Cavitator design for a supercavitating torpedo using evidence theory for reliability estimation

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Abstract

Recently the US Navy has invested in supercavitating torpedo research and development. The supercavitating torpedo is a new technology to the United States and many challenges are associated with its design. One such challenge is the uncertainties involved. Furthermore, there are no full data sets to create probability distributions for classical reliability based analysis. Most information is in the form of expert opinion from the designers. Thus, evidence theory is a natural choice to determine reliability of the system. In this paper a cavitator structure is presented and optimized for shape and structural thickness with respect to stress, buckling and weight. Finally, evidence theory is used to handle the limited data situation as an alternative to classical probability theory for reliability assessment.

Keywords: Reliability; Evidence theory; Supercavitating; Cavitator; Multi-point approximation

1. Introduction

A supercavitating torpedo is a high-speed underwater vehicle that is completely surrounded by a cavitation bubble or cavity. This cavity eliminates the viscous drag associated with underwater motion and enables the torpedo to obtain high speeds. The cavitator, located at the front of the torpedo, initiates the cavity and is thus very important. Currently the US Navy is in the developmental stages of designing a supercavitating torpedo. To explore new design ideas many researchers have developed mathematical models for computer simulation. Much work has been done with regard to the structural design by Ruzzene [1] and Alyanak et al. [2]. Further work has been done with regard to the cavitator shape design by Alyanak et al. [3]. However, in all cases no uncertainty information was incorporated. Likely this is because no clear data sets are available to apply classical probability theory without making gross assumptions. To overcome this, recent work by Oberkampf and Helton [4] is utilized. They have categorized uncertainty into two distinct types: aleatory and epistemic uncertainty. Aleatory uncertainty is known as irreducible or inherent uncertainty and can be handled with classical probability theory. However, epistemic uncertainty is subjective and comes from lack of knowledge or incomplete data sets. Evidence theory has been applied to structural design problems by Bae et al. [5], and shown to be able to handle both epistemic and aleatory uncertainty. In this paper evidence theory is utilized to estimate reliability for a cavitator structure that is optimized for shape and structural thickness with respect to stress, buckling and weight requirements.

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2. Evidence theory

Evidence theory (ET) was developed by Shafer [6] from Dampster's work. Due to this ET is also known as Dampster–Shafer theory. ET is characterized by two distinct measures that bound the uncertainty: belief (BEL) and plausibility (PL). These are formulated from the basic belief assignment (BBA) which is developed from expert opinion for each uncertain parameter in question. The BBA information is contained in the function $m(A)$, where A is a possible event. Thus, BBA is a mathematical representation of partial belief for a set of possible events. From the BBA, belief and plausibility can be defined by:

$$BEL(A) = \sum_{B \subset A} m(B) \quad (1)$$

$$PL(A) = \sum_{B \cap A \neq \emptyset} m(B) \quad (2)$$

where A and B are events, and BEL and PL can be interpreted as upper and lower bounds of probability. Because no assumptions were made to obtain these quantities, they are consistent with the given partial evidence.

3. Cavitator problem

The cavitator is subjected to extreme forces during operation. It is desired to design a cavitator for a supercavitating torpedo considering shape, structure, buckling performance, stress limitations and weight. The optimal structure must be thin walled because the US Navy wishes to include radar arrays inside the cavitator.

To accomplish this task, a supercavitating flow prediction method, based on potential flow theory, developed by Kirschner et al. [7], is utilized to determine the fluid characteristics for a given cavitator shape. Using this fluid flow analysis capability the pressure profile along the cavitator and the overall coefficient of drag can be determined. A finite element (FE) model, composed of plate elements, is then constructed of the required shape and the calculated pressure is applied. Using this FE model a stress distribution can be predicted for the given load. This stress distribution can then be utilized to determine the buckling stability based on the bifurcation buckling problem defined in Eq. (3).

$$\left([K] + \lambda_{cr}[K_\sigma]_{ref}\right)\{\phi\} = \{0\} \quad (3)$$

Here $[K]$ is the global stiffness matrix, $[K_\sigma]_{ref}$ is the global stress stiffness matrix with respect to the pressure load, λ_{cr} is the first eigenvalue of the problem or critical multiplier, and $\{\phi\}$ is the eigenvector associated with λ_{cr}. The value of λ_{cr} is used to define a state of stress at which the system becomes unstable, $\lambda_{cr} < 1.0$.

4. Optimization problem

The objective of the problem is to minimize the drag due to fluid flow and structural weight associated with the cavitator shape. Mathematically this is done by:

$$\min\left\{\frac{C_D}{C_{D0}} + \frac{M}{M_0}\right\} \quad (4)$$

where C_D is the drag coefficient and M is the cavitator mass. The nominal values C_{D0} and M_0 are defined such that the weighting of each ratio are equal to each other. The design variables considered in the problem become: two variables to define the axisymmetric cavitator shape shown in Fig. 1 and nine variables that defined different skin thicknesses within the cavitator structure. The skin thickness variables define the skin thickness for each ring of elements shown in the deterministic optimal solution in Fig. 2. The constraints on the problem are developed to avoid unrealistic cavitator shapes, ensure $\lambda_{cr} \geq 1.1$, and have a maximum Von-Mises stress less than 15 ksi in every element.

