Filtering and regularization techniques in shape optimization with CAD-free parametrization

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Abstract

In this contribution an innovative method for shape optimization with FE-parametrization is proposed. The major shortcomings of CAD-free parametrization are discussed and filter techniques are presented in this context. A global filter for shape optimization with FE-parametrization is developed and applied to numerical examples of basic structures. Finally, some further regularization techniques depending on curvature terms are mentioned.

Keywords: Shape optimization; CAD-free parametrization; FE-parametrization; Filter techniques; Sensitivity analysis; Locking; DSG shell element; Regularization techniques

1. Introduction

Parametrization of the design for shape optimization can be based on CAD models or directly on the FE mesh (CAD-free parametrization).

In CAD-based design optimization, design variables are chosen among the parameters related to the underlying CAD model. During the optimization process, update of the finite element mesh is due to changes of the CAD model.

In FE-based design optimization, nodal coordinates are taken as design variables. This parametrization gives more freedom to the optimization process, since the result of the optimization problem is not restricted to the preselected CAD design space. However, there are many technical facts that in the past made this natural choice of optimization appear prohibitive, as there are a large number of design variables, and the wiggly shape and the inherent problems of FE discretization (e.g. locking) that may lead to mechanically wrong answers. This contribution presents an innovative approach to overcoming these shortcomings by a combination of several techniques which results in a powerful and flexible tool for the preliminary design of free-form shells. The problem of the large number of variables is overcome by the use of adjoint sensitivity analysis. From the mechanical point of view, locking-free elements (shell elements) are indispensable. Not only does the correct system response depend on such element formulation, but also the feasible domain of the optimization problem is severely affected by locking phenomena, cf. Camprubi et al. [7], Bischoff et al. [4].

The main problem of the FE-based design is that wiggly shapes may be obtained as optimal designs. These wiggles are mesh dependent and obviously it is not desirable that the optimum design depends on the coarseness of the mesh used. Therefore, the aim of this research is to provide a tool to control the smoothness of the designs obtained as a result of shape optimization and to avoid mesh dependency. To control the shape, different approaches are considered.

Based on the filter proposed by Sigmund in [9] to overcome checkerboard and mesh dependency problems in topology optimization, a filter for shape optimization is proposed. Through this sensitivity filter, not only are high-frequency waves dependent on the mesh smoothed, but also the global wave length of the final design can be controlled by adjusting the filter radius or the filter weights.

Focusing the problem from the mathematical point of view, wiggles in designs may be due to the ill-posedness of the minimization problem or to the presence of local optima. To overcome this problem, regularization techniques are applied. In some engineering problems, it occurs that the solution to a problem is difficult to handle because of its discontinuities or singularities. In those cases, regularization is applied to the problem

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obtaining smooth and regularized solutions, cf. Betytschko et al. [2]. The theory of regularization of functionals has been widely studied and some interesting applications in the optimization field have been presented over recent years. In acoustics, regularization has already been successfully used in Bängtsson et al. [1] for shape optimization of curves. There are also other successful applications in aerodynamics. In our contribution, regularization is applied for shape optimization of shells minimizing strain energy.

2. Filter techniques

In general, a shape can be understood as the superposition of several principal shape functions each of which is characterized by a certain wave length. The shortest is represented by the element mesh. Results of shape optimization – often of local nature – are generated by arbitrary combinations of principal shape functions. As a consequence, successful optimization methods must be able to keep control of the desired mesh independent wave lengths of the resulting shapes by means of filter techniques. Nowadays filter techniques are well established procedures both in shape and topology optimization and in many other disciplines, cf. Bendsoe and Sigmund [3], Bängtsson et al. [1]. These methods are by far more than just pragmatic tools, which do a good job in practice. The research in the last two decades especially in mathematics gives us many technically mature methods (for more details see Tikhonov et al. [10]).

