Comparative studies of two POD methods for airfoil design optimization

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Abstract

Proper-orthogonal-decomposition (POD) methods have recently been developed for the design of airframe components. In this paper, two POD-based approaches have been studied: the gappy POD reconstruction procedure [1], and the gradient approach [2]. Both methods do not require a projection onto the computational fluid dynamics (CFD) governing equations but are, instead, a collection of flow snapshots that covers the parameter ranges of interest. Their performance on the inverse design of airfoil shapes have been compared and evaluated. Our studies show that while both methods are efficient and accurate, once appropriate flow snapshots have been collected, the gradient-based method is generally more accurate.

Keywords: Gappy POD; Gradient POD; Airfoil shape design; Multi-disciplinary flow

1. Introduction

Aerodynamic shape optimization is receiving attention in the aerospace industry as a useful tool for the design of airframe components. The analysis usually involves a gradient-based optimizer and an adjoint solver, which can then be coupled with a computational fluid dynamics (CFD) solver that provides the required gradients.

Significant progress has been reported on the application of this approach to realistic design of complex geometries and viscous flows [3]. However, the use of the adjoint-solver approach for situations involving multiple disciplines and a large number of design constraints has been somewhat limited. The fundamental reason for this might be related to the fact that the adjoint equations, boundary conditions, and gradient calculation formulae are cost-function dependent, and therefore need to be rederived every time the cost function changes. Moreover, it is not possible to treat arbitrary forms of the cost functions. In this work, alternative approaches for aerodynamic design of wings based on the proper-orthogonal-decomposition (POD) method have been evaluated by the application to inverse design of a series of airfoil shapes.

POD has been used in reduced-order methods for aeroelasticity-based aircraft design [4]. The work by Bui-Thanh et al. [1] and LeGresley and Alonso [2] have demonstrated that the POD method could also be used for low-cost aerodynamic shape optimization. These methods do not require a projection onto the governing equations for CFD, but are instead a collection of flow snapshots that covers the parameter ranges of interest. The method proposed by Bui-Thanh et al. [1] was based on the gappy reconstruction procedure [5], while that used by LeGresley and Alonso [2] was based on the gradient approach to cost function optimization. In both cases, conventional CFD methods were used to generate the data ensemble (snapshots), from which the POD process computes a set of optimal eigenfunctions. The two methods differ in the way the cost function is evaluated and the optimal solutions are approached. In the current paper, the effectiveness and the accuracy of the two methods discussed above are evaluated as functions of the size of the snapshots, the size of the reduced POD modes, and the values of the design variables for geometry optimization.

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2. Inverse design

The POD procedure used here relies on the Karhunen–Loève expansion for the data ensembles which span a range of airfoil geometries [6]. Once the POD basis functions, ψ, have been obtained, we can expand the flow solution about an arbitrary airfoil shape (δ), i.e.

$$U^{\delta} \approx \sum_{j=1}^{p} a_j^{\delta} \psi_j \tag{1}$$

The accuracy of the expansion depends on the scope of the database and the number of POD eigen-modes ($p, 1 \leq p \leq N$). We investigate the effects of these two factors on the performance of two previously reported POD-based inverse design procedures which are described below, within the framework of airfoil shape design.

Given a target pressure distribution $\mathbf{P}^*$, the inverse design problem is to find an optimal airfoil shape whose surface pressure distribution $\mathbf{P}$ minimizes the cost function

$$J = \|\mathbf{P}^* - \mathbf{P}\|_2 \tag{2}$$

In addition to containing the flow variables, the snapshot in the implemented gappy procedure is augmented to also contain airfoil coordinates. The minimal solution of a cost function

$$J = \|\mathbf{V}^* - \mathbf{V}\|_2 \tag{3}$$

is sought, where the new target vector $\mathbf{V}^*$ contains a surface pressure distribution $\mathbf{P}^*$ and the corresponding airfoil coordinates $\mathbf{C}^*$. As the target pressure distribution $\mathbf{P}^*$ is known to the designer, the new target vector $\mathbf{V}^*$ then contains both known and unknown components. Everson and Sirovich [5] reported on reconstructing the missing (gappy) data by assuming that

$$\mathbf{V}^* \approx \overline{\mathbf{V}} = \sum_{n=1}^{M} \bar{a}_n \psi_n \tag{4}$$

