Stress concentration near the corner point of a magnetoactive 2D compound wedge

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Abstract

It is well known that the stress state at the surface edges of joined bodies can have a significant effect on the strength of the connection. In this paper, a magnetoactive connection that can be modeled as a piecewise homogeneous two-dimensional (2D) wedge is assumed, and the magnetoelastic stresses and the magnetic field near the corner point of this wedge are investigated. By using eigenfunction series expansions, all components of the magnetoelastic stresses and the perturbed magnetic field near the corner point are analyzed. It is shown that the applied magnetic field has a significant influence on the stress concentration in the vicinity of the contact surface edge.

Keywords: Compound wedge; Magnetoelasticity; Stress concentration; Contact surface

1. Introduction

Recent years have witnessed the proposal of a new concept of multifunctional materials/structures and the strong growth of the techniques associated with this concept. The concept is especially targeted to provide broader capabilities to the next generation of aerospace vehicles/spacecraft. The underlying idea of this concept is to exploit multiphysical and/or multiscale properties of materials or structures in such a way that besides its major designated functionality, the same structural component should accomplish at least one more function. An example of such a design is a smart load-carrying structure that can conduct non-destructive crack diagnosis or health monitoring by itself. This can lead to truly integrated structures, being able to perform multiple structural as well as electromagnetic and electromechanical functions. To implement this concept in various contexts, such as in aerospace vehicles and nuclear reactor constructions, a better understanding of the static and dynamic behaviors of the elastic structures subjected to the simultaneous action of mechanical, thermal, electrical, magnetic, and other fields becomes necessary.

In the past, much research effort has been devoted to, without considering the interaction of mechanical and other (e.g. magnetic, electrical) applied fields, the study of stress concentration near the corner point of joined linear elastic bodies [1]. In the present work, a body made of magnetosoft ferromagnetic materials is assumed to be immersed in a stationary magnetic field. The governing equations and surface conditions can be found in the theory of magnetoelasticity of magnetosoft ferromagnetic bodies [2]. Local solutions are constructed for the formulated boundary problem. All the components of the magnetoelastic stresses and the perturbed magnetic field near the wedge corner point are analyzed using eigenfunction series expansions. This leads to a set of ordinary differential equations. The unknown coefficients depend on the boundary and contact conditions, and the local characteristics of the stresses near the corner point can be predicted by solving an eigenvalue problem of a system of linear equations.

Depending on the eigenvalues, the stress concentration near the contact surface edge could be either increased or decreased by the applied magnetic field, in comparison with the pure elastic case. The numerical calculations show that the account of the magnetic field and the interaction of mechanical and magnetic fields can essentially change the magnitude of the stress state. This makes it possible to control stress concentration in

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the vicinity of the contact surface edge of the composite body by applying an external magnetic field.

2. Modeling the problem

Consider the plane problem of magnetoelasticity for an isotropic prismatic compound body made of magnetosoft ferromagnetic materials and having different magnetoelastic properties for each constituent component. In order to investigate the local stress state and local magnetic field near the corner point of the body, P, the cylindrical coordinate system (z, r, θ) is adopted so that the origin of the polar coordinate system (r, θ) is located at the point P. The angle θ is measured counterclockwise from the division line (Fig. 1). To be more specialized, it is assumed that the compound wedge associated with the point P consists of two components perfectly bonded together. The wedge is immersed in an external stationary magnetic field and the medium surrounding the wedge is assumed to be the vacuum.

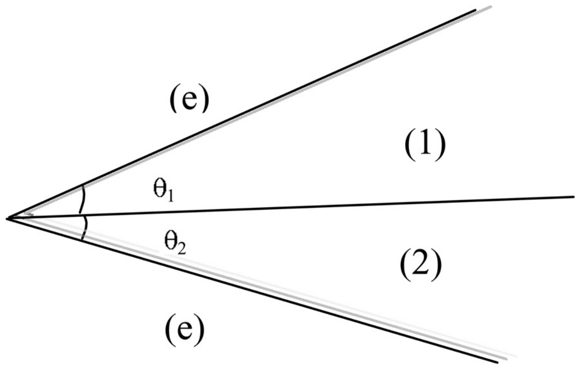

Fig. 1. Geometry of a compound wedge.

