Benchmark experimental data for radiative heat transfer prediction

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Abstract

The computation of the fraction of the thermal radiation that leaves one surface and arrives at another is largely determined from the geometrical view factor and is central to a radiative heat transfer simulation. Several methods can be used to calculate view factors, such as direct integration, Monte-Carlo and the Hemi-Cube method. These methods can be tested on several benchmarks for which analytical view factor equations exist, such as parallel plates, hinged plates and parallel circular discs. However, there are no analytical solutions that combine radiative, conductive and convective heat transfer. Furthermore, basic experimental data for benchmarking computational solutions is also scarce.

This paper describes the construction of a simple testing rig that allows experiments to be performed that combine view factor effects with measured heat transfer. Two different cases are studied in this paper: parallel plates and parallel disc-to-plate cases.

Keywords: Heat transfer; Monte-Carlo; Radiation; Finite Element Method (FEM); Radiative flux

1. Introduction

Radiative heat transfer becomes of relatively greater importance over the convective and conductive modes at higher temperatures. Greybody surface-to-surface thermal radiation is given by Stefan-Boltzmann's equation, which defines the net heat exchange between two surfaces A_1 and A_2, as:

$$Q_{1\leftrightarrow 2} = \sigma\left(\varepsilon_1 T_1^4 - \alpha_2 T_2^4\right) A_1 F_{1-2} = \sigma\left(\varepsilon_1 T_1^4 - \alpha_2 T_2^4\right) A_2 F_{2-1} \quad (1)$$

where ε is the emissivity, σ is the Stefan-Boltzmann constant, T is the absolute temperature of the greybody, and A is the surface area. F_{1-2} is the view factor and is defined as:

$$F_{1-2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2 dA_1 \quad (2)$$

View factors can be calculated from Eq. (2) by any of the methods mentioned earlier, but in this paper we have used the Monte-Carlo method as implemented by Baranoski et al. [1] or Loehrke et al. [2].

The heat transfer calculations used for this work are transient, three-dimensional and based on the Galerkin FEM form of the conductivity equation, using eight-noded hexahedral elements. For the view factor calculation, external radiation-exchanging faces are further sub-divided into triangles.

In the following sections, a sensitivity analysis is used to examine the relationship between view factor accuracy, mesh division and the number of rays used in the Monte-Carlo method, and their effect on the heat exchanged between surfaces, as compared to experimental data.

2. Experimental rig

An experiment rig was built at the University of Wales Swansea, according to an original design presented in the paper by Vujičić et al. [3]. It consists of a track (contained within a heat-resistant low-reflectivity painted box), which can hold an emitter and receivers of various shapes (disk, square, cylindrical), sizes (thicknesses from 3 to 15 mm) and constitutive materials

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Table 1
The values of view factor obtained for parallel plates for the ratio $c = 0.2$

Mesh 10 × 10 rays/el.	Model	Error %	Mesh 25 × 25 rays/el	Model	Error %	Mesh 40 × 40 rays/el	Model	Error %
100	0.6863	−0.567	100	0.6908	0.0824	100	0.6892	−0.154
1,000	0.6894	−0.118	1,000	0.6913	0.146	500	0.6878	−0.3483
10,000	0.6901	−0.024	5,000	0.6898	−0.0724	1,000	0.6851	−0.7494
20,000	0.6901	−0.015	10,000	0.691	0.1153	2,000	0.6851	−0.7747

Table 2
The values of view factor obtained for parallel discs for the ratio $c = 1$

Mesh 4 × 10 rays/el.	Model	Error %	Mesh 4 × 20 rays/el.	Model	Error %	Mesh 4 × 40 rays/el.	Model	Error %
100	0.1723	0.4074	100	0.1711	−0.254	100	0.1716	−0.004
1,000	0.1722	0.3439	1,000	0.1714	−0.0787	500	0.1714	−0.1299
2,000	0.1716	0.0111	2,000	0.171	−0.3072	1,000	0.1718	0.1451
4,000	0.1709	−0.3637	4,000	0.1713	−0.1521	2,000	0.1717	0.005
16,000	0.1708	−0.4727	16,000	0.1712	−0.2028	4,000	0.1716	0.0048

(steel, brass and aluminium), as well as at various rotational orientations to each other. The ratio c between emitter and receivers can be varied from $c = \text{distance}/D = 0.2$ to 4 (disc emitter), and from $c = \text{distance}/L = 0.2$ to 1 (square emitter). For this paper we have used steel for both the emitter (197 × 197 mm) and the receiver, with the following properties:

(i) Thermal conductivity of 16 W/mK.
(ii) Emissivity of 0.81.
(iii) Specific heat capacity of 503 J/kgK.
(iv) Density of 8030 kg/m^3.

Temperatures were measured using thermocouples and an infra-red thermal imaging camera (Thermo Tracer TH7102).

3. Parameters varied in the sensitivity analysis

(i) The emitter (2 × 197 × 197 mm) was divided into 10 × 10 × 4; 25 × 25 × 4 and 40 × 40 × 4 elements.
(ii) The receiver (5 × 197 × 197 mm) was divided into 10 × 10 × 10; 25 × 25 × 10 and 40 × 40 × 10 elements.
(iii) The distance ratio between the plates was varied from 0.2 to 4.
(iv) 100, 1000, 5000, 10,000, 20,000 and 50,000 rays were used in the Monte-Carlo algorithm to calculate the view factors.