The coefficient $\bar{a}_n$ satisfies

$$\mathbf{M} \cdot \bar{\mathbf{a}} = \mathbf{f} \tag{5a}$$

$$M_{kn} = (\psi_k, \psi_n)_{s[\hat{\mathbf{V}}]} \tag{5b}$$

$$f_k = (\mathbf{V}, \psi_k)_{s[\hat{\mathbf{V}}]} \tag{5c}$$

where the inner product is over the support $s[\hat{\mathbf{V}}]$ of $\hat{\mathbf{V}}$. The vector $\hat{\mathbf{V}}$ is defined as

$$\hat{\mathbf{V}}(\mathbf{x}) = m(\mathbf{x})\mathbf{V}^*(\mathbf{x}) \tag{6}$$

where m is zero on the missing data, and unity elsewhere.

In addition to the gappy procedure, we also implemented the gradient optimization approach. For the latter, the cost function is as defined in Eq. (2), but $\mathbf{P}$ is represented by the POD modes, i.e.

$$\mathbf{P} = \sum_i b_i \psi_i \tag{7}$$

The coefficients b_i are functions of the design variables which are chosen as the amplitudes of bumps that are added to the basic airfoil geometries. In the current work, the bump functions are a series of Hicks–Henne functions [7]:

$$b(x) = \left\{ \sin\left[\pi x^{\log 0.5/\log(t_1)}\right]\right\}^{t_2}, \quad 0 \leq x \leq 1 \tag{8}$$

Gradients of the cost function with respect to the design variables have also been obtained by finite-differencing of the POD coefficients in this paper.

3. Results

The results of our implementation of the gappy POD-reconstruction procedure is exemplified in Fig. 1, using the NACA0012 airfoils at Mach number, $Ma = 0.75$, and attach angle, $\alpha = 0.75$. Various numbers of POD eigen-modes have been used to reconstruct the pressure fields. As expected, the larger the number of eigen-modes used, the more accurate the reconstruction. For inverse design, the snapshots were generated for RAE 2822 airfoil to which a series of Hicks–Henne bump

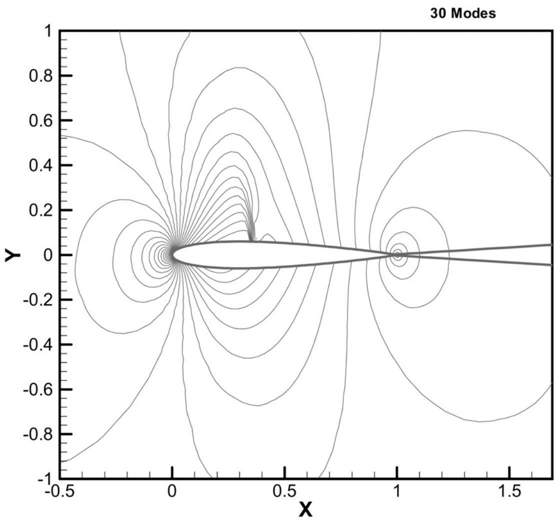

Fig. 1. Reconstruction of the pressure field by the gappy POD method (dashed) compared to the original CFD contours (solid) ($Ma = 0.75$, $\alpha = 0.75$, with 30 POD modes).

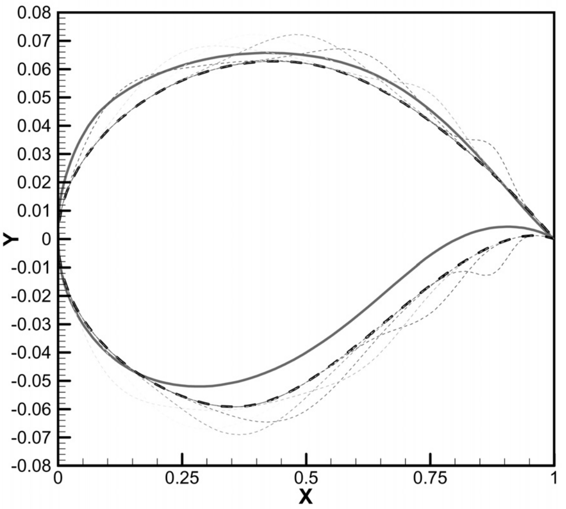

Fig. 2. Parameterized airfoils based on RAE 2822 (dash) and the Korn airfoil (solid).

