Developments in extended differential quadrature-based discrete element analysis methods and time integration schemes

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Abstract

The extended differential quadrature has been used to develop discrete element analysis methods and direct time integration schemes for solving continuum mechanics problems having arbitrary domain configuration. The concepts are stated briefly. Certain numerical results are presented.

Keywords: Differential quadrature; Generic differential quadrature; Extended differential quadrature; DQ-generated EDQ model; Differential quadrature element method; Generalized differential quadrature element method; Differential quadrature finite difference method

1. Introduction

The method of differential quadrature (DQ) defines a set of nodes in a problem domain. Then a derivative or partial derivative of a variable function at a node with respect to a coordinate is approximated as a weighted linear sum of all the function values at all nodes along that coordinate direction [1]. The original DQ can be used only to solve problems having a regular domain. Consequently, its application is very limited.

The author has generalized the DQ, which leads to the generic differential quadrature (GDQ) [2]. The weighting coefficients for a grid model defined by a coordinate system having arbitrary dimensions can also be generated. The configuration of a grid model can be arbitrary. In the GDQ, a certain-order derivative or partial derivative of the variable function with respect to the coordinate variables at a node is expressed as the weighted linear sum of the values of function and/or its possible derivatives or partial derivatives at all nodes.

The DQ and GDQ have been extended by the author, resulting in the extended differential quadrature (EDQ) [3]. In solving a problem, a discrete fundamental relation can be defined at a point that is not a node. The points for defining fundamental relations are discrete points. A node can also be a discrete point. Then, a certain-order derivative or partial derivative, of the variable function existing in a fundamental relation, with respect to the coordinate variables at an arbitrary discrete point can be expressed as the weighted linear sum of the values of function and/or its possible derivatives or partial derivatives at all nodes. Thus, in solving a problem, a discrete fundamental relation can be defined at a discrete point that is not a node. If a point used for defining discrete fundamental relations is also a node, then it is not necessary that the number of discrete fundamental relations at that node equals the number of degrees of freedom attached to it. This concept has been used to construct the discrete inter-element transition conditions and boundary conditions in the differential quadrature element analyses of the beam-bending problem, frame problem, and warping torsion bar problem [4–6]. In the EDQ discretization, the number of total degrees of freedom attached to the nodes is the same as the number of total discrete fundamental relations required. Some EDQ models can also be generated by using the DQ model through a transformation operation. They are DQ-generated EDQ models. The author has used DQ, GDQ, and EDQ to develop the differential quadrature element method (DQEM) [4,7], the generalized differential quadrature element method (GDQEM) [8], the differential quadrature finite element method (DQFEM) [9], and the differential quadrature finite difference method (DQFDM) [10].

In DQEM and GDQEM, the problem domain is separated into a finite number of subdomains or elements. The DQ, GDQ, and EDQ techniques are used to

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discretize the differential or partial differential governing equations defined in all elements, the transition conditions defined on the inter-element boundaries of two adjacent elements, and the boundary conditions of the problem. The DQEM adopts the mapping technique to develop irregular elements, while the GDQEM develops irregular elements without introducing this mathematical technique.

In using the DQEM, GDQEM, and DQFDM to the spacial discretization to solve generic engineering or scientific problems, the interior elements can be regular. However, in order to solve a problem having an arbitrary analysis domain configuration, elements connected to or near the analysis domain boundary might need to be irregular. The theoretical basis of DQEM, GDQEM, and DQFDM is rigorous, since all fundamental relations are satisfied locally.

The EDQ also has been used to develop direct time integration schemes for the transient analysis [11]. The developed special discretization techniques and the temporal discretization method can be used to effectively solve generic continuum mechanics problems. Numerical results of some sample problems solved by using these methods are presented. They demonstrate these novel numerical methods.

