The hypersonic quasineutral gas discharge plasma in a magnetic field

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Abstract

The computing model of quasineutral gas discharge plasma in a magnetic field is developed and applied to predict a viscous-inviscid interaction in a hypersonic boundary layer. Numerical simulations are performed for a range of magnetic field flux densities ($B = 0.1$–1 tesla) imposed transversely to gas flow. The numerical result demonstrates a strong influence of the magnetic field to electrodynamic and gas-dynamic structures of weakly ionized gas flow.

Keywords: Quasi-neutral plasma; Viscous-inviscid interaction; Hypersonic boundary layer

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1. Introduction

A numerical analysis of a magneto-aerodynamic actuator has shown that the ignited surface plasma can modify the boundary-layer displacement thickness and is further amplified by viscous-inviscid interaction [1]. The plasma actuator was simulated by solving the two-dimensional magneto-aerodynamic equation including Lorentz force and Joule heating. The drift-diffusion plasma model [2,3] was adopted to describe the detailed electrodynamics structure of the glow discharge. However, the Hall parameters of electrons and ions were neglected by the earlier investigation in the presence of an applied external magnetic field [1]. This paper remedies this shortcoming and expands the predictive capability by developing a two-component plasma model including the Hall effect.

2. Analysis

In the low magnetic Reynolds number limit, the Faraday's induction law can be uncoupled from the governing equation. The simplified magneto-aerodynamic equations consist of the compressible Navier–Stokes equation containing the Lorentz acceleration and Joule heating in the momentum and energy equation [4].

$$\partial\rho/\partial\mathrm{t} + \mathrm{div}(\rho\mathbf{v}) = 0 \tag{1}$$

$$\partial\rho\mathbf{v}/\partial\mathrm{t} + \mathrm{div}(\rho\mathbf{v}\mathbf{v}+\boldsymbol{\tau}) = \mathbf{J}\times\mathbf{B} \tag{2}$$

$$\partial\rho\mathrm{e}/\partial\mathrm{t} + \mathrm{div}(\rho\mathrm{e}\mathbf{v}+\mathbf{q}+\mathbf{v}\cdot\boldsymbol{\tau}) = \mathbf{E}\cdot\mathbf{J} \tag{3}$$

A quasineutral model of direct current discharge (DCD) in a magnetic field is formulated to provide the continuity equation for species concentration and to ensure the compatible electrical field intensity, as follows:

$$\frac{\partial n}{\partial t} + \mathrm{div}(\mathbf{V}n) = \mathrm{div}\left(\mu_e^* D_a \mathrm{grad}\, n\right) + \dot{\omega}_e, \tag{4}$$

$$\mathrm{div}\left[(\mu_+ + \tilde{\mu}_e)n\mathbf{E} + \left(\tilde{D}_e - D_+\right)\mathrm{grad}\, n\right] = 0, \tag{5}$$

where

$$\tilde{\mu}_e = \frac{\mu_e}{1+b_e^2}, \mu_e^* = \frac{\mu_e + \mu_+}{\left(1+b_e^2\right)\mu_+ + \mu_e} \cong \frac{\mu_e}{\left(1+b_e^2\right)\mu_+ + \mu_e},$$

$$\tilde{D}_e = \frac{D_e}{1+b_e^2}, D_a = \frac{\mu_+ D_e + \mu_e D_+}{\mu_e + \mu_+}, \tag{6}$$

where n is the concentration of charged particles, μ_+ and μ_e are the mobilities of ions and electrons, respectively, D_+ and D_e are the diffusion coefficients of ions and electrons, respectively, and b_i and b_e are the Hall parameters of ions and electrons, respectively.

The source term in Eq. (4) must be modified in the presence of an externally applied magnetic field:

$$\dot{\omega}_e = \left(\frac{\alpha}{p^*}\right) p^* E \frac{\mu_e(p^*)}{1+b_e^2} - \beta n^2. \tag{7}$$

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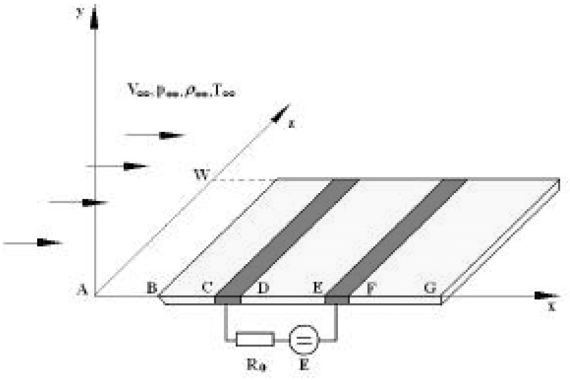

Fig. 1. Schematic of electromagnetic-fluid-dynamic actuator.