The above-mentioned problem of high-frequency waves in the optimization results arise from the many possible scales of the analyzed problem. On the one hand there exist many local minima, on the other hand during the optimization process long waves of possible solutions compete with diverse short waves or wiggles (of other scales). Exactly at this point the filter techniques are engaged to selectively affect the desired wave length and to filter it out. The aim is to develop a procedure, by which smoothness and curvature of the optimized structures are controlled, involving some user-defined geometrical measurement of the modeled structure. The influence of the high-frequency waves, also called disturbance or noise, can be demonstrated on the basis of the one-dimensional case. A cut-out (Fig. 1) shows a long wave as well as other superposed short ones.

If the oscillating part of the solution is undesirable, then it is obvious that the wiggles (the superposed short waves) influence the sensitivity severely, e.g. at point P in Fig. 1 we even notice a change of sign. It is obvious that the sensitivity of the undisturbed curve deviates from the value of the disturbed one, which definitely affects the

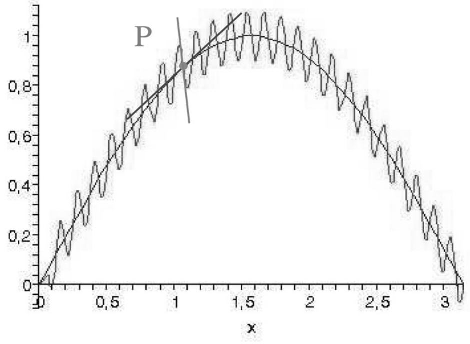


Fig. 1. Influence of high-frequency waves on gradients.

whole optimization procedure, exceedingly in gradient based algorithms. The task now is to get rid of the noisy wiggles systematically.

Similar problems are faced in signal technology, where the waves, before transmission, are transformed by means of Fourier transformation or wavelet transformation. It is a common technique to approximate a function f by superposing periodical or compact shape functions $N_i(x)$ and the corresponding amplitudes or function values f_i, respectively

$$\tilde{f}(x) = \sum_i N_i(x) \, . f_i \tag{1}$$

Besides the shape functions, there are also the dual functions $D_i(x)$ that filter out the f_i values. The dual function is defined as follows:

$$\frac{1}{\|\Omega\|} \int\limits_{\Omega(x)} N_i(\xi) \cdot D_j(\xi) d\Omega = \delta_{ij} \tag{2}$$

The integration domain Ω depends on the used shape and dual functions. It usually represents the smallest wave length described in the approximation. If we adopt the dual function on the approximation $\tilde{f}(x)$, the evaluations of the original function at the sampling points f_i are exactly filtered out. On the other hand, if the dual function is adopted on the original (noisy) function $f(x)$, the oscillations, which cannot be represented by the shape functions, will be eliminated, as their wave length is smaller than the radius of Ω. A smoothed function evaluation f_m is produced.

$$\frac{1}{\|\Omega\|} \int\limits_{\Omega(x)} f(\xi) \cdot D(\xi) d\Omega = f_m(x) \tag{3}$$

The integration, Eq. (3), over the domain Ω can be interpreted as a way to determine the weighted mean value at each sampling point. This issue motivates a smoothing method for shape optimization. The choice of a suitable dual function and the discretization of Eq. (3), where the sensitivities are filtered instead of the function itself, yields to

$$\frac{\partial f_m}{\partial x_e} = \frac{\sum_{i=1}^{node} D_i \cdot \frac{\partial f}{\partial x_i}}{\sum_{i=1}^{node} D_i} \tag{4}$$

D_i $= R - R_{ei}$ weighting function concerning spatial **and** topological distance
R smoothing radius representing Ω
R_{ei} distance between sampling point e and node i.

This approach is applied in a similar manner in topology optimization, cf. Bendsoe and Sigmund [5].

The radius of Ω controls the smallest desired wave length (size of wiggles) in the optimized structure. An additional difficulty in shape optimization concerns the fast evaluation of distances on the curved surfaces, as the spatial distance 'only' is not enough to determine the sphere of influence of the filter radius. This fact can be demonstrated by means of an intersection curve on a curved surface (Fig. 2). Taking point A as a sampling point (point at which the sensitivity is filtered), we notice that node 1, from the topological point of view, is closer to node A than node 2, and accordingly node 1 should possess a greater weighting factor. However, if only the direct distance is considered, then node 2 will get the bigger weight.