We denote, near the corner point P of the wedge, by $u_r^{(i)}$, $u_\theta^{(i)}$ ($i = 1, 2$) the components of the displacement vector $\mathbf{U}^{(i)}$ in the directions r and θ, while $\varphi^{(i)}$ is the magnetic potential induced within the wedge. For the problem being considered herein, with the help of the field equations of magnetoelasticity of dielectric magnetosoft ferromagnetic media [2,3], the following system of equations can be derived:

Equations in the domain Ω_i ($i = 1, 2$):

$$\left(\nabla^2 u_r^{(i)} - \frac{u_r^{(i)}}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta^{(i)}}{\partial\theta}\right) + k_i \frac{\partial}{\partial r}\left(\frac{\partial u_r^{(i)}}{\partial r} + \frac{u_r^{(i)}}{r} + \frac{1}{r}\frac{\partial u_\theta^{(i)}}{\partial\theta}\right) + \lambda^{(i)} H_\theta^{(i)}\left(\frac{1}{r}\frac{\partial^2\varphi^{(i)}}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial^2\varphi^{(i)}}{\partial\theta^2}\right) = 0 \quad (1a)$$

$$\left(\nabla^2 u_\theta^{(i)} - \frac{u_\theta^{(i)}}{r^2} + \frac{2}{r^2}\frac{\partial u_r^{(i)}}{\partial\theta}\right) + k_i \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\partial u_r^{(i)}}{\partial r} + \frac{u_r^{(i)}}{r} + \frac{1}{r}\frac{\partial u_\theta^{(i)}}{\partial\theta}\right) + \lambda^{(i)} H_\theta^{(i)}\left(\frac{1}{r^2}\frac{\partial^2\varphi^{(i)}}{\partial\theta^2} + \frac{1}{r}\frac{\partial\varphi^{(i)}}{\partial r}\right) = 0 \quad (1b)$$

$$\nabla^2\varphi^{(i)} = 0, \quad (1c)$$

where $\nabla^2 \equiv \partial^2/\partial r^2 + 1/r\cdot\partial/\partial r + 1/r^2\cdot\partial^2/\partial\theta^2$ is the Laplace operator in the polar coordinate system; $\lambda^{(i)} \equiv 2\mu_0\chi_i/\mu_i$; $k_i \equiv 1/(1-2\nu_i)$; $\chi_i = \mu_r^{(i)} - 1$.

The induced magnetic field outside the body can be written as

$$\nabla^2\varphi^{(e)} = 0. \quad (2)$$

The boundary conditions on the surfaces $\theta = \theta_1$ and $\theta = -\theta_2$ can be expressed as

$$\tau_{r\theta}^{(i)} + \tau_{r\theta}^{m(i)} = \tau_{r\theta}^{m(e)}; \qquad \tau_{\theta\theta}^{(i)} + \tau_{\theta\theta}^{m(i)} = \tau_{\theta\theta}^{m(e)}, \quad (3a,b)$$

$$\varphi^{(i)} - \varphi^{(e)} + \mathcal{X}_i H_{02}^{(i)} u_\theta^{(i)} = 0; \qquad \mu_r^{(i)}\frac{\partial\varphi^{(i)}}{\partial\theta} - \frac{\partial\varphi^{(e)}}{\partial\theta} = 0. \quad (3c,d)$$

Eqs (3a–d) state that the surfaces $\theta = \theta_1$ and $\theta = -\theta_2$ are free of total stresses.

The contact conditions on the surface $\theta = 0$ can be expressed in the following form:

$$\mathbf{U}^{(1)} = \mathbf{U}^{(2)}; \quad \tau_{r\theta}^{(1)} + \tau_{r\theta}^{m(1)} = \tau_{r\theta}^{(2)} + \tau_{r\theta}^{m(2)}; \quad \tau_{\theta\theta}^{(1)} + \tau_{\theta\theta}^{m(1)} = \tau_{\theta\theta}^{(2)} + \tau_{\theta\theta}^{m(2)}, \quad (4a-c)$$

$$\varphi^{(1)} - \varphi^{(2)} + \left[\frac{\mathcal{X}_1}{\mu_r^{(1)}} - \frac{\mathcal{X}_2}{\mu_r^{(2)}}\right] H_0\, u_\theta^{(1)} = 0; \quad \mu_r^{(1)}\frac{\partial\varphi^{(1)}}{\partial\theta} - \mu_r^{(2)}\frac{\partial\varphi^{(2)}}{\partial\theta} = 0. \quad (4d-e)$$

