A probabilistic approach to improve the static performance of a composite wing

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Abstract

Composite materials are being used more and more in the aerospace industry, to reduce aircraft weight while maintaining or even improving strength and stiffness. Accurate design predictions and improved reliability can only be obtained when inherent variability is included in the numerical modeling process. This paper outlines several reliability analysis methods, and demonstrates their use in the aerospace industry. In particular, the reliability of a composite wing structure with stochastic material properties is assessed.

Keywords: Composite material; Aerospace; Wing design; Variability; Reliability analysis; FORM

1. Introduction

Two parallel trends largely determine the innovations in today's aerospace engineering. The first is the increased use of numerical prediction models to reduce the need for expensive physical prototypes, so that the design cycle can be shortened and the time-to-market reduced. The second is the utilization of strategically placed materials such as composites in order to manufacture lighter aircraft. Compared to metal sheets, composite materials have a higher number of design parameters (e.g. fiber directions, layer thicknesses, material properties, etc.) with higher variability in these parameters. Traditionally, the natural scatter in composite properties is taken into account by applying reduction coefficients to the nominal strength. This results in a smaller weight reduction than strictly possible, without a quantifiable increase in structural reliability. Clearly, much can be gained when the composite materials, with their inherent variability, are accurately represented in the numerical modeling phase. Better design predictions then allow reduction of the knockdown factors without losing reliability.

By incorporating a probabilistic approach in the numerical analysis process, the design engineer obtains a powerful tool to assess the reliability of aerospace design structures, taking into account the variability in material and geometric properties. This allows reduction of weight and costs, while improving the design reliability.

2. Problem definition: composite wing

The considered structure is a real prototype composite wing for a high altitude, long endurance (HALE) unmanned air vehicle (UAV). Figure 1 shows the wing geometry and outlines the characteristics. The wing has been made of AS4 12k/3502 unitape material. The stochastic material parameters, as characterized in [1], are given in Table 1: the Young's moduli along the fiber direction (E_1), perpendicular to the ply (E_2) and along the fiber direction of the stringers (SE_1); the shear modulus (G_{12}); and the mass density (ρ). The Tsai-Hill failure criterion has been used as performance function for the analysis [2].

The composite wing has been examined in a static analysis under gust load conditions following JAR 25 specifications and by clamping it in the section that connects to the fuselage. The intensity of the load configuration has been stochastically modeled through a Load Coefficient to take into account the probability of particularly strong load conditions. A Finite Element (FE) model of the wing has been created, and a particular framework for the necessary computations has been set up. More specifically, NASTRAN [3] has been

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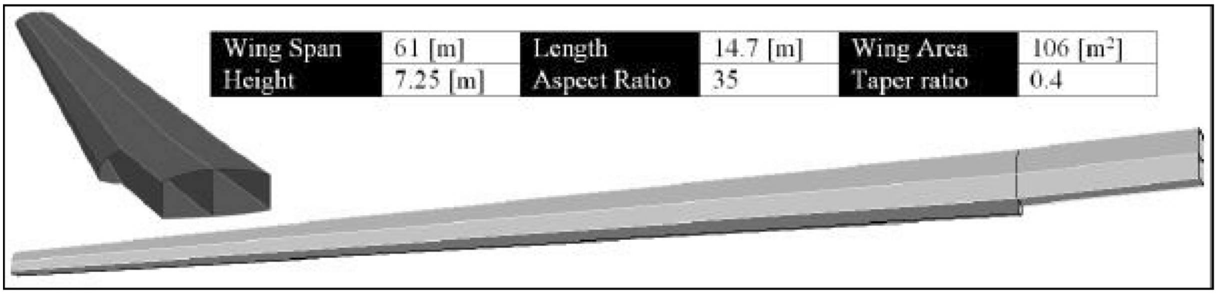

Fig. 1. The composite wing structure.