2. Extended differential quadrature

In using the EDQ to solve a problem, the number of total degrees of freedom attached to the nodes is the same as the number of total discrete fundamental relations required for solving the problem. A discrete fundamental relation can be defined at a point that is not a node. Then, a certain order of derivative or partial derivative, of the variable function existing in a fundamental relation, at an arbitrary point with respect to the coordinate variables can be expressed as the weighted linear sum of the values of variable function and/or its possible derivatives at all nodes [3]. Thus, in solving a problem, a discrete fundamental relation can be defined at a point that is not a node. If a point used for defining discrete fundamental relations is also a node, then it is not necessary that the number of discrete fundamental relations at that node equals the number of degrees of freedom attached to it. This concept has been used to construct the discrete inter-element transition conditions and boundary conditions in the differential quadrature element analyses of the beam-bending problem, and the warping torsion bar problem.

Let $\pi(\xi)$ denote the variable function associated with a one-dimensional problem with ξ the space coordinate or time variable. The EDQ discretization for a derivative of order m at discrete point α can be expressed by

$$\frac{d^m\pi_\alpha}{d\xi^m} = D^{\xi^m}_{\alpha i}\tilde{\pi}_i, \quad i = 1,2,\ldots,\bar{N} \tag{1}$$

where $\bar{N}$ is the number of degrees of freedom and $\tilde{\pi}_{\bar{\alpha}}$ is the values of variable function and/or its possible derivatives at the N nodes. The variable function can be a set of appropriate analytical functions denoted by $\Upsilon_p(\xi)$. The substitution of $\Upsilon_p(\xi)$ in Eq. (1) leads to a linear algebraic system for determining the weighting coefficients $D^{\xi^m}_{\alpha i}$. The variable function can also be approximated by

$$\pi(\xi) = \psi_p(\xi)\tilde{\pi}_p, \quad p = 1,2,\ldots,\bar{N} \tag{2}$$

where $\psi_p(\xi)$ are the corresponding interpolation functions of $\tilde{\pi}_p$. Adopting $\psi_p(\xi)$ as the variable function $\pi(\xi)$ and substituting it into Eq. (1), a linear algebraic system for determining $D^{\xi^m}_{\alpha i}$ can be obtained. The mth-order differentiation of Eq. (2) at discrete point α also leads to the EDQ discretization in Eq. (1) in which $D^{\xi^m}_{\alpha i}$ is expressed by

$$D^{\xi^m}_{\alpha i} = \frac{d^m\psi_i}{d\xi^m}|_\alpha \tag{3}$$

Using this equation, the weighting coefficients can be obtained easily by simple algebraic calculations.

The variable function can also be approximated by

$$\pi(\xi) = \Upsilon_p(\xi)c_p, \quad p = 1,2,\ldots,\bar{N} \tag{4}$$

where $\Upsilon_p(\xi)$ are appropriate analytical functions and c_p are unknown coefficients. The constraint conditions at all nodes can be expressed as

$$\tilde{\pi}_p = \mathcal{X}_{p\bar{p}}c_{\bar{p}} \tag{5}$$

where $\chi_{p\bar{p}}$ are composed of the values of $\Upsilon_p(\xi)$ and/or their possible derivatives at all nodes. Solving Eq. (5) for $c_{\bar{p}}$ and then substituting it in Eq. (4), the variable function can be rewritten as

$$\pi(\xi) = \Upsilon_p(\xi)\mathcal{X}^{-1}_{\bar{p}p}\tilde{\pi}_{\bar{p}} \tag{6}$$

Using this equation, the weighting coefficients can also be obtained:

$$D^{\xi^m}_{\alpha i} = \frac{\partial^m\Upsilon_{\bar{p}}}{\partial\xi^m}|_\alpha\mathcal{X}^{-1}_{i\bar{p}} \tag{7}$$

Various analytical functions, such as sinc functions, Lagrange polynomials, Hermite polynomials, Chebyshev polynomials, Bernoulli polynomials, Euler polynomials, rational functions, etc., can be used to define the weighting coefficients. To solve problems having singularity properties, certain singular functions can be used for the EDQ discretization. The problems having infinite domains can also be treated. Hermite polynomials have been used to solve certain structural problems with the convergence assured [3]. The convergence characteristics of EDQ adopting Chebyshev polynomials, Bernoulli polynomials, and Euler polynomials also has been carried out [12].

