvent this, we propose a new optimization strategy for unsteady problems. Here GA is applied over a running time interval (T, $T + \Delta T$) – termed here the GA time-horizon. At the beginning of this interval we start computations with a number of parameter combinations. Rather than finding the optimal solution

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over a large time interval, we look for the best population member of the current GA generation. This GA procedure on successive time-horizons eventually converges to optimal decision variables after a finite number of generations. We demonstrate the success of this approach by reporting the results of a rotary oscillation problem that match well with the values obtained experimentally [1]. It is to be noted that this is the only computational result solving Navier-Stokes equations at $Re = 15000$.

2. Governing equations and numerical method

2D Navier-Stokes equations solved here are given by

$$\nabla^2 \psi = -\omega \tag{2}$$

$$\frac{\partial \omega}{\partial t} + \mathbf{V} \cdot \nabla \omega = \frac{1}{Re} \nabla^2 \omega \tag{3}$$

where ω is the out-of-plane component of vorticity and the velocity is related to the stream function by $\mathbf{V} = \nabla \times \Psi$ where $\Psi = (0, 0, \psi)$. $(\psi - \omega)$ formulation is preferred here due to its inherent accuracy and computational efficiency [5]. The no-slip boundary condition on the cylinder wall is satisfied by

$$\left(\frac{\partial \psi}{\partial \eta}\right)_{body} = \frac{h_2 d\Omega}{2} \tag{4}$$

$$\psi = \text{constant} \tag{5}$$

where h_2 is a scale factor [5] of the orthogonal grid transformation used. At the outer boundary, uniform flow conditions are applied at the inflow (Dirichlet condition on ψ) and convective boundary conditions on radial velocity at the outflow, as shown in Fig. 1.

3. Optimization problem formulation and genetic algorithm

The functional $\overline{C_D}$, for a time interval $[T, T + \Delta T]$ is the time-averaged drag coefficient (C_D] of the cylinder, where

$$C_D(\Omega, S_f) = \frac{\text{Drag of cylinder}}{1/2 \rho U_\infty^2 d} \tag{6}$$

Hence,

$$\overline{C_D} = \frac{1}{\Delta T} \int_T^{T+\Delta T} C_D dt \tag{7}$$

where ΔT is the time-horizon of GA generation. The above equations along with boundary conditions define the system to be controlled with input as (Ω_1, S_f) and

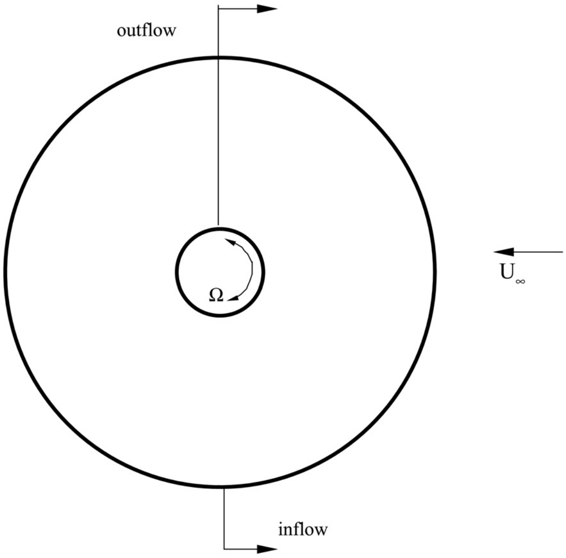

Fig. 1. Schematic of the flow configuration. A 150×50 grid has been used for the presented results for $Re = 15000$.

output as $\overline{C_D}$, that is minimized. An efficient GA of [9] is implemented for the optimization. A population of input parameter vectors (Ω_1, S_f) is selected at each generation using three GA operators (details as given in Deb [9]):

- **Selection**: an operation through which members are selected for reproduction according to their fitness (objective function value). In the present paper, tournament selection, with the participation size of two, is used.
- **Crossover**: an operation through which new members are created from the members selected by the *Selection* operator. A Simulated Binary Crossover (SBX) algorithm has been used in this work.
- **Mutation**: This operation leads to random changes in the newly created members. Polynomial mutation has been used in this study.