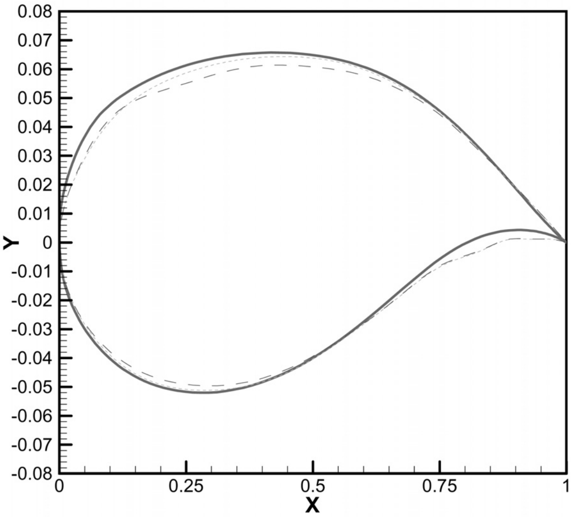

Fig. 3. Inverse design of the Korn airfoil (solid) using the gappy POD method (long dash) and the gradient-based optimization (short dash). 15 snapshots based on RAE 2822 used.

functions is added. First, 14 bump functions were added to the basic RAE 2822 airfoil, where seven each were distributed uniformly on the upper and lower surfaces, respectively, and are shown in Fig. 2 with dashed lines. Flow solutions for the original airfoil plus the 14 modified airfoils are computed using the commercial software AEROFLO [8], which is a high-order flow solver for multi-disciplinary aeroelasticity problems. With the ensemble of the generated flow solutions, the POD basis with a complete set of modes is generated and used for the following inverse design problems.

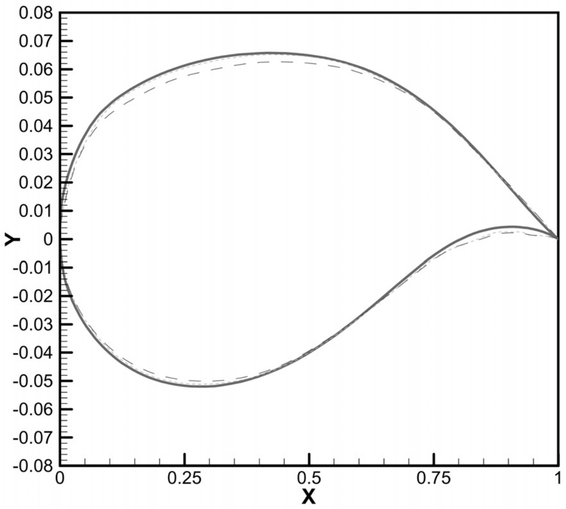

Fig. 4. Inverse design of the Korn airfoil (solid) using the gappy POD method (long dash) and the gradient-based optimization (short dash). 20 snapshots based on RAE 2822 used.

The surface pressure distribution for the Korn airfoil, whose geometry is also shown in Fig. 2 with solid lines, is specified as the design target. It can be seen in this figure that, while the Korn airfoil shares some similarities with the RAE 2822-based snapshot set, its camber and thickness distributions are quite different. This example thus represents a challenge for both POD-based methods. Figure 3 compares the exact Korn airfoil (solid line), the POD design results using gappy POD method (long dashed lines), and gradient optimization method (short dashed lines). It can be seen that the POD design results are close to those of the target except in a few regions. Also, the gradient cost-function optimization POD approach produces better results than the gappy method. However, on the lower-side trailing region and the upper-side leading edge region, both methods produce significant errors relative to the target shape, implying the need for improved methods.

One approach to improve inverse design is to increase the richness of the subspace spanned by the POD basis vectors. This can be achieved by increasing the number of snapshots in the ensemble. To test this, four more bumps (t_1 = 0.05, 0.10, 0.20, 0.40 in the Hicks–Henne functions) have been added on the top part of the RAE 2822 airfoil, and one more bump (t_1 = 0.925) added on the bottom part. Figure 4 compares the exact Korn airfoil geometry to the POD design results using the gappy POD method (long dashed lines) and cost-function optimization method (short dashed lines) using the extended snapshot database. Improved results are evident for both POD methods. Note that the gradient-based cost function optimization method almost recovers the target airfoil shape and the surface pressure distribution (not shown).

4. Conclusions

We have shown that both the gappy POD method and the gradient-based POD optimization method can be used for accurate inverse design of airfoil shapes. With the same ensembles, the gradient-based method shows more accurate results. These observations are based on the results for one airfoil shape. Extension to other interesting airfoils, such as NACA65A004, are being carried out.

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