In Eqs (4b–e), $\tau_{r\theta}^{(i)}$ and $\tau_{\theta\theta}^{(i)}$ are the components of elastic stresses; $\tau_{r\theta}^{m(i)}$, $\tau_{\theta\theta}^{m(i)}$, $\tau_{r\theta}^{m(e)}$ and $\tau_{\theta\theta}^{m(e)}$ are the components of the Maxwell stress tensor within the wedge and the vacuum, respectively. It is recalled that in Eq. (4d), the relation $\mu_r^{(1)}H_{02}^{(1)} = \mu_r^{(2)}H_{02}^{(2)} = H_0$ is used.

3. Solution of the problem

The displacement field and magnetic potential in each of the domains Ω_I ($I = 1, 2$) and the magnetic potential in the external domain are found by assuming the following form of the solution [1, 5]:

$$u_r^{(i)}(r,\theta) = r^\alpha \bar{u}_r^{(i)}(\theta); \qquad u_\theta^{(i)}(r,\theta) = r^\alpha \bar{u}_\theta^{(i)}(\theta), \quad (5a,b)$$

$$\varphi^{(i)}(r,\theta) = H_\theta r^\alpha \bar{\varphi}^{(i)}(\theta); \qquad \varphi^{(e)}(r,\theta) = H_\theta r^\alpha \bar{\varphi}^{(e)}(\theta), \quad (5c,d)$$

where $\bar{u}_r^{(i)}(\theta)$, $\bar{u}_\theta^{(i)}$ (θ), $\bar{\varphi}^{(i)}(\theta)$, and $\bar{\varphi}^{(e)}(\theta)$ are unknown functions; α is a parameter that needs to defined by the problem itself.

Substituting Eqs (5a–d) into Eqs (1) and (2) the unknown functions can be represented as:

$$\bar{u}_r^{(i)}(\theta) = A_i \sin[(\alpha-1)\theta] + B_i \cos[(\alpha-1)\theta] + C_i \sin[(\alpha+1)\theta] + D_i \cos[(\alpha+1)\theta] + \gamma_i E_i \cos(\alpha\theta) + \gamma_i F_i \sin(\alpha\theta) \quad (6a)$$

$$\bar{u}_\theta^{(i)}(\theta) = \gamma_{2i} A_i \sin[(\alpha-1)\theta] - \gamma_{2i} B_i \cos[(\alpha-1)\theta] + C_i \cos[(\alpha+1)\theta] - D_i \sin[(\alpha+1)\theta] + \gamma_{1i} E_i \sin(\alpha\theta) + \gamma_{1i} F_i \sin(\alpha\theta), \quad (6b)$$

$$\bar{\varphi}^{(i)}(\theta) = E_i \sin(\alpha\theta) + F_i \cos(\alpha\theta), \quad \bar{\varphi}^{(e)}(\theta) = E_e \sin(\alpha\theta) + F_e \cos(\alpha\theta), \quad (6c, d)$$

in which

$$\gamma_i \equiv \frac{\lambda^{(i)} H_\theta^2 \alpha(\alpha-1)(k_i\alpha + 2\alpha + 1)}{(k_i+1)(1-4\alpha^2)}, \quad (7a)$$

$$\gamma_{1i} \equiv \frac{\lambda^{(i)} H_\theta^2 \alpha(\alpha-1)(k_i\alpha + 2\alpha + k_i + 1)}{(k_i+1)(1-4\alpha^2)}; \quad \gamma_{2i} \equiv \frac{k_i\alpha + k_i + 2}{k_i\alpha - k_i - 2}. \quad (7b, c)$$

In Eqs (6a–d), A_i, B_i, C_i, D_i, E_i, F_i ($i = 1, 2$), E_e, and F_e are unknown constants, which are to be determined by imposing the boundary and contact conditions from Eqs (3) and (4). As a result, a homogeneous system of linear algebraic equations for the unknown constants is obtained. The condition of existence of a non-trivial solution of this algebraic system determines the unknown parameter α, i.e. α should fulfill the following transcendental equation:

$$\Delta(\alpha, b_c^2, \theta_1, \theta_2, \mu_r^{(1)}, \mu_r^{(2)}, \mu_2, \nu_1, \nu_2) = 0, \quad (8)$$

where $b_c \equiv H_0/\sqrt{\mu_0\mu_2}$.

