Bounds on structural system reliability in the presence of interval variables

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Abstract

The failure of a structural system is usually governed by multiple failure criteria, all of which are to be taken into consideration for the reliability estimation. If all the uncertain parameters are defined as random variables, then the reliability of a structural system can be estimated accurately by using the existing techniques. But when the knowledge about some of the variables is limited to lower and upper bounds, the entire range of these bounds should be explored while estimating the bounds on the reliability. The computational cost involved in estimating these bounds increases tremendously because a single reliability analysis, which is a computationally expensive procedure, is performed multiple times for each configuration of the interval variables. To reduce the computational cost involved, high-quality function approximations for each of the limit states and the joint failure surface are considered in this paper. The proposed technique is demonstrated with a numerical example.

Keywords: System reliability; Random variables; Uncertain intervals; Fast Fourier transforms

1. Introduction

A structure consists of many individual components, all of which have the potential to fail. The failure of any of these individual components might lead to structural collapse. The reliability analysis of structural systems involves the consideration of multiple limit states from different disciplines, which might be correlated. Each limit state is an implicit function and requires expensive computations to evaluate the function value and the gradients that are needed for reliability analysis. Therefore, in the presence of multiple limit states the computational effort involved in estimating the failure probability increases tremendously.

For only random variables, the failure probability of a structural system is obtained by solving the multi-dimensional integral

$$p_{\mathrm{f}} = \int_{\Omega} f_x(\mathbf{X}) d\mathbf{X} \qquad (1)$$

where p_{f} is the probability of failure and $f_x\,(\mathbf{X})$ denotes the joint probability density function of the vector for the basic random variables, $\mathbf{X} = (x_1, x_2, \ldots, x_n)^T$, which represent uncertain quantities. Furthermore, $\boldsymbol{\Omega}$ is the joint failure region modeled by all of the limit-state functions. Monte-Carlo simulation can be used to evaluate this multi-fold integration. However, it requires a large number of samples to accurately estimate the small order of failure probabilities. To reduce the computational cost, algorithms [1,2] were developed that make use of surrogate representations of the failure surface and compute the failure probability.

Moller et al. [3] developed an algorithm to estimate the bounds on the reliability for problems with mixed variables and a single failure criterion. But methods need to be developed for mixed variable problems with multiple failure criteria.

In the presence of both random and interval variables, every combination of the interval variables has an unknown probability. At each combination, a reliability analysis needs to be carried out. This increases the computational cost exponentially with the increase in interval variables. Moreover, for each combination a new joint failure region should be modeled accurately for the prediction of the reliability. To reduce the computational cost involved, high-quality approximations are used in this paper for modeling the implicit limit-state functions and the joint failure region.

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2. Proposed methodology

For modeling the joint failure region using an approximation, the limit-state functions should be available in closed form so that the points on the joint failure surface can be sampled. In the case of an implicit function, several local approximations can be constructed, with sample points around the most probable point (MPP) for each limit-state function, and blended into a multi-point approximation (MPA) [4]. The MPA can be regarded as a weighted sum of several local approximations. It can be written using the general formulation as:

$$\tilde{F}(\mathbf{X}) = \sum_{k=1}^{K} W_k(\mathbf{X})\tilde{F}_k(\mathbf{X}) \tag{2}$$

where $\tilde{F}_k(\mathbf{X})$ is a two-point local approximation, k is the number of local approximations, and W_k is a weighting function that adjusts the contribution of $\tilde{F}_k$ $(\mathbf{X})$ to $\tilde{F}$ $(\mathbf{X})$ in Eq. (2). As the accuracy of the MPA is based on the accuracy of the local approximations, two-point adaptive nonlinear approximations (TANA2) [5] are used as local approximations to construct the MPA for each limit-state function. As TANA2 can capture the information of the limit-state function around the vicinity of the points used, MPA can retain the information for each of the failure surfaces without increasing the computational effort. Since each of the limit-state functions are modeled using high-quality approximations, these approximations can be used as closed-form expressions for sampling the points on the joint failure surface.

The details of the algorithm and its implementation are presented below:

1. Estimate the MPP of each of the limit-state functions with the interval variables at their central values. The MPP is obtained by using the modified Hasofer Lind –Rackwitz Fiessler (HL–RF) algorithm with TANA2 approximate models. This reduces computational time and is efficient for highly nonlinear problems with a large number of random variables.
2. Design points are sampled within the vicinity of each MPP using a Latin hypercube sampling technique. The bounds on the random variables are taken to be two standard deviations on either side of each MPP.
3. Local TANA2 approximations are constructed for the set of design points sampled around each MPP. These local TANA2 approximations are blended into a multi-point approximation, which captures the behavior of the limit-state function around the MPP. Using the same procedure, an MPA is constructed for each of the limit-state functions.
4. For each combination of the interval variables, points are sampled on the joint failure surface using surrogate representations of each of the limit states. Multiple response surface models are constructed using these sampled points. In order to improve the accuracy of each of the response surface models, the design space is sub-divided until the regression sum of squares value of the approximations in each subdivision is acceptable.
5. The convolution integral is solved using fast Fourier transforms (FFT) based on the response surface models to estimate the failure probability of the system [2].
6. Steps 4 and 5 are repeated for every combination of the interval variables and the bounds on the failure probability are the minimum and maximum values obtained from all of the resulting combinations. Figure 1 illustrates the methodology discussed above.