Consider the one-dimensional discretization using only one degree of freedom (DOF) of the variable function at the node to define the DQ. Lagrange interpolation functions can be used to explicitly express the weighting coefficients. For the Lagrange DQ model, the number of DOF $\bar{N}$ is equal to the number of nodes N. Lagrange interpolation functions $\hat{L}_\beta(\xi)$ can be expressed by

$$\hat{L}_\beta(\xi) = \frac{M(\xi)}{(\xi - \xi_\beta)M^{(1)}(\xi_\beta)} \tag{8}$$

where

$$M(\xi) = \prod_{\gamma=1}^{N} (\xi - \xi_\gamma), \quad M^{(1)}(\xi_\beta) = \prod_{\gamma=1,\gamma\neq\beta}^{N} (\xi_\beta - \xi_\gamma)$$

Then, the weighting coefficients can be derived

$$\begin{aligned} D^{\xi}_{\alpha\beta} &= \frac{d\hat{L}_B}{d\xi}\Big|_\alpha \\ &= \frac{(\xi_\alpha-\xi_\beta)M^{(1)}(\xi_\beta)M^{(1)}(\xi_\alpha)-M(\xi_\alpha)M^{(1)}(\xi_\beta)}{[(\xi_\alpha-\xi_\beta)M^{(1)}(\xi_\beta)]^2} \\ &= \begin{cases} \frac{M^{(1)}(\xi_\alpha)}{(\xi-\xi_\beta)M^{(1)}(\xi_\beta)]}, & \text{for} \quad \alpha \neq \beta \\ -\sum_{\gamma=1,\gamma\neq\alpha}^{N} D_{\alpha\gamma}, & \text{for} \quad \alpha = \beta \end{cases} \end{aligned} \tag{9}$$

When the uniform grid is used, Eq. (9) is reduced to

$$D^{\xi}_{\alpha\beta} = (-1)^{\alpha+\beta} \frac{(\alpha-1)!(N-\alpha)!}{\Delta\xi(\alpha-\beta)(\beta-1)!(N-\beta)!} \quad \text{for} \quad \alpha \neq \beta \tag{10}$$

where $\Delta\xi = \xi_\alpha - \xi_\beta$. The mth higher-order weighting coefficients $D^{\xi^m}_{\alpha\beta}$ can be calculated by using the first-order weighting coefficients $\mathrm{D}^{\xi}_{\alpha\beta}$

$$\begin{aligned} D^{\xi^2}_{\alpha\beta} &= \sum_{\gamma=1}^{N} D^{\xi}_{\alpha\gamma} D^{\xi}_{\gamma\beta}, \\ D^{\xi^3}_{\alpha\beta} &= \sum_{\gamma=1}^{N} D^{\xi^2}_{\alpha\gamma} D^{\xi}_{\gamma\beta}, \\ &\;\;\vdots \\ D^{\xi^m}_{\alpha\beta} &= \sum_{\gamma=1}^{N} D^{\xi^{m-1}}_{\alpha\gamma} D^{\xi}_{\gamma\beta} \end{aligned} \tag{11}$$

3. Generation of extended differential quadrature using differential quadrature

Some EDQ models can be generated by using DQ through the establishment of a transformation relation between the set of discrete function variables π_α of the equivalent DQ element and the set of discrete EDQ parameters $\tilde{\pi}_\alpha$. The weighting coefficients of these EDQ models are calculated by using the related transformation matrices and the weighting coefficients of the equivalent DQ model. For illustration, the C^1–C^0–C^1 and C^2–C^0–C^2 EDQ models used in the spacial discretization of DQEM analysis, and the C^1–C^0 EDQ model used in the temporal discretization of EDQ-based direct time integration method for structural dynamic problems, are stated.

3.1 C^1–C^0–C^1 extended differential quadrature model

Consider the C^1–C^0–C^1 EDQ model with each of the two boundary nodes having two DOF of the function variable and its first-order physical derivative, and each of the interior nodes having only one DOF of the function variable. This EDQ model is compatible and conformable and can automatically set the kinematic transition conditions on the inter-element boundary of two adjacent elements and the kinematic boundary conditions for the DQEM analysis of flexural deformation problems of structures.

