Sliding mode control for civil structures based on complex Fourier coefficients of the earthquake

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Abstract

An improved control algorithm based on the theory of variable structural control or sliding mode control is presented. A method for determining the sliding surface by pole assignment algorithm using complex Fourier coefficients of the incoming earthquake signal applied to the structure is proposed. The design response spectrum is used for estimation of maximum control force for saturated control. Furthermore, a more general theoretical description of the sliding surface of the system is presented. The potential applications and the effectiveness of the proposed control algorithm are demonstrated by numerical examples. The simulation results indicate that a trade–off between the control forces and the structural response is possible.

Keywords: Sliding mode control; FFT; Complex Fourier coefficients; Pole placement; Saturated control

1. Introduction

The main purpose in structural control theory is to determine a control strategy that uses the measured structural response and the excitation signal to calculate appropriate control forces that will enhance the structural safety and serviceability against dynamic excitation like wind or earthquake. Over the past few decades various control algorithms and control devices have been developed, modified and investigated by various groups of researchers. The works of Yao [1], Housner et al. [2], Kobori et al. [3], Spencer et al. [4], Yang et al. [5], Soong, [6] and Connor and Klink [7] are the most representative. While many of these structural control strategies have been successfully applied, challenges as cost, reliance on external power and mechanical intricacy have delayed their widespread use.

Among various control algorithms sliding mode control (SMC) has demonstrated its potential of becoming a dominant control algorithm for civil engineering structures. Various details of the theory and implementation of SMC have been presented, among others by Utkin [8], Yang et al. [5], Cai et al. [9], Lee et al. [10].

In SMC the critical issues are, first the determination of the sliding surface where the system trajectory remains stable, and secondly the determination of the control force, usually using Lyapunov stability theory, which drives the trajectory to the sliding surface and forces it to stay there. To find the sliding surface the pole assignment and linear quadratic regulator LQR methods can be used. The pole assignment method requires prespecified values of the poles of the system on the sliding surface. In this paper the design of the sliding surface is performed by the pole assignment method, where the poles of the system on the sliding surface are chosen based on the complex Fourier coefficients of the incoming earthquake signal. Furthermore, a more general theoretical description of the sliding surface of the system is presented. Finally, the estimation of the maximum control force in saturated control is suggested based on the design response spectrum and the mode superposition method.

2. Improved sliding mode control algorithm

Sliding mode control or variable structure strategies were developed specifically for robust control of uncertain nonlinear systems by Utkin [8]. The equation of motion of a structural system with n degrees of freedom

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controlled by m forces and subjected to an earthquake excitation a_g is:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = -\mathbf{M}\mathbf{E}a_g + \mathbf{E}_f\mathbf{F} \tag{1}$$

where **M**, **C**, **K** denote the mass, damping and stiffness matrices of the structure, respectively, **F** is the control force matrix and **E**, $\mathbf{E}_f$ are the location matrices for the earthquake and the control forces on the structure. In the state space approach the above equation (1) can be written as follows:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}_g a_g + \mathbf{B}_f\mathbf{F} \tag{2}$$

The matrixes **X**, **A**, $\mathbf{B}_g$, $\mathbf{B}_f$ are given by

$$\mathbf{X} = \begin{bmatrix} \mathbf{U} \\ \dot{\mathbf{U}} \end{bmatrix}_{2nx1}, \mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}_{2nx2n},$$
$$\mathbf{B}_g = \begin{bmatrix} \mathbf{O} \\ -\mathbf{E} \end{bmatrix}_{2nx1}, \mathbf{B}_f = \begin{bmatrix} \mathbf{O} \\ \mathbf{M}^{-1}\mathbf{E}_f \end{bmatrix}_{2nxm} \tag{3}$$

The first step in SMC is to design the sliding surface on which the response is stable and the second step is the determination of the control demand which will drive the response trajectory into the sliding surface and force it to stay there all the subsequent time.