Table 1
The composite wing in Fig. 1 consists of AS4 12k/3502 unitape material. The material properties have been statistically characterized [1] at 75 °F (23.89 °C)

Property	Distribution	Mean	Variance
Fiber E_1	Normal	1.330688E + 11 [Pa]	7.638150E + 09 [Pa]
Fiber E_2	Normal	9.307922E + 09 [Pa]	3.965175E + 08 [Pa]
Fiber G_{12}	Normal	3.743853E + 09 [Pa]	1.931828E + 08 [Pa]
Density ρ (Rho)	Normal	1575 [kg/m^3]	2.5 [kg/m^3]
Stringer Fiber E_1 (SE_1)	Normal	1.330688E + 11 [Pa]	7.638150E + 09 [Pa]
Load Coefficient	Normal	1	0.1

used for the FE computations and a connection between MATLAB [4] and OPTIMUS [5] has been established for the necessary computations of the reliability algorithms.

3. Reliability analysis methods

The main target of a reliability analysis is to assess the failure probability of a structural design, as a result of variability in input parameters. The exploration of the stochastic domain can be carried out with various methodologies. The Monte-Carlo (MC) approach is quite straightforward: generate random parameter combinations and perform a simulation for each combination to verify if this results in a failure. The method is always applicable but is typically very costly. Approximate but faster methods have therefore been developed, such as the First Order Reliability Method (FORM). To improve algorithm efficiency and applicability, the design variable distributions are transformed into standard normal distributions with the Nataf model [6]. To compute the probability of failure, the Limit State Function (LSF), defined as the boundary between safe and failure solutions in the parameter space, is replaced by a linearized hyperplane in the point of minimum distance to the origin. This optimal point is called the *most probable point* (MPP) and its distance to the origin *reliability index* β (Fig. 2). To locate this point, FORM algorithms solve a constrained or unconstrained optimization problem with the appropriate algorithm and boundary limit [7]. The constrained FORM problem in the transformed space is usually given by

$$\begin{cases} \min\limits_{y} \left[f(y) = \frac{1}{2} y^T y \right] \\ \text{s.t.} G(y) = 0 \end{cases} \quad \text{with} \quad \{ y \in \Re^n, f : \Re^n \to \Re \} \tag{1}$$

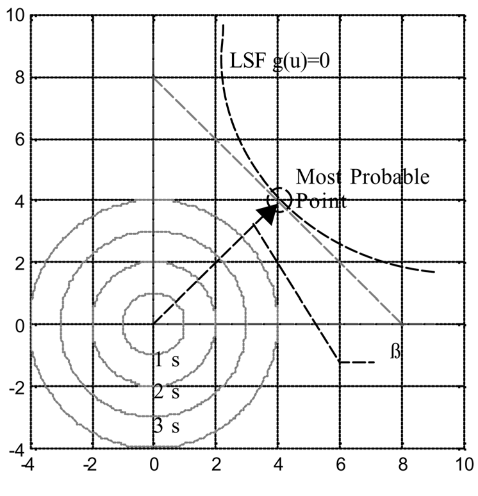

Fig. 2. FORM approximation.

where $G(y)$ is the Limit State Function (LSF) in the transformed space. This constrained problem can be solved directly or transformed to an unconstrained problem. Once the MPP has been located, the probability of failure P_f can be computed using Eq. (2a) or (2b), depending on β:

$$\Phi(-\beta) = \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{-\beta}{\sqrt{2}}\right) \quad \text{for} \quad \beta \leq 7.9 \tag{2a}$$

$$\Phi(-\beta) \approx \frac{1}{\beta\sqrt{2\pi}} e^{-\frac{1}{2}\beta^2} \quad \text{for} \quad \beta > 7.9 \tag{2b}$$

All FORM algorithms used in this paper consist of two parts: a direction search (to find the direction to the MPP) and a subsequent line search (to estimate the distance along the current direction toward the MPP). Four methods have been used for direction search:

1. the modified HL-RF algorithm [6];
2. the classical steepest descent method with the unconstrained exact penalty form of Eq. (1).
3. an SQP implementation with Lagrangian formulation [6,8,9];
4. a Quasi-Newton Trust Region method with quadratic penalty term [10].

Combined with the direction search methods, four line search methods have been implemented [11]:

(a) merit function with an exact penalty method;
(b) linear polynomial interpolation;
(c) quadratic polynomial interpolation;
(d) the first Wolfe condition with quadratic model.

In this paper, reported combinations of the above direction search and line search methods are: 1(a), 1(b), 1(c), 2(c), 3(b) and 4(d). Results will be given in the next section.