Let $D^{\xi^m}_{\alpha i}$ denote the weighting coefficients for the equivalent DQ model defined on the natural space, which is a C^0–C^0–C^0 model. For the DQEM analysis of flexural deformation problems of structures using the DQ model, the DOF assigned to the first two and last two nodes are used to define either the transition conditions at the inter-element boundary of two adjacent real physical elements or boundary conditions at the real physical domain boundary. One of the two nodes at or next to one boundary node of the equivalent DQ element is at the inter-element boundary of two real physical elements used to separate the analysis domain or at the real physical domain boundary, while the other one can be either outside or inside the real physical elements and next to the element boundary of the real physical element. If the two extra nodes are inside the real physical element, then the equivalent DQ element coincides with the real physical element. If the two extra nodes are outside the real physical element, then the equivalent DQ element extends over the two element boundary points of the real physical element. Consequently, for the DQEM analysis of flexural deformation problems using DQ elements, the two adjacent equivalent DQ elements are overlapped partially and the equivalent DQ element containing the domain boundary is extended over the physical domain boundary.

Let $\hat{N}$ denote the number of the two boundary nodes of the C^1–C^0–C^1 EDQ element plus the other $\bar{N}-4$ interior nodes with $\bar{N}=N$. Also, let $\bar{l}$ and l denote the physical element lengths of the equivalent physical DQ element and the physical C^1–C^0–C^1 EDQ element. $\bar{l}=l$ for the transformation model with the physical EDQ element coinciding the physical DQ element, while $\bar{l}>l$ for the transformation model with the two extra nodes of the equivalent physical DQ element outside the physical EDQ element.

For the DQEM analysis of flexural deformation problems using EDQ elements, the DOF assigned to an auxiliary node of the equivalent DQ element can be transformed to obtain one rotational DOF assigned to the related element boundary node of the EDQ element. Fig. 1 shows the C^1–C^0–C^1 EDQ model generated by using the equivalent DQ model with the two auxiliary nodes inside the physical EDQ element. Assume that ξ is a natural coordinate having one unit length for both the C^1–C^0–C^1 EDQ model and the equivalent DQ model defined on the natural space. The transformation relation between the C^1–C^0–C^1 EDQ model and the equivalent DQ model can be represented by the following equation

$$\{\tilde{\pi}\}=[T]\{\pi\} \tag{12}$$

where

$$\{\pi\}=\begin{bmatrix}\pi_1 & \pi_2 & \pi_3 & \cdots & \pi_{N-2} & \pi_{N-1} & \pi_N\end{bmatrix}^T \tag{13}$$

For this transformation, $\{\tilde{\pi}\}$ and $[T]$ in Eq. (12) are expressed by

$$\{\tilde{\pi}\}=\begin{bmatrix}\pi_1 & \frac{d\pi_1}{d\xi} & \pi_3 & \cdots & \pi_{N-2} & \pi_N & \frac{d\pi_N}{d\xi}\end{bmatrix}^T \tag{14}$$

and

$$[T]=\begin{bmatrix}1 & 0 & 0 & \cdots & 0 & 0 & 0\\ \bar{D}^{\xi}_{11} & \bar{D}^{\xi}_{12} & \bar{D}^{\xi}_{13} & \cdots & \bar{D}^{\xi}_{1(N-2)} & \bar{D}^{\xi}_{1(N-1)} & \bar{D}^{\xi}_{1N}\\ 0 & 0 & 1 & \cdots & 0 & 0 & 0\\ . & . & . & \cdots & . & . & .\\ . & . & . & \cdots & . & . & .\\ . & . & . & \cdots & . & . & .\\ 0 & 0 & 0 & \cdots & 1 & 0 & 0\\ 0 & 0 & 0 & \cdots & 0 & 0 & 1\\ \bar{D}^{\xi}_{N1} & \bar{D}^{\xi}_{N2} & \bar{D}^{\xi}_{N3} & \cdots & \bar{D}^{\xi}_{N(N-2)} & \bar{D}^{\xi}_{N(N-1)} & \bar{D}^{\xi}_{NN}\end{bmatrix} \tag{15}$$