The population for the first generation that involves calculation of fitness of members and a single application of all three GA operators, is taken randomly in the allowed decision variable space. The fitness of the members is calculated for a time-horizon ΔT, and instead of repeating the calculations with new population members for the same interval, we move onto the next time-horizon with the best solution of the previous one being the initial solution for all the members. The best member is continued through to the next generation, by including it as a member of the new population.

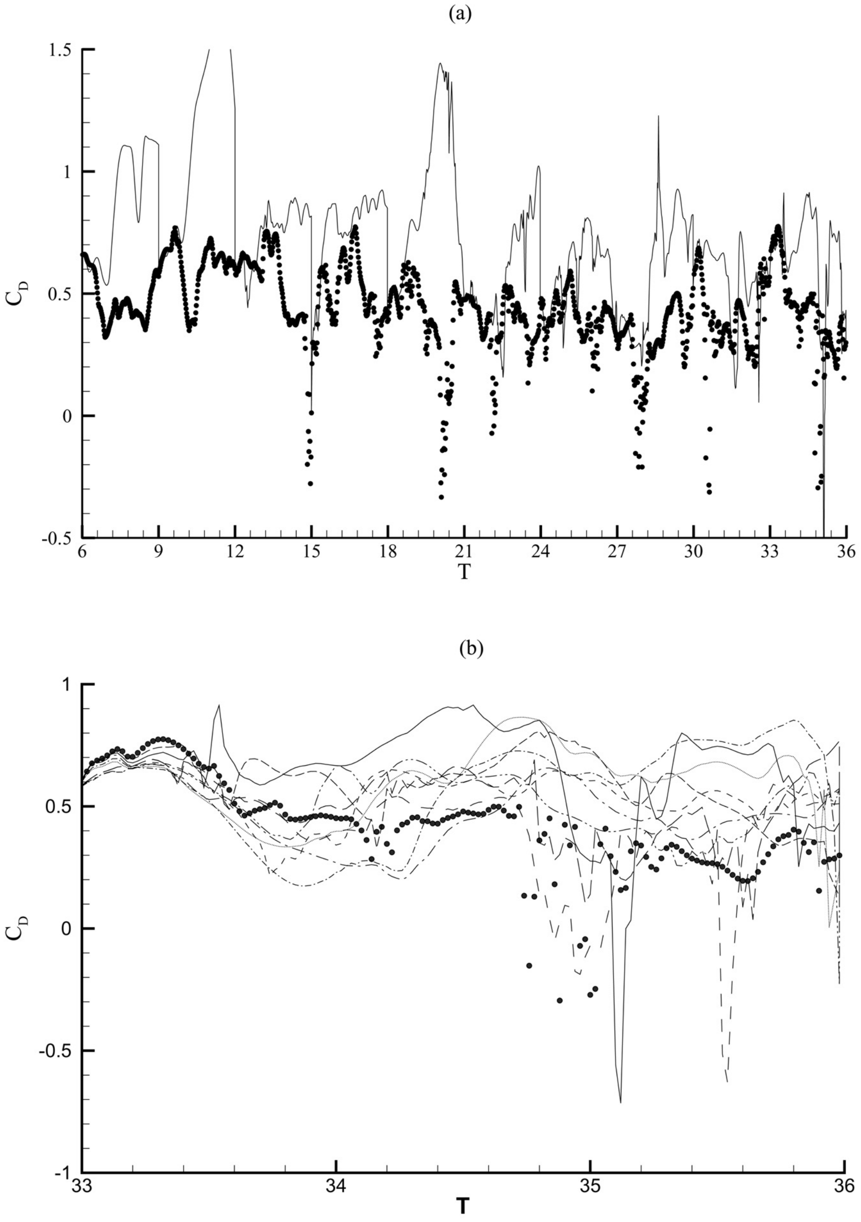

Fig. 2. Variation of drag coefficient with time for $Re = 15000$. (a) Shown are the best (discrete symbols) and the worst (solid line) cases over the first ten generations; (b) drag coefficient variation during the tenth time-horizon is shown for all eleven members of the GA.