2.1 Design of sliding surface

In most studies the sliding surface is defined as a linear combination of the state vector. In the work of Slotin and Li [11] a more general approach is proposed. Let $\mathbf{U}_d$ be the desired response of the system. Usually in the control of buildings this quantity is zero or, if more relaxed criteria are preferred, $\mathbf{U}_d$ could be the response of the system just before yielding of some members. $\mathbf{U}_d$ could also be the response of a reference model, the behavior of which we would like our systems to follow. Let $\bar{\mathbf{U}}$ be the error defined as:

$$\bar{\mathbf{U}} = \mathbf{U} - \mathbf{U}_d \tag{4}$$

The sliding, time varying, surface is defined as:

$$\mathbf{s}(\bar{\mathbf{U}}, t) = 0, \quad \text{where } \mathbf{s}(\bar{\mathbf{U}}, t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \bar{\mathbf{U}} \tag{5}$$

where $\boldsymbol{\lambda}$ is a symmetric positive defined matrix or, more generally, $\boldsymbol{\lambda}$ is a Hurwitz matrix. If $n = 2$ then **s** is the weighted sum of the position and velocity error:

$$\mathbf{s} = \dot{\bar{\mathbf{U}}} + \boldsymbol{\lambda}\bar{\mathbf{U}} = [\boldsymbol{\lambda}\ \mathbf{I}]\begin{bmatrix} \bar{\mathbf{U}} \\ \dot{\bar{\mathbf{U}}} \end{bmatrix} = \mathbf{P}\mathbf{X} \tag{6}$$

If $n = 3$ the surface becomes as follows:

$$\mathbf{s} = \ddot{\bar{\mathbf{U}}} + 2\boldsymbol{\lambda}\dot{\bar{\mathbf{U}}} + \boldsymbol{\lambda}^2\bar{\mathbf{U}} \tag{7}$$

Thus, the problem of our system having the same response as the response of the ideal reference model is equivalent to that of the system remaining on the surface **s**. In the case where **s** is a linear combination of the states the matrix **P** is defined by pole assignment method. Successful application of the method requires judicious placement of the poles of the system on the sliding surface. In the work of Pnevmatikos et al. [12] the feedback matrix is estimated by a pole placement algorithm and a selection of the poles of the controlled system is based on the frequency content of the incoming earthquake. A method of estimation of matrix **P** by pole assignment method based on the complex form of the Fourier transform is proposed.

The complex form of the Fourier transform of the incoming earthquake signal $a_g(t)$ is :

$$a_g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} a_g(t)e^{-i\omega t}dt \tag{8}$$

Using the known trigonometric equation

$$e^{-i\omega t} = \cos(\omega t) + i\sin(\omega t) \tag{9}$$

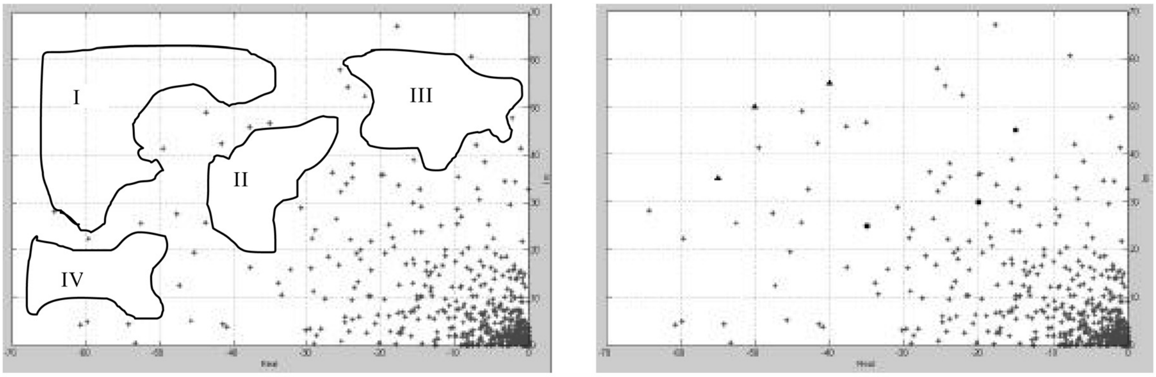
the complex Fourier coefficients $A(\omega)$ and $B(\omega)$ are obtained:

$$a_g(\omega) = A(\omega) - iB(\omega), A(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} a_g(t)\cos(\omega t)dt,$$
$$B(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \alpha_g(t)\sin(\omega t)dt \tag{10}$$

The complex coefficients of an Athens 1999 earthquake record are shown in Fig. 1a. By observing this figure one can identify some regions I, II, III or IV where the poles can be placed. However, one should be careful not to move too far from the center of axes because this will cause high control effort. If we take a closer look near the origin (Fig. 1b), we can observe some more regions where the poles can be placed. Generally, if the control devices are limited, then locations in the regions of Fig. 1b are suggested, while when the control devices are installed on every floor then locations in the regions of Fig. 1a are preferable. After choosing the poles of the system on the sliding surface, the matrix **P** in Eq. (6) is obtained by pole assignment method and consequently the sliding surface **s** is determined.