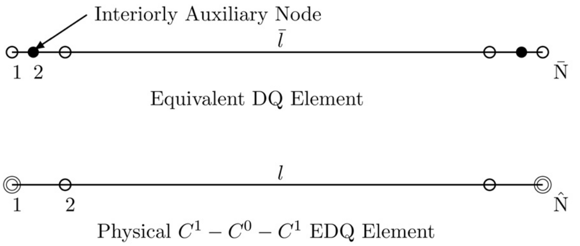

Fig. 1. Close boundary nodes of the C^1–C^0–C^1 EDQ model generated by using the equivalent DQ model with two auxiliary nodes inside the physical EDQ model.

Fig. 2 shows the C^1–C^0–C^1 EDQ model generated by using the equivalent DQ model with the two auxiliary nodes outside the physical EDQ element. For this transformation, $\{\tilde{\pi}\}$ and $[T]$ in Eq. (12) are expressed by

$$\{\tilde{\pi}\}=\begin{bmatrix}\pi_2 & \frac{d\pi_2}{ld\xi} & \pi_3 & \cdots & \pi_{N-2} & \pi_{N-1} & \frac{d\pi_{N-1}}{d\xi}\end{bmatrix}^T \tag{16}$$

and

$$[T]=\begin{bmatrix}0 & 1 & 0 & \cdots & 0 & 0 & 0\\ \bar{D}^{\xi}_{21} & \bar{D}^{\xi}_{22} & \bar{D}^{\xi}_{23} & \cdots & \bar{D}^{\xi}_{2(N-2)} & \bar{D}^{\xi}_{2(N-1)} & \bar{D}^{\xi}_{2N}\\ 0 & 0 & 1 & \cdots & 0 & 0 & 0\\ . & . & . & \cdots & . & . & .\\ . & . & . & \cdots & . & . & .\\ . & . & . & \cdots & . & . & .\\ 0 & 0 & 0 & \cdots & 1 & 0 & 0\\ 0 & 0 & 0 & \cdots & 0 & 1 & 0\\ \bar{D}^{\xi}_{(N-1)1} & \bar{D}^{\xi}_{(N-1)2} & \bar{D}^{\xi}_{(N-1)3} & \cdots & \bar{D}^{\xi}_{(N-1)(N-2)} & \bar{D}^{\xi}_{(N-1)(N-1)} & \bar{D}^{\xi}_{(N-1)N}\end{bmatrix} \tag{17}$$

Assume that $D^{\xi^m}_{\alpha i}$ are weighting coefficients for the C^1–C^0–C^1 EDQ model defined on the natural space. By using the fact that the derivatives with respect to the local element-based physical coordinates at the $\hat{N}$ grid nodes are the same for both the C^1–C^0–C^1 EDQ model and the DQ model, then the following relation can be obtained

$$\bar{D}^{\xi^m}_{\alpha i}\pi_i=D^{\xi^m}_{\alpha i}\tilde{\pi}_i,\quad \alpha=1,2,\ldots,\hat{N};\, i=1,2,\ldots,\bar{N} \tag{18}$$

The introduction of Eq. (12) into Eq. 18 leads to the

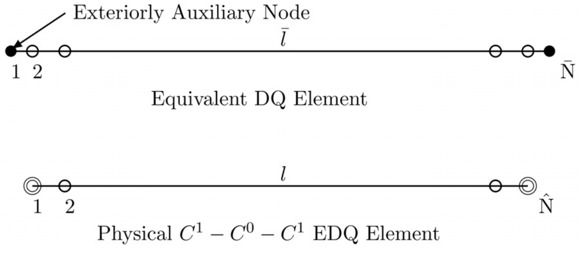

Fig. 2. Close boundary nodes of the C^1–C^0–C^1 EDQ model generated by using the equivalent DQ model with two auxiliary nodes outside the physical EDQ model.

following transformation equation for calculating the weighting coefficients for the C^1–C^0–C^1 EDQ model

$$D^{\xi^m}_{\alpha i} = \bar{D}^{\xi^m}_{\alpha j} T^{-1}_{ji}, \quad \alpha = 1,2,\ldots,\hat{N};\ i = 1,2,\ldots,\bar{N} \tag{19}$$