4. Results

All the computations were done on an orthogonal grid of size (150×450), with 150 points in the azimuthal direction and 450 points in radial direction. The efficiency and accuracy of the CFD technique used here can be seen by its applications in [5,10]. A population size of eleven was used for the GA. Experimental observations in [1] reveal that there exist a local and a global minimum for drag for this flow configuration. Both the minima lie in the space $0 \leq \Omega_1 \leq 4$ and $0 \leq S_f \leq 3.5$, and these define the decision variable space here. A time-horizon of $\Delta T = 3$ has been used for the present exercise. The flow is computed up to $t = 6$ without any control and subsequently the GA procedure begins. This is done to reduce the undue transient effects following the impulsive start of the flow past the cylinder. The initial solution for the population members for any

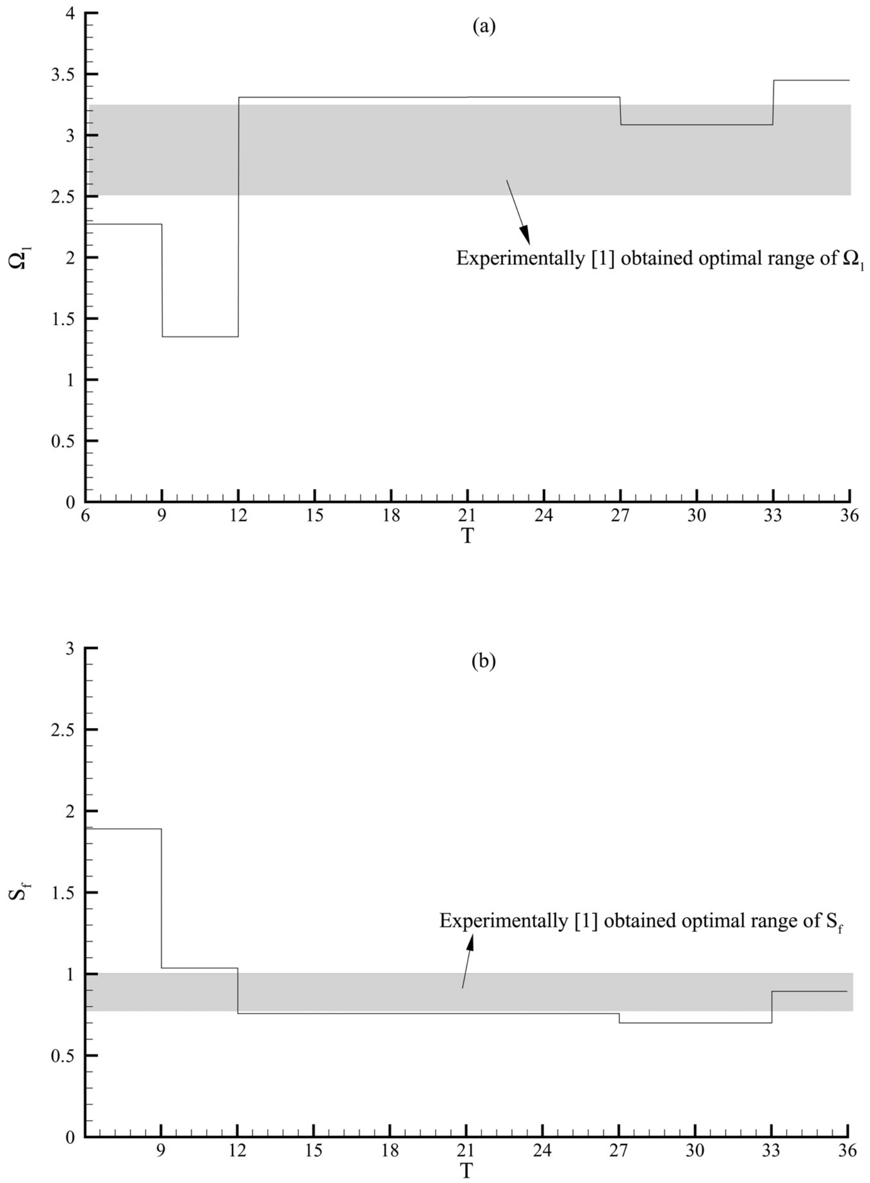

Fig. 3. Variation of decision variables of the best member with time. (a) Peak rotation rate Ω_1, (b) forcing frequency f. Here $[T, T + 3]$ corresponds to one generation of GA, with the first generation starting at $T = 6$. One can clearly see the proper match of these variables with the optimal ranges found experimentally – shown by the shaded regions.