4. Results

In a static analysis, the external load configuration has been applied to the composite wing FE model. The probability of failure according to the Tsai-Hill criterion, resulting from input variability, has then been computed. As in most aerospace industry cases, the probability of failure is very low, in the order of 10^{-7} to 10^{-9}. Such low values indicate an LSF boundary that is quite distant from the origin. For such problems, accurate reliability predictions require either a huge number of Monte-Carlo samples or a FORM algorithm that accurately converges to points far from the origin. To prevent having to deal with an infinite design space, the transformed standard normal space has been bounded in a hypercube between -10σ and $+10\sigma$. As the expected $P_f \geq 10^{-19}$, the hypercube used has the closest boundary at 10σ, which corresponds to $P_f \leq 1.998{\cdot}10^{-23}$. This means that, in the case of an elliptic LSF as usually generated by the Tsai-Hill criterion, the closest point of the ellipse is located inside the hypercube.

Two test cases have been considered, with a deterministic and a stochastic Load Coefficient characterization. In the first case, the Load Coefficient has been set up deterministically to a high value, representative of a strong gust load situation. For this case, the relevant parameters that quite strongly influence the system probability of failure are the Young's moduli E_1 and E_2 of the ply and E_1 of the stringers. Parameters G_{12} and ρ have little effect on the performance function. Thus, particular attention should be paid to the correct optimization of the stringers and their properties. The use of a high Load Coefficient for the second case emphasizes the non-linearities of the performance surface defined by the Tsai-Hill criterion.

Table 2
Reliability analysis results. For direction search and line search algorithms refer to Section 3

Test-case	Direction search	Line search	Iterations	LSF evaluations	Beta	Prob. of failure
Deterministic Load Coefficient	1	a	8	66	4.31231919E+00	8.07754912E−06
	1	b	13	104	4.30408621E+00	8.38381824E−06
	1	c	13	110	4.30408621E+00	8.38381824E−06
	2	c	12	94	4.31069221E+00	8.13721505E−06
	3	b	4	34	4.31285291E+00	8.05806719E−06
	4	d	12	173	4.34813545E+00	6.86499022E−06
Stochastic Load Coefficient	1	a	12	140	7.14870122E+00	4.38038494E−13
	1	b	13	113	7.14860371E+00	4.38316050E−13
	1	c	13	119	7.14860371E+00	4.38316050E−13
	2	c	Not Converged			
	3	b	13	209	7.14023273E+00	4.65849581E−13
	4	d	7	122	7.16591230E+00	3.86357613E−13

The analysis of the first case shows, however, that characterization of the probability of a strong load coefficient pulls the MPP a little further in respect to the first deterministic and conservative case. This also exhibits the load capability of amplifying nonlinearities in the FE response (Table 2).

When comparing the various algorithms, the modified HL-RF algorithm shows a good performance. This algorithm, although not always being convergent, is a specific iterative scheme rather than a class of algorithms and it only solves problems having the form of Eq. (1). The other algorithms are generic iterative algorithms used to solve the general unconstrained problem; their performance is usually worse than the HL-RF algorithm. The lower performance can be explained by considering the particular form used to convert the problem from its constrained to its unconstrained form. Three conversion methods have been used in this paper: the pure Lagrangian form, the exact and the quadratic penalty form, which require at each iteration the evaluation of the Lagrangian multiplier or the penalty coefficient, respectively. The quadratic approximation of the performance surface drives direction search algorithms 3 and 4 quickly to a point on the LSF, but then has the result that minimization of the distance function $f(x)$ in Eq. (1) may take considerable time. As a result, algorithm 3 takes a lot of iterations to improve the estimate of the Lagrangian multiplier, while algorithms 2 and 4 get stuck in the curvature discontinuities of the specific penalty form used.

5. Conclusions and discussion

Not all unconstrained minimization algorithms can be used for the analysis of the composite wing, as they suffer from convergence problems due to the particular unconstrained formulation. Only the HL-RF algorithm performs well and is recommended for most cases. However, the performance may be less accurate when a closed LSF (such as the elliptic Tsai-Hill criterion) is used for a highly nonlinear problem [2].

One should trust the FE results only in the feasible domain, but not completely outside this domain [2]. Care should be taken to consider only physically meaningful results and to correctly define the probabilistic distributions and the bounded stochastic domain. This attention is essential to correctly define the failure region and to accurately estimate the probability of failure.

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