$D^{\xi^m}_{\alpha i}$ is an $N \times \bar{N}$ matrix with $N = \bar{N} - 2$. For applying the C^1–C^0–C^1 EDQ model to the DQEM analysis, the physical first-order derivatives at the two EDQ element boundary nodes are discrete EDQ parameters. Consequently, the elements of the second and last columns of the related EDQ weighting coefficient matrix $D^{\xi^m}_{\alpha i}$ need to be multiplied by l.

3.2. C^2–C^0–C^2 extended differential quadrature model

Consider the C^2–C^0–C^2 EDQ model with each of the two boundary nodes having three DOF of the function variable and its first- and second-order derivatives, and each of the interior nodes having only one DOF of the function variable. This EDQ model can automatically set the kinematic transition conditions and the continuity of the second-order derivatives of displacements on the inter-element boundary of two adjacent elements and the kinematic boundary conditions and the bending-moment-related natural boundary condition for the DQEM analysis of flexural deformation problems of structures.

To obtain the C^2–C^0–C^2 EDQ model, two DOF of two nodes of the equivalent DQ model close to a boundary node of the physical C^2–C^0–C^2 EDQ model have to be transformed to the two DOF representing the first-order and second-order derivatives at the related EDQ element boundary node. These two nodes can be either inside or outside the physical C^2–C^0–C^2 EDQ model. For illustration, the one of transforming four DOF assigned to four equivalent physical DQ nodes inside the related physical C^2–C^0–C^2 EDQ model is stated. Fig. 3 shows the C^2–C^0–C^2 EDQ model generated by using the equivalent DQ model with the four auxiliary nodes inside the physical EDQ element. The transformation relation between the C^2–C^0–C^2 EDQ model and the equivalent DQ model is also represented by Eq. (12) with

$$\{\tilde{\pi}\} = \begin{bmatrix} \pi_1 & \frac{d\pi_1}{d\xi} & \frac{d^2\pi_1}{d\xi^2} & \pi_4 & \cdots & \pi_{N-3} & \pi_N & \frac{d\pi_N}{d\xi} & \frac{d^2\pi_N}{d\xi^2} \end{bmatrix}^T \tag{20}$$

and

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \bar{D}^{\xi}_{11} & \bar{D}^{\xi}_{12} & \bar{D}^{\xi}_{13} & \bar{D}^{\xi}_{14} & \cdots & \bar{D}^{\xi}_{1(N-3)} & \bar{D}^{\xi}_{1(N-2)} & \bar{D}^{\xi}_{1(N-1)} & \bar{D}^{\xi}_{1N} \\ \bar{D}^{\xi^2}_{11} & \bar{D}^{\xi^2}_{12} & \bar{D}^{\xi^2}_{13} & \bar{D}^{\xi^2}_{14} & \cdots & \bar{D}^{\xi^2}_{1(N-3)} & \bar{D}^{\xi^2}_{1(N-2)} & \bar{D}^{\xi^2}_{1(N-1)} & \bar{D}^{\xi^2}_{1N} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ . & . & . & . & \cdots & . & . & . & . \\ . & . & . & . & \cdots & . & . & . & . \\ . & . & . & . & \cdots & . & . & . & . \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 \\ \bar{D}^{\xi}_{N1} & \bar{D}^{\xi}_{N2} & \bar{D}^{\xi}_{N3} & \bar{D}^{\xi}_{N4} & \cdots & \bar{D}^{\xi}_{N(N-3)} & \bar{D}^{\xi}_{N(N-2)} & \bar{D}^{\xi}_{N(N-1)} & \bar{D}^{\xi}_{NN} \\ \bar{D}^{\xi^2}_{N1} & \bar{D}^{\xi^2}_{N2} & \bar{D}^{\xi^2}_{N3} & \bar{D}^{\xi^2}_{N4} & \cdots & \bar{D}^{\xi^2}_{N(N-3)} & \bar{D}^{\xi^2}_{N(N-2)} & \bar{D}^{\xi^2}_{N(N-1)} & \bar{D}^{\xi^2}_{NN} \end{bmatrix} \tag{21}$$