As a special case, in the absence of the external magnetic field, i.e. $b_c = 0$, Eq. (8) splits into two independent transcendental equations:

$$\Delta_{0y} = 0, \quad \Delta_{0m} = 0. \quad (9a, b)$$

Eq. (9a) has been obtained by Chobanyan [4], and the characteristics of elastic stresses near the corner point of a compound body have been investigated. Eq. (9b) has been obtained by Mittra and Li [5], and the behavior of the magnetic field on the top of a piecewise homogeneous ferromagnetic wedge was studied. It can be proved that all components of the magnetoelastic stresses near the corner point of the wedge can assume the following form:

$$\tau = r^{\alpha \mathrm{Re}^{-1}} R(r,\theta), \quad (10)$$

where $\alpha_{\mathrm{Re}} \equiv \mathrm{Re}(\alpha)$, and $R(r, \theta)$ is bounded in amplitude and in general does not vanish when $r \to 0$.

From Eq. (10), it can be seen that the characteristics of the magnetoelastic stress state near the contact surface edge are determined by α_{Re}. If $\alpha_{\mathrm{Re}} \geq 1$, then in the local area of the wedge, we have zero stress state. If $\alpha_{\mathrm{R}} < 1$, then the magnetoelastic stresses increase without bound at point P (i.e. strong concentration of stresses occurs). It is recalled that due to the finiteness of the induced magnetoelastic strain energy [4,5], $\alpha_{\mathrm{Re}} > 0$.

4. Results and discussion

The secant method for two functions is employed to obtain the numerical solution of α_{Re} and α_{Im} ($\equiv \mathrm{Im}(\alpha)$) in Eq. (8). Table 1 and Figs 2 and 3 show some results of α. It can be seen that:

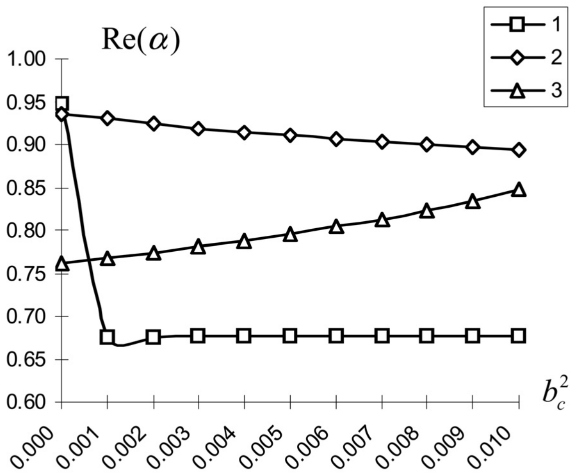

Fig. 2. Influence of the applied magnetic field on the stress concentration at a wedge: Re(α) versus b_c^2. Curve 1: $\mu = 0.0625$, $\nu_1 = 0.28$, $\nu_2 = 0.32$, $\mu_r^{(1)} = 10^2$, $\mu_r^{(2)} = 10^4$, $\theta_1 = 3\pi/4$, $\theta_2 = \pi/4$. Curve 2: $\mu = 0.0625$, $\nu_1 = \nu_2 = 0.3$, $\mu_r^{(1)} = 10^3$, $\mu_r^{(2)} = 10^2$, $\theta_1 = 17\pi/24$, $\theta_2 = 7\pi/24$. Curve 3: $\mu = 0.0625$, $\nu_1 = 0.32$, $\nu_2 = 0.28$, $\mu_r^{(1)} = 10$, $\mu_r^{(2)} = 10^4$, $\theta_1 = \theta_2 = \pi/2$.