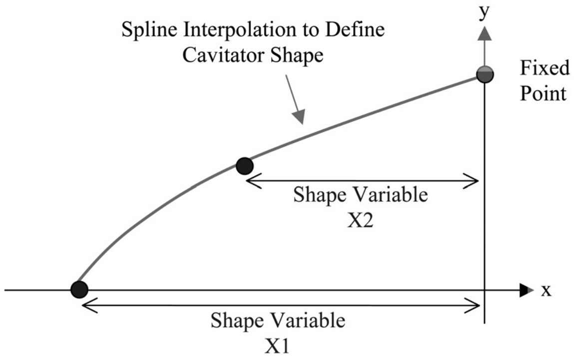

Fig. 1. Shape definition for axisymmetric cavitator shape.

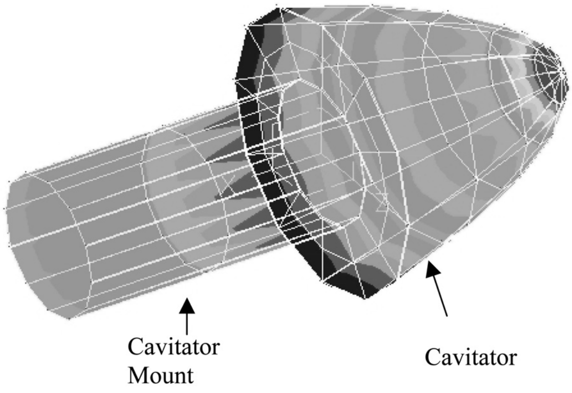

Fig. 2. Optimal structural shape.

5. Proposed evidence theory methodology

The problem depicted is computationally extensive. To perform evidence theory reliability analysis it is necessary to reduce the computational expense of the problem. The proposed algorithm begins at the deterministic optimum shown and explores the design space defined by the BBA for each variable and constraint. To accomplish this, the cavitator problem can be reduced to the function:

$$\{Y\} = f(\{X(\xi_i)\}) \quad (5)$$

For each value in the output vector $\{Y\}$ a multi-point

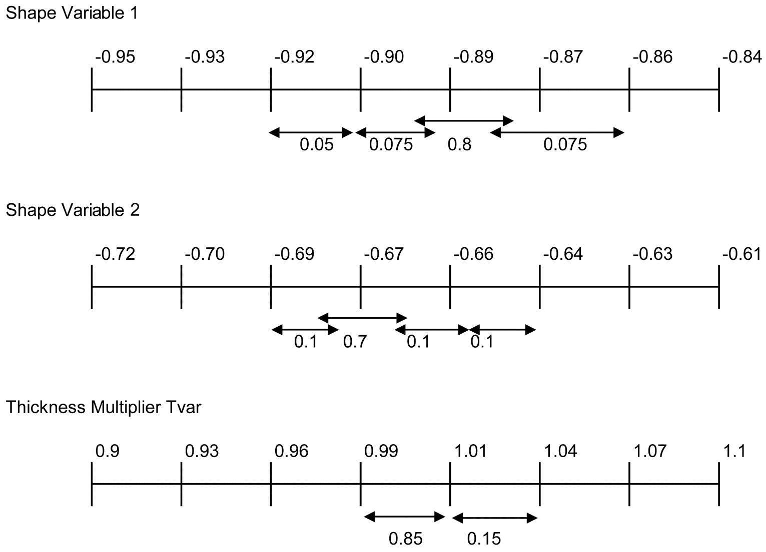

Fig. 3. BBA for three design variables.

Table 1
Results of cavitator problem

Deterministic design point				Constraint values at optimum				
Shape Variable 1: -0.8903				λc: 70.8				
Shape Variable 2: -0.6676				Max Von-Mises Stress (ksi): 14.96				
T var: 1.000				Wt (lbs): 3.80				
Variable Thickness Values (inches)				Cd: 0.1547				
T1	T2	T3	T4	T5	T6	T7	T8	T9
0.0313	0.0313	0.1093	0.0387	0.0335	0.4980	0.4743	0.3925	0.0651
Reliability Analysis								
With MPA 120 f(x) calls		No MPA 544 f(x) calls						
BEL 0.0064		BEL 0.0064						
PL 0.8898		PL 0.8950						

approximation (MPA) can be developed with respect to the design variables {X}, that are functions of the random parameters ξ_i, to approximate the design space of interest. This approximation is developed by combining local two-point approximations, TANA2 [5], with a weighted sum technique.

The epistemic uncertainty defined in the BBA is expressed by intervals (upper and lower bounds). These intervals can be scattered, nested or overlapped. The assignment value given to each interval represents the imprecise statistics of the parameter. As the dimension of the problem increases, the computational expense for calculating BEL and PL from the BBA increases substantially. The proposed method utilizes the MPA to reduce the cost. It then solves sub-optimization problems to identify the failure surface boundaries, which may be highly nonlinear. These boundaries become the bounds of integration or numerical summation as defined in Eqs. (1) and (2).

BBA for this problem is simplified by considering the variability in skin thickness as one design variable: the percent change for each of the previously defined nine variables is equivalent. BBA for the three remaining variables is defined in Fig. 3. Notice, the defined intervals may partially overlap each other and the overlapping parts are completely independent of each other. The complete results are contained in Table 1. The main active constraint becomes the minimum stress constraint while the weight and drag are minimized. The reliability assessment was carried out at the deterministic optimum (i.e. the point in question was directly on failure boundaries) yielding a very high PL value. The BEL is significantly less than the PL because of the very broad BBA for each variable. As the BBA becomes more defined the BEL and PL converge to the true probabilistic value.

6. Summary

A supercavitating torpedo cavitator was designed. Reliability assessment was determined for a given set of BBA using ET. Because of the flexibility of ET the very limited information case given in Fig. 3 can be handled without making assumptions. The computational expense of solving the reliability problem was reduced by developing global approximations of the required portion of the design space by combining local TANA2 approximations.

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