Using $[T]$ in Eq. (19), the weighting coefficients for the C^2–C^0–C^2 EDQ model can be obtained with $\hat{N} = \bar{N} - 4$. For applying the C^2–C^0–C^2 EDQ model to the DQEM analysis, the physical first- and second-order derivatives at the two EDQ element boundary nodes are discrete EDQ parameters. Consequently, the elements of the second and $(\bar{N} - 1)$th columns of the related EDQ weighting coefficient matrix $\mathrm{D}^{\xi^m}_{\alpha i}$ need to be multiplied by l, and the elements of the third and last columns need to be multiplied by l^2.

3.3. C^1–C^0 extended differential quadrature model

Consider the C^1–C^0 EDQ model with the first node having two DOF of the function variable and its first derivative, and each of the other nodes having only one DOF of the function variable. This EDQ model can automatically set the initial conditions of each integration step for the EDQ-based step-by-step direct time integration method for solving transient structural problems.

To obtain the C^1–C^0 EDQ model, one DOF of one node of the equivalent physical DQ model close to the first node of the related physical EDQ model has to be transformed to the DOF representing the first-order derivative at the first node of the physical C^1–C^0 EDQ element. For illustration, the one of transforming one DOF assigned to one equivalent physical DQ model outside the physical C^1–C^0 EDQ model is stated. Fig. 4 shows the C^1–C^0 EDQ node with the related location outside the related C^2–C^0–C^2 EDQ model is stated. The transformation relation between the C^1–C^0 EDQ model and the DQ model is also represented by Eq. (12) with

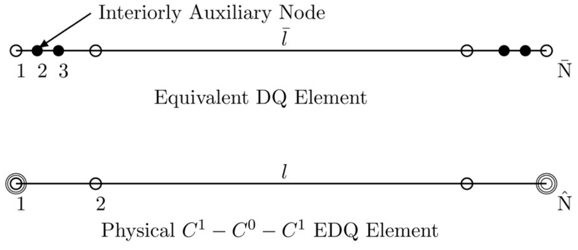

Fig. 3. Close boundary nodes of the C^2–C^0–C^2 EDQ model generated by using the equivalent DQ model with four auxiliary nodes inside the physical EDQ model.

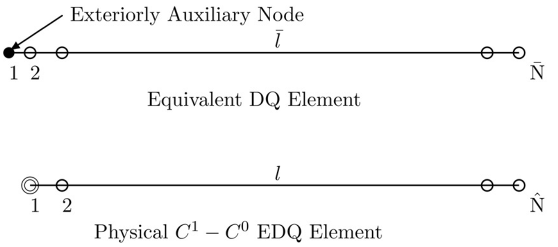

Fig. 4. Close boundary nodes of the C^1–C^0 EDQ model generated by using the equivalent DQ model with the auxiliary node outside the physical EDQ model.

$$\{\tilde{\pi}\} = \begin{bmatrix} \pi_2 & \frac{d\pi_2}{d\xi} & \pi_3 & \cdots & \pi_{N-1} & \pi_N \end{bmatrix}^T \tag{22}$$

and

$$[T] = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ \bar{D}^{\xi}_{N21} & \bar{D}^{\xi}_{N22} & \bar{D}^{\xi}_{N23} & \cdots & \bar{D}^{\xi}_{2(N-1)} & \bar{D}^{\xi}_{2N} \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{23}$$

Using $[T]$ in Eq. (19), the weighting coefficients for the C^1–C^0 EDQ model can be obtained with $\hat{N} = \bar{N} - 1$. For applying the C^1–C^0 EDQ model to the transient structural analysis, the physical first-order derivative at the first EDQ element boundary node is a discrete EDQ parameter. Consequently, the elements of the second column of the related EDQ weighting coefficient matrix $D^{\xi^m}_{\alpha i}$ need to be multiplied by l.