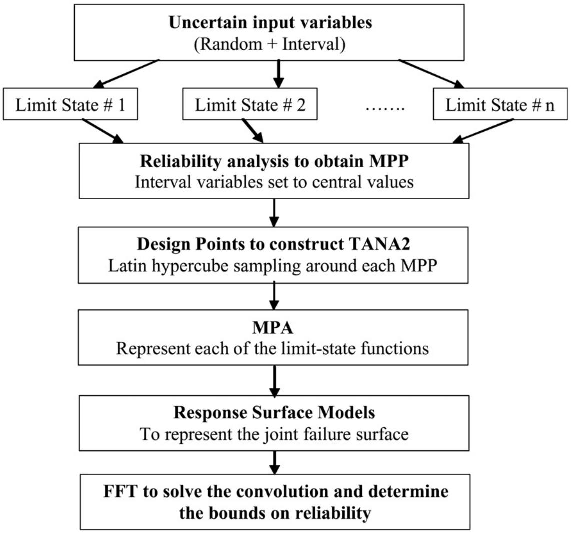

Fig. 1. Proposed algorithm details

3. Numerical example

A numerical example is presented to show the applicability of the proposed method. This methodology could be applied to problems with multiple non-normal random variables and implicit or explicit limit-state functions. The estimates obtained by using the above-mentioned method are compared with the results obtained from Monte-Carlo simulations.

3.1. Wing structure example

A wing structure, as shown in Fig. 2, is considered to estimate the bounds on the failure probability. Two

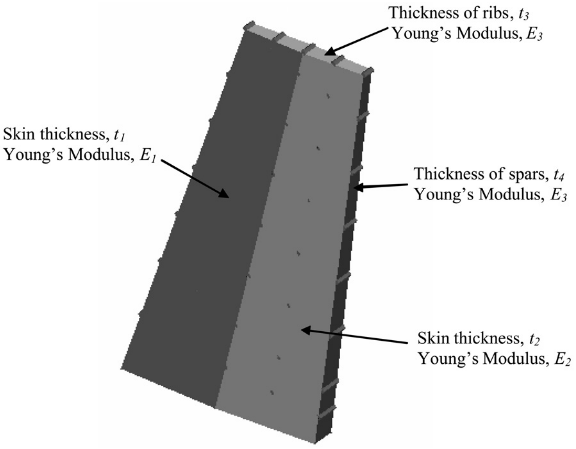

Fig. 2. Wing structure.

failure criteria of the system are considered: the displacement at the tip of the wing when subject to aerodynamic loading less than 1.6 in Eq. (3) and the fundamental natural frequency of the wing more than 1.52 Hz (Eq. 4).

$$g_1(\mathbf{X}) = D_{\text{tip}}(\mathbf{X}) - 1.6 \leq 0 \tag{3}$$

$$g_2(\mathbf{X}) = 1.52 - \omega_1(\mathbf{X}) \leq 0 \tag{4}$$

The thicknesses of the first two span-wise skins are physically linked to have the same value, which is modeled as a normal distribution with a mean of 1.5 in. The same is done with the other two span-wise skins. All of the spars are linked to have the same thickness and the same is done with the ribs. These are also modeled as normally distributed random variables with a mean of 0.5 in. Physical linking results in four random variables and the coefficient of variation is assumed to be 10% for all of these variables. The Young's moduli of the two physically linked skins are modeled as interval variables. Moreover, the Young's modulus of the spars and ribs is also modeled as an interval variable. These variables were considered to be in the interval of [1.04E7, 1.06E7] psi.

Table 1
Comparison of failure probability bounds for the wing structure

Methodology	Failure probability bounds	% Difference	
		Lower limit	Upper limit
Monte Carlo	[0.00132, 0.04522]	—	—
Proposed algorithm	[0.00125, 0.04692]	−5.30	3.76

Table 1 shows the comparison of the reliability estimated using the proposed methodology and Monte-Carlo simulation. As this is a problem with implicit limit-state functions, multi-point approximations were constructed for each of the limit states which were used as closed form expressions in estimating the bounds on the reliability. The bounds obtained by using the proposed technique were conservative with a difference of around 5% on the lower bound and 4% on the upper bound. The proposed methodology required only 81 exact simulations as opposed to 1.2 million simulations for Monte Carlo.

4. Summary

Due to the vagueness in the available information, all of the uncertain parameters in a problem cannot be assumed to be random in nature. When dealing with a combination of random and nonrandom variables, the computational cost increases exponentially. To reduce the computational cost without a loss of accuracy, a methodology to efficiently deal with mixed variables is presented in this paper.

The accuracy of the proposed methodology depends entirely on the accuracy of the MPA constructed for each of the limit-state criteria as the points on the joint failure region are sampled based on these MPA. Once a good approximation is constructed for the joint failure region, the convolution integral is be solved accurately by using FFT.

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