2.2 Design of the controller

The controllers are designed to drive the response trajectory into the sliding surface $\mathbf{s} = 0$. To achieve this goal a Lyapunov function V is chosen and the control



(a) (b)

Fig. 1. The complex coefficients (*) of the Athens 1999 earthquake and the suitable regions for poles of the system on sliding surface (a) and (b) the two cases of location of poles case I: (●) and case II (▼).

forces are obtained for the condition that the derivative of the function V should be negative. According to Yang et al. [5] the control forces $\mathbf{F}$ are given by:

$$\mathbf{F} = \mathbf{G} - \boldsymbol{\delta}\cdot\boldsymbol{\lambda}^{\mathrm{T}}, \mathbf{G} = -(\mathbf{P}\mathbf{B}_{\mathrm{f}})^{-1}\mathbf{P}\big(\mathbf{A}\mathbf{X} + \mathbf{B}_{\mathrm{g}}a_{\mathrm{g}}\big), \boldsymbol{\lambda} = \mathbf{s}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{\mathrm{f}} \tag{11}$$

If the control force of Eq. (11) is replaced into Eq. (2) we obtain:

$$\dot{\mathbf{X}} = -\mathbf{B}\boldsymbol{\delta}\mathbf{B}^{\mathrm{T}}\mathbf{P}^{\mathrm{T}}\mathbf{P}\mathbf{X} = -\mathbf{A}_{\mathrm{c}}\mathbf{X} \tag{12}$$

The poles of the controlled system are the eigenvalues of matrix $\mathbf{A}_c$. The last step is to check that the above eigenfrequencies of $\mathbf{A}_c$ are not located within the main frequency window of the incoming earthquake signal or, otherwise, the poles of $\mathbf{A}_c$ are not near the poles of the earthquake.

Yang et al. [5] proved that when each degree of freedom is implemented by a controller ($n = m$) the external earthquake excitation can be completely compensated. However, because $\mathbf{G}$ includes the restoring, damping, inertial and seismic forces, the magnitude of $\mathbf{G}$ is very large for controlling conventional civil engineering structures. Thus, the control force should be restricted to a certain level and a saturated controller should be considered in the design of SMC. In this case, full compensation of the response cannot be achieved. If the maximum control force is bounded by $\pm\,\mathbf{f}_{nax}$ a control force is estimated as follows:

$$\mathbf{F} = \begin{cases} \mathbf{G} - \boldsymbol{\delta}\cdot\boldsymbol{\lambda}^{\mathrm{T}}, & \left|\mathbf{G} - \boldsymbol{\delta}\cdot\boldsymbol{\lambda}^{\mathrm{T}}\right| \le \mathbf{f}_{\max} \\ \mathbf{f}_{\max} sign\big(\mathbf{G} - \boldsymbol{\delta}\cdot\boldsymbol{\lambda}^{\mathrm{T}}\big), & \left|\mathbf{G} - \boldsymbol{\delta}\cdot\boldsymbol{\lambda}^{\mathrm{T}}\right| > \mathbf{f}_{\max} \end{cases} \tag{13}$$

The force $\mathbf{f}_{max}$ is specified by the device capacity. Instead of the maximum control force of the device in a saturated controller, a percentage of the seismic loads could be used. The maximum seismic force for each eigenmode is given by:

$$\mathbf{f}^i_{\max} = a\mathbf{M}\boldsymbol{\Phi}^i{}_i\Psi_d(T_i, \zeta_i) \tag{14}$$

where a is a percentage of the seismic loads ($0 < a < 1$), while $\boldsymbol{\Phi}^{\mathrm{i}}$, ψ_{i}, Φ_{d} are the i eigenmode, the corresponding participation factor and the acceleration for the i eigenperiod and pertinent damping ratio, obtained from the design response spectrum. Having a monitoring system which measures on-line the incoming signal and the response of the structure, with appropriate identification techniques one can detect which einenmodes will contribute to the vibration of the building and use only them to obtain $\mathbf{f}_{max}$. Otherwise, $\mathbf{f}_{max}$ can be estimated a priori using such a number of eigenmodes as prescribed by the design code (eigenmass contribution equal to 90% of total mass) and superimposing them by the complete quadratic combination (CQC) or square root of sum of squares (SRSS) method. If a specific value of a is used, then in the design process a scaled response spectrum can be used, obtained from the original one using a as scale factor. Thus, the structure will be more cost effective since smaller seismic forces are applied.

