Application of fast multipole methods to the analysis of MEMS

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Abstract

The analysis of micro-electro-mechanical structures (MEMS) is addressed. Focus is set on the evaluation of damping forces exerted on moving parts through a fast multipole accelerated boundary element method (BEM) technique, assuming a rigid-body behaviour of the structural parts.

Keywords: Boundary elements; Fast multipole; MEMS; Stokes flow; Unbounded domain; Dirichlet problem

1. Introduction

Micro-electro-mechanical-structures (MEMS) though being a promising and revolutionary technology, still suffer several design difficulties in view of their intrinsic complexity which is due to the interaction of various physical phenomena (electromagnetism, solid mechanics, fluid dynamics) and to extremely elaborate geometries with moving boundaries. The already available finite element techniques are in general valid tools for the analysis of MEMS, however the quality of the analysis of some of the involved phenomena could be greatly enhanced using boundary element techniques.

One of the open issues in MEMS design is the efficient simulation of the low Reynolds and Knudsen numbers fluid flow around moving parts, which can be modelled by the incompressible steady state Stokes approximation (see [1]). The adoption of fast multipole accelerated boundary-element techniques has the advantage of both allowing for the solution of large-scale external problems and requiring a negligible amount of remeshing under a rigid body motion of any of the movable parts.

2. Formulation and examples

In order to evaluate the total drag force due to the surrounding gas, the MEM structure can be modelled as a multibody system composed by several movable parts, subject to large displacements and rotations and to small deformations.

The rate of deformation is, however, small compared to the rigid-body velocity, so the fluid flow problem can be efficiently approximated by a Dirichlet problem, where the boundary geometry is given by the deformed configuration and the boundary conditions are given by the rigid body velocities g of the moving bodies.

This problem is known in literature as the *resistance problem*, and it can be formulated in terms of a Fredholm integral equation of the first kind in the form

$$\underline{g}\,(x) = [\underline{Vt}]\,(x) \tag{1}$$

where

$$[\underline{Vt}]_i(\underline{x}) = \int_\Gamma G_{ij}(\underline{x},\underline{y})\; t_j(\underline{y})\; \mathrm{d}S_y \tag{2}$$

and G_{ij} is a *stokeslet* (see [2]).

However, the single-layer formulation, in the case of the external Dirichlet problems, admits infinite solutions differing by a set of hydrostatic pressures over each immersed body. Moreover, the particular geometry of MEMS leads to extremely ill-conditioned operators.

To overcome these difficulties the mixed velocity-traction equation, developed by Frangi and Tausch ([3]) can be used. This approach starts from the consideration that a solution of Eq. (1) is also a solution of the traction equation

$$\left[\left(\frac{1}{2} + \underline{K}'\right)\underline{t}\right](x) = 0 \tag{3}$$

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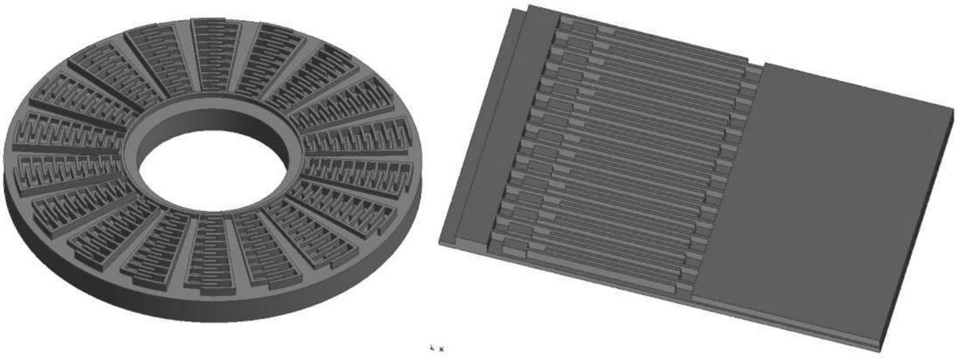

Fig. 1. Comb finger rotational accelerometer and parallel plate accelerometer.

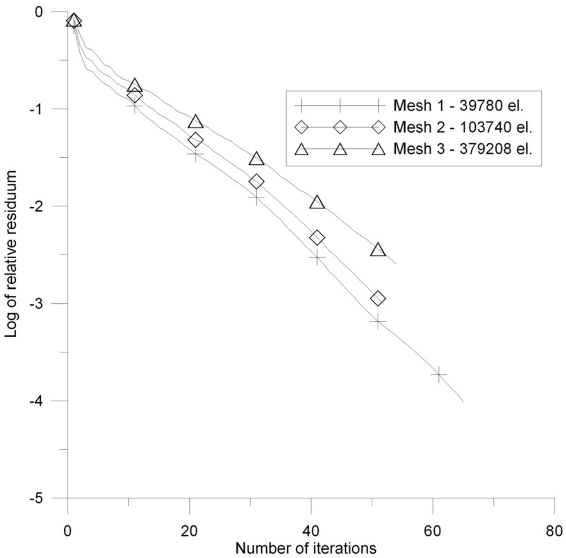

Fig. 2. Rotational accelerometer: convergence history.

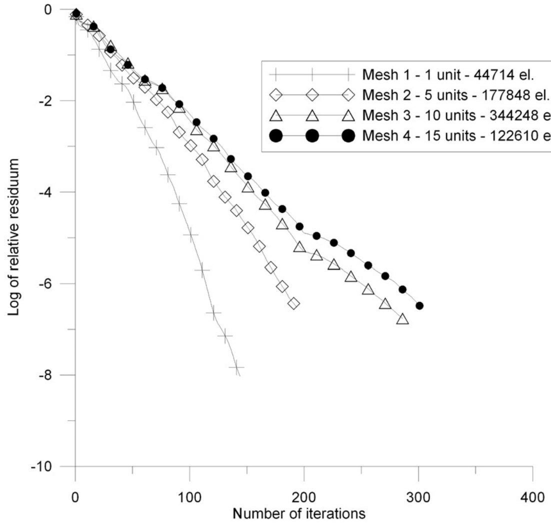

Fig. 3. Parallel plate accelerometer: convergence history.

where

$$[\underline{K}'\underline{t}]_j(\underline{x}) = \int_\Gamma T_{ijk}(\underline{x},\underline{y})\; n_k(\underline{x})\; t_j(\underline{y})\; \mathrm{d}S_y \tag{4}$$

is the adjoint of the double-layer operator, and T_{ijk} is a *stresslet*.

Due to the properties of the null-space of the traction equation, a linear combination of the two equations of the form

$$[\underline{Vt}](x) + \frac{\gamma}{\mu}\left[\left(\frac{1}{2}+\underline{K}'\right)\underline{t}\right](x) = \underline{g}\,(x) \tag{5}$$

still delivers a valid solution for the velocity equation subject to the constraints defined by the traction equation. A qualocation approach [4] for the discretization of the two operators allows to preserve the orthogonality of the ranges of the corresponding matrices, such orthogonality being an essential property for obtaining a correct approximate solution.

The formulation developed proves useful in the range of limited Knudsen numbers where it has received strong experimental confirmations. Its range of applicability can, however, be further extended through the straightforward implementation of appropriate slip boundary conditions [1].

Two realistic benchmarks are given by the rotational comb finger accelerometer and the parallel plate accelerometer shown in Fig. 1. Tractions are modelled as piecewise constant over each triangular surface element. The convergence history of the generalized minimal residual (GMRES) iterative solver, run on a standard desktop PC, is plotted in Figs 2 and 3 for different meshes up to more than one million degrees-of-freedom.

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