generation is chosen as the best solution obtained from the previous generation. Figures 2 and 3 summarize the results obtained through the application of this algorithm. Figure 2(a) shows the variation of drag coefficient for the best and worst solution for the first ten generations out of the eleven solutions obtained in all time-horizons. In Fig. 2(b) all eleven members of the solution are shown during the tenth generation. The time-averaged $\overline{C_D}$ is brought down from about 0.6 (during $t = 6$ to 9) to 0.4 (during the tenth generation) in ten generations. Figure 3 provides the time history of the best design variables for all the generations. One can clearly see that the optimal solution obtained by the GA stays very close to the experimentally obtained optimal solution shown in the figure by the shaded bands. In Fig. 4, the vorticity distribution in the flow field is demonstrated during the ninth and tenth generations for the best and worst combination of the design variables. The same contour levels are shown in all the frames of the figure. The fact that the best and the worst solutions

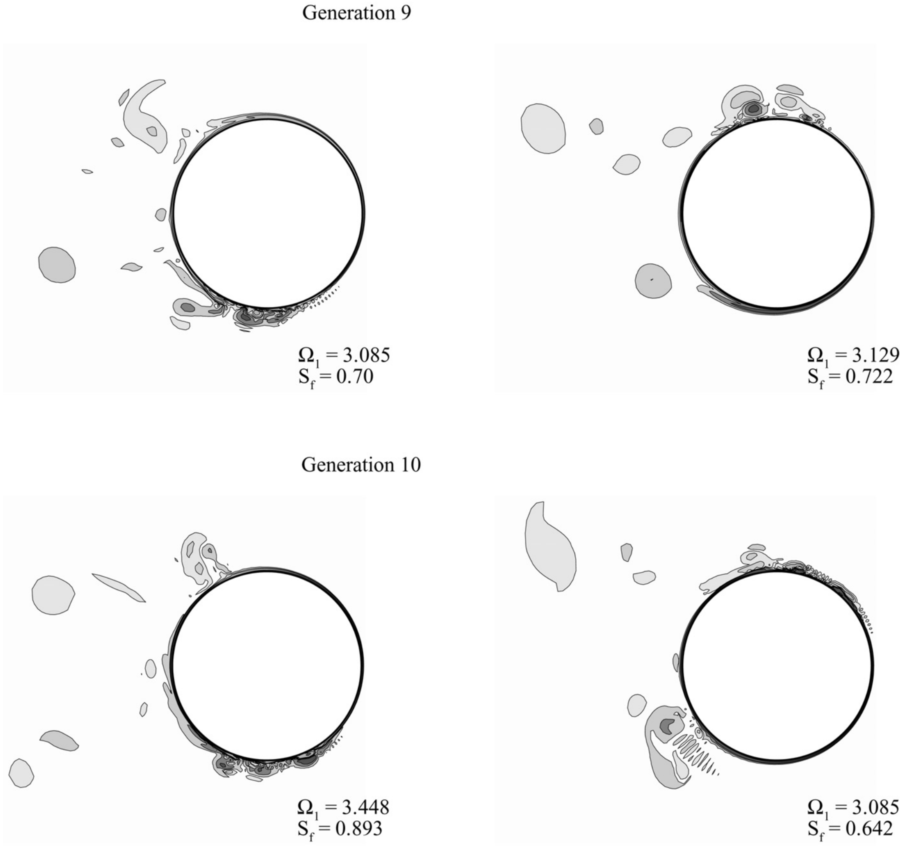

Fig. 4. Vorticity development close to the cylinder surface for the best (left) and the worst (right) members of the GA generations, as shown at the end of the time-horizon of indicated generations.

at the end of the ninth and tenth generations vary marginally, testifies to the fact that the GA achieves near optimality within a few generations itself.

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