The efficiency of the above control algorithm depends on the estimation of matrix $\mathbf{P}$. Numerical models using these control procedures have been designed using the Simulink of Matlab and are shown in Fig. 2.

3. Examples and numerical experiments

The proposed approach is demonstrated by means of numerical examples, where a three-story building described in [5] is analyzed, subjected to an Athens 1999 earthquake record. The signal is scaled to have maximum acceleration of 0.12 g. Two schemes of control

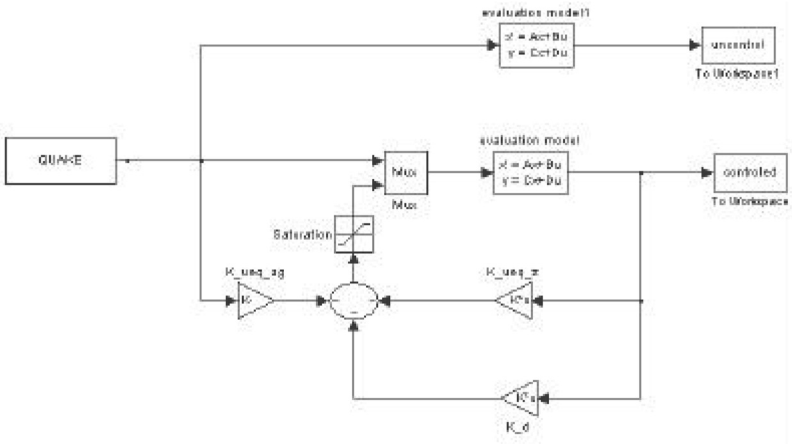


Fig. 2. The Simulink model for the control forces.

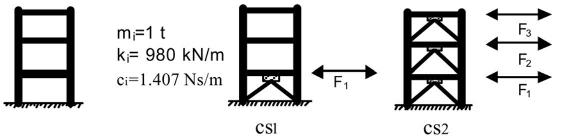


Fig. 3. The parameters of a three-story building and the two different control schemes cs1 and cs2.

force locations are considered: the first scheme (cs1), with only one force at the first story, and the second scheme, (cs2), with one force at each story, are illustrated in Fig. 3. For cs1 two cases of the location of the poles of the system on the sliding surface are examined, shown in Fig. 1b. In case I the locations of the poles are: $-35 + 25i$, $-20 + 30i$, $-15 + 45i$ and the conjugate ones, while in case II they are: $-55 + 35i$, $-50 + 50i$, $-40 + 55i$ and the conjugate ones. Because of the resulting low control forces, saturation controller was not taken into account. The results of the response and the control effort are shown in Table 1a. For cs2, if case I location of the poles is chosen, the system becomes unstable, due to the fact that matrix $\mathbf{A}_c$ from equation (12) has eigenfrequencies located near the main frequency window of the incoming earthquake signal. For case II this does not happen, the results are shown in Table 1b.

4. Summary and conclusions

From the above examples it is clear that the more control positions exist for the structure, the better the response is controlled. The success of the application of the algorithm depends on the well-chosen position of the poles of the system on the sliding surface and on checking that the eigenfrequencies of the controlled system are not located near the main frequency window of the incoming earthquake signal. The more control devices we have, the further away we can choose the poles on the sliding surface.

Table 1
The response (displacement and acceleration) and the control effort for every control scheme and for every case of location of poles on sliding surface

Controlled	cs1				
	(mm)		(m/sec^2)		(kN)
	u_1	u_3	$\ddot{u}_1$	$\ddot{u}_3$	F_1
Case I	2.20	3.20	2.49	2.31	5.72
Case II	2.40	2.00	5.16	1.95	7.55
Uncontrolled	6.90	15.55	2.20	3.30	

Controlled	cs2						
	(mm)		(m/sec^2)		(kN)		
	u_1	u_3	$\ddot{u}_1$	$\ddot{u}_3$	F_1	F_2	F_3
Case II	0	0	1.29	1.29	13.2	13.1	12.8
Uncontrolled	7.10	15.55	2.22	1.57			

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