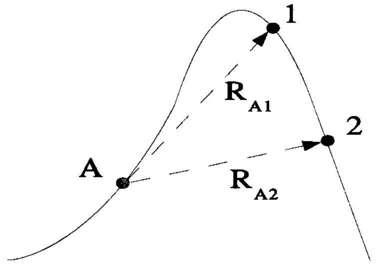


Fig. 2. Spatial (geometrical) and topological distance.

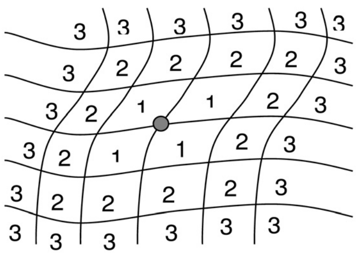


Fig. 3. Element and node levels.

One possibility is to measure the distances on the curved surface, which is difficult to perform, particularly for discretized structures. To avoid measuring on the curved surface, we introduce a hierarchical weighting system. The FE nodes around the sampling point are classified in levels. Each level gets a special topological weighting factor, as depicted in Fig. 3. The final weighting factor is a combination including both information on the topological distance and the spatial one. In this way we keep the algorithmic effort low, by scanning the a priori created node map (in the input phase) within the filter radius. Moreover, the smallest (shortest) waves in the optimized structures are controlled by user defined measurements not by discretization parameters, namely the size of the finite elements.

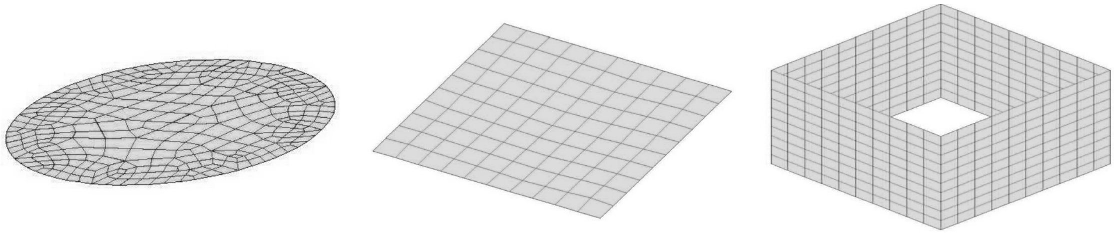
3. Regularization techniques

A whole set of problem classes in structural and fluid mechanics suffer from numerical instabilities. This manifests itself in singularities e.g. formfinding of membranes (Bletzinger and Wüchner [7]), or in terms of badly conditioned or unstable sets of equations e.g. in fluid mechanics.

In shape optimization of free-form shell structures, we are confronted with most of those difficulties, where additionally many local minima occur as solutions of optimization. One method of resolution with the filter techniques was proposed in section 2. Another mathematically established approach is based on regularization techniques, cf. Tikhonov et al. [8]. Because of their high efficiency these methods are often applied in shape optimization.

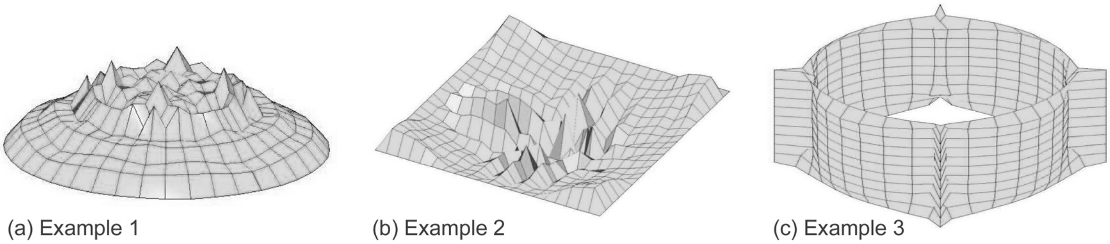
Most numerical instabilities in shape optimization are traced back to the high-frequency waves appearing during optimization. Therefore it is obvious to formulate the stabilization terms in the form of curvature information to get smoother surfaces. A successful implementation for the one-dimensional case can be found in Bängtsson et al. [6]. The expansion for two dimensions (shell surface) is more complicated. An important step towards generalization for multidimensional problems is the determination of the curvature tensor at each FE node of the discrete geometry (more details in Camprubi and Bletzinger [9]). Based on the curvature tensor the objective function (e.g. strain energy) can be modified as follows:

$$Obj_{mod} = Obj + \epsilon \int_{\Omega} \kappa^2 d\Omega \tag{5}$$



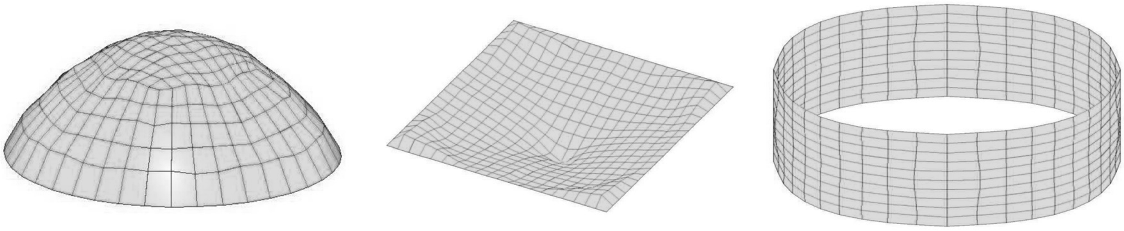
(a) Example 1 (b) Example 2 (c) Example 3

Fig. 4. Start geometry.



(a) Example 1 (b) Example 2 (c) Example 3

Fig. 5. Optimization results without filter.



(a) Example 1 (b) Example 2 (c) Example 3

Fig. 6. Optimization results with filter radius equal half the span length.

where κ is the mean curvature at each node. The stabilizing term $\epsilon \int_{\Omega} \kappa^2 d\Omega$ in Eq. (5) competes with the high-frequency solutions (local minima) and enforces a smoother solution.

The next step, using gradient algorithms, is to adapt the derivatives of the sensitivity analysis. The derivative of the stabilizing term in Eq. (5) with respect to the design variables s is illustrated in Camprubi and Bletzinger [9]. First results show a significant improvement in smoothness of optimal structures.

4. Numerical examples

The effects of the presented filter will be demonstrated on several exemplary but meaningful benchmarks of minimizing strain energy (compliance). The examples deal with some basic shapes to show the shortcomings of CAD-free shape optimization without filter techniques or stabilizing terms. The comparison with the results after adopting the proposed filter shows the benefits of the method, Figs. 4–6. The following examples are restricted to the static and linear case.

In the first example, a Navier-supported circular plate under self-weight is optimized. As the whole energy is produced in terms of bending, some disturbance in the form of start deflection is necessary. The problem involves 294 design variables which are the Z-coordinates (vertical movement) of all nodes except for the supported ones. As the state of minimal strain energy is reached when the load is carried by membrane forces, we expect a shape which has a tendency towards minimal surfaces. Locking-free DSG linear shell elements are used, cf. Bletzinger et al. [10]. The sensitivity analysis is performed by an adjoint method which reduces the calculation cost severely. In the second example a Navier-supported square plate under point loading in the center is optimized. The same remark about start geometry applies in this case. For parametrization of the geometry 436 design variables are used. In the third example the start geometry is a rectangular container under internal pressure, where we expect a cylindrical shape as a solution of optimization. In this example 438 design variables are used.

The optimization results in Fig. 6 show the performance of the proposed method. The wiggles of wave lengths smaller than the filter radius (half the span length) are eliminated, as they cannot be represented by the chosen shape functions. The smoothness of the resulting structures is controlled by means of a user-defined parameter derived from the optimized model (filter radius) and is mesh independent which is the main aim of this approach.

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