1. The presence of the magnetic field can not only increase the level of a stress concentration (see Table 1, columns 3–5, and Fig. 2) but also decrease the concentration level (see Table 1, columns 1 and 2, and Fig. 2). In some cases, the imaginary part α_{Im} vanishes, depending on the presence of magnetic field (see Table 1, column 2).
2. There are cases when the imaginary part is zero in the pure elastic case but different from zero in the

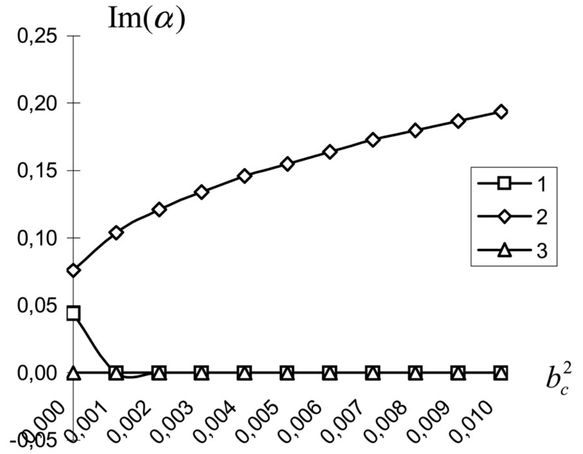

Fig. 3. Influence of the applied magnetic field on the stress concentration at a wedge: Im(α) versus b_c^2. Curve 1: $\mu = 0.0625$, $\nu_1 = 0.28$, $\nu_2 = 0.32$, $\mu_r^{(1)} = 10^2$, $\mu_r^{(2)} = 10^4$, $\theta_1 = 3\pi/4$, $\theta_2 = \pi/4$. Curve 2: $\mu = 0.0625$, $\nu_1 = \nu_2 = 0.3$, $\mu_r^{(1)} = 10^3$, $\mu_r^{(2)} = 10^2$, $\theta_1 = 17\pi/24$, $\theta_2 = 7\pi/24$. Curve 3: $\mu = 0.0625$, $\nu_1 = 0.32$, $\nu_2 = 0.28$, $\mu_r^{(1)} = 10$, $\mu_r^{(2)} = 10^4$, $\theta_1 = \theta_2 = \pi/2$.

presence of a magnetic field (see Table 1, columns 3 and 4).

If a crack is on the boundary of two piecewise homogeneous half planes (i.e. $\theta_1 \to \pi$, $\theta_2 \to -\pi$), then for the solution of transcendental Eq. (8), we have $\alpha_{\mathrm{Re}} = 0.5$, while α_{Im} depends on the physicomechanical and geometrical parameters of the problem (the result is in accordance with the results of Shindo [3], Hasanyan et al. [6], and Bagdasaryan et al. [7]).

5. Conclusions

From the results obtained we can conclude that the external magnetic field can change the stress concentration level near the corner point of a compound wedge. Compared with the pure elastic case, it can either increase or decrease the stress concentration. Depending on the amplitude of the applied magnetic field and the geometrical as well as physical characteristics of the wedge, the change of the amplitude of the stress concentration in comparison with the pure elastic case can reach about 25% (see Table 1 and Figs 2 and 3). Hence, the applied magnetic field can be instrumental toward controlling the degree of concentration of magnetoelastic stresses in the vicinity of the corner point of a compound wedge.

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Table 1
Numerical solutions of α when $\nu_1 = 0.32$, $\nu_2 = 0.28$, $\mu_r^{(1)} = 10$, $\mu_r^{(2)} = 10^4$, and $\theta_1 = \pi - \theta_2$

b_c^2	θ_2				
	$\pi/6$	$7\pi/24$	$\pi/3$	$9\pi/24$	$\pi/2$
0	0.738	$0.946+0.059\jmath$	0.837	0.784	0.762
0.001	0.616	0.754	0.863	0.797	0.768
0.002	0.616	0.757	$0.896+0.039\jmath$	0.816	0.774
0.003	0.616	0.760	$0.893+0.065\jmath$	0.842	0.781
0.004	0.617	0.763	$0.890+0.082\jmath$	$0.850+0.078\jmath$	0.788
0.005	0.617	0.765	$0.888 + 0.096\jmath$	$0.842 + 0.088\jmath$	0.795
0.006	0.617	0.767	$0.886 + 0.107\jmath$	$0.836 + 0.097\jmath$	0.804
0.007	0.617	0.769	$0.885 + 0.118\jmath$	$0.832 + 0.104\jmath$	0.813
0.008	0.618	0.771	$0.883 + 0.127\jmath$	$0.829 + 0.112\jmath$	0.823
0.009	0.618	0.773	$0.882 + 0.136\jmath$	$0.827 + 0.118\jmath$	0.834
0.010	0.618	0.774	$0.881 + 0.144\jmath$	$0.825 + 0.124\jmath$	0.848

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