Here, we used the definition of recombination coefficient β and ionization frequency

$$\nu_i = \left(\frac{\alpha}{p^*}\right) \cdot v_{e,d} = \frac{\alpha}{p}\mu_e E, \tag{8}$$

where $\frac{\alpha}{p^*}$ is the first Townsend coefficient and p^* is the effective pressure.

In view of the great disparity of characteristic velocity scales, the sonic speed in fluid dynamics, and speed of light in electromagnetics, the two groups of equations are solved loosely coupled. For the electrodynamic equations, the pentadiagonal matrix system is solved by the successive over-relaxation scheme.

The quasineutral model was used for prediction of the magneto-fluid-dynamic interaction for hypersonic flow control. A schematic of the problem is shown in Fig. 1. In the present simulation, the cathode is placed upstream of the anode according to an experimental in a hypersonic magnetohydrodynamics (MHD) channel.

Initial conditions for the calculations were the same as in [1], but the Hall effect is taken into account in the present analysis. The unperturbed gas stream is characterized by a static pressure of 78.4 Pa, a temperature of 43 K, and a velocity of 675.5 m/s. Under this flow condition, the Reynolds number per meter is 1.615×10^5 and the Mach number is about 5.15. The surface plasma is generated by DCD with an emf of 1.2 kV.

In the absence of an externally applied magnetic field, the predicted electron number density over the anode spans a range from $10^{10}/\mathrm{cc}^3$ near the electrode and decays rapidly to a value of $8.7 \times 10^8/\mathrm{cc}^3$ at a distance of 2.5 cm from the electrode. This distribution reaches a very good agreement with experimental observation [1].

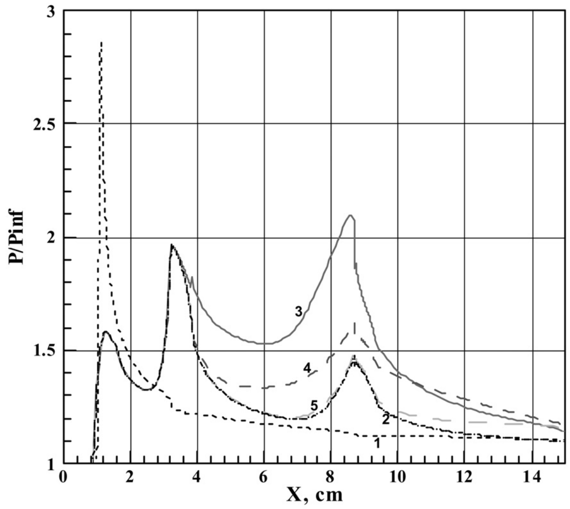

Fig. 2. Induced surface pressure by electromagnetic field (p_{inf} is the undisturbed gas pressure). 1, asymptotic theory of the weak interaction for isolated surface; 2, Navier–Stokes prediction for heated electrodes without direct current discharge (DCD); 3, DCD with magnetic field $B = 0.1\,\mathrm{T}$; 4, DCD with magnetic field $B = 0.5\,\mathrm{T}$; 5, DCD with magnetic field $B = 1\,\mathrm{T}$.

Figure 2 shows the induced surface pressure predicted by the quasineutral plasma model in the magnetic field. The Lorentz force according to the polarity of the imposed magnetic field exerts an upward acceleration to the charged particles away from the model. Indeed, the present results support the theoretical deduction for the lower magnetic flux density ($B = 0.1\,\mathrm{T}$) and induce a higher pressure rise through the increased displacement of the displacement boundary thickness. However, the strong magnetic fields ($B = 0.5$ and $1.0\,\mathrm{T}$) eventually suppress the ionization process and reverse the behavior.

3. Conclusion

A diffusion-drift model for DCD in the presence of an external magnetic field to take into account the Hall effect has been developed and studied numerically. The two-component plasma model provides a reasonable description for the charged particle number distribution of DCD. It is also shown that magnetic field of high inductivity suppresses effect of electric gas discharge on the viscous-inviscid interaction.

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