4. Numerical examples

The free vibration of the clamped square plate was analyzed by using one DQEM element. The problem was solved by using the C^1–C^0–C^1 EDQ model generated by using both of the equivalent DQ models with equally spaced nodes and with the two auxiliary nodes inside and outside the physical EDQ model. Let a, δ, D, and ρ denote the edge length, thickness, flexural rigidity, and mass density, respectively, of a square plate. Also, let the natural frequency ω_n of the nth vibration mode be expressed by $\omega_n = \frac{C_n}{a^2}\sqrt{\frac{D}{\rho\delta}}$, with C_n being the frequency factor. The first four frequency factors obtained by the two DQEM models are listed in Tables 1 and 2, separately. They are compared with the results obtained by Leissa [13]. They all show excellent convergence property.

The analysis of buckling of a simply supported beam resting on a Winkler elastic foundation and subjected to a compressive force was also carried out. The Young's modulus E, second moment of inertia I, and beam length L are all equal to 1. The foundation constant k is

Table 1
The first four frequency factors of a clamped square plate solved by the DQEM using the C^1–C^0–C^1 EDQ model generated by using the equivalent DQ model with two auxiliary nodes inside the physical EDQ model

Order of approximation	C_1	C_2	C_3	C_4
6	0.368344×10^2	0.650341×10^2	0.650341×10^2	0.901151×10^2
8	0.360286×10^2	0.796985×10^2	0.796985×10^2	
10	0.359929×10^2	0.730596×10^2	0.730596×10^2	0.105345×10^3
Leissa's solutions	0.35992×10^2	0.73413×10^2	0.73413×10^2	0.10827×10^3

Table 2
The first four frequency factors of a clamped square plate solved by the DQEM using the C^1–C^0–C^1 EDQ model generated by using the equivalent DQ model with two auxiliary nodes outside the physical EDQ model

Order of approximation	C_1	C_2	C_3	C_4
4	0.3475629×10^2			
6	0.3630833×10^2	0.7160144×10^2	0.7160144×10^2	0.1022950×10^3
8	0.3600669×10^2	0.7433538×10^2	0.7433538×10^2	0.1097962×10^3
10	0.3599020×10^2	0.7335022×10^2	0.7335022×10^2	0.1082930×10^3
Leissa's solutions	0.35992×10^2	0.73413×10^2	0.73413×10^2	0.10827×10^3

Table 3
The first three critical loads of a simply supported beam resting on a Winkler foundation subjected to a compressive force, solved by the DQEM using the C^2–C^0–C^2 EDQ model generated by using the equivalent DQ model with four auxiliary nodes inside the physical EDQ model

Number of elements	Order of approximations	$P_{cr,1}$	$P_{cr,2}$	$P_{cr,3}$
1	9	0.852698524×10^1	0.272684597×10^2	
	11	0.902698246×10^1	0.356982561×10^2	0.855320102×10^2
	13	0.979825438×10^1	0.377536249×10^2	0.852650326×10^2
	15	0.980216983×10^1	0.390026895×10^2	0.880869515×10^2
3	9	0.992172463×10^1	0.395073043×10^2	0.888569559×10^2
	11	0.992035566×10^1	0.394968576×10^2	0.888342912×10^2
	13	0.992030431×10^1	0.394968036×10^2	0.888338560×10^2
	15	0.992028329×10^1	0.394967956×10^2	0.888336930×10^2
5	9	0.992034204×10^1	0.395267305×10^2	0.888340231×10^2
	11	0.992027013×10^1	0.394939899×10^2	0.888336810×10^2
	13	0.992026889×10^1	0.394939052×10^2	0.888330992×10^2
	15	0.992026512×10^1	0.394917598×10^2	0.888320693×10^2
Analytical solutions		0.992026499×10^1	0.394910827×10^2	0.888320685×10^2

equal to 0.5. The C^2–C^0–C^2 EDQ model generated by using the equivalent DQ model with equally spaced nodes and with the four auxiliary nodes inside the physical EDQ model is used to model the beam structure. Let $P_{cr,n}$ denote the critical loads. Numerical results of the first three critical loads are summarized and listed in Table 3. The DQEM results are compared with the analytical solutions [14].

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