4. View factor calculations

Two examples are used to verify the view factor calculations:

- **Case 1: two parallel plates**
  For this case, and $c = 0.2$, the analytical equation gives a view factor value of 0.690245. Computed view factor values by Monte-Carlo are presented in Table 1 together with error estimates based on the analytical value. For a 40 × 40 × 4 mesh with 1000 rays, the CPU time was 1114 seconds.
- **Case 2: parallel circular discs with centres along the same normal**
  For this case, and $c = 1.0$, the analytical equation gives a view factor value of 0.171573. Computed view factor values by Monte-Carlo are presented in Table 2 together with error estimates based on the analytical value.

The results indicate that the Monte-Carlo method can be used for the view factor calculation. Increasing the number of rays gives higher accuracy, but is accompanied by a corresponding increase in CPU time, a well-documented problem with surface-to-surface radiative heat transfer calculations.

It should be pointed out that the view factor values listed in Tables 1 and 2 are the total accumulated value from emitter to receiver, summed over individual view factor values of elements on the external surfaces, and the accuracy of this value can be misleading. For example, in the parallel plate case, a surface consisting of 25 × 25 elements with a separation ratio $c = 0.2$ and 100 rays, gives a difference between the computed total

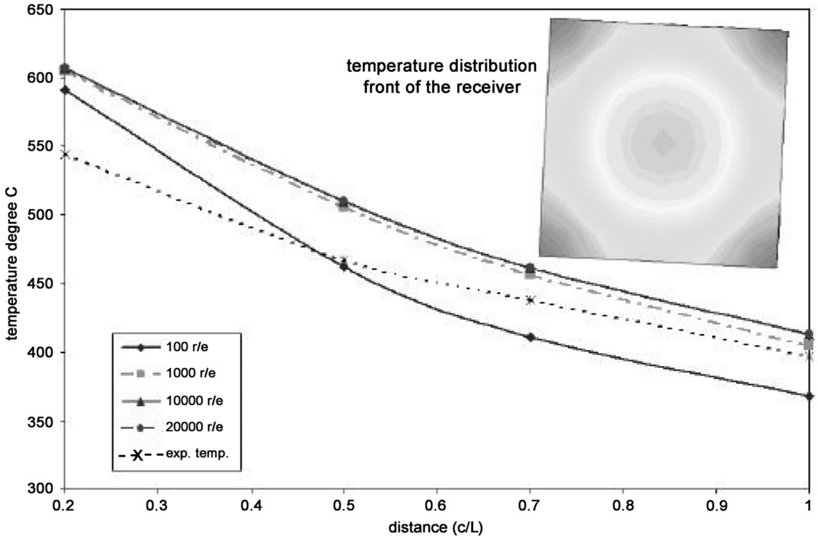

Fig. 1. Average temperatures of the receiver for the mesh 10×10.

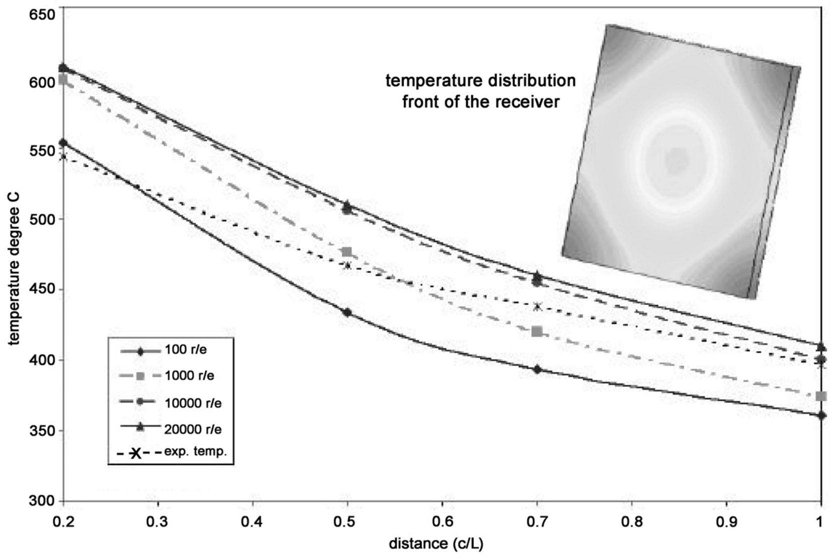

Fig. 2. Average temperatures of the receiver for the mesh 25×25.

view factor and the analytical view factor to be 0.0824%. However, taking a single element on this surface, the local view factor value difference to its analytical value is 20.93%, while for 10,000 rays this difference is reduced to 3.66%.

This means that while the average radiative energy gained by the receiver can be relatively accurate with only a few rays, the actual distribution of the radiative flux requires many more rays for an adequate resolution, as highlighted by the radiative flux contours embedded in Fig. 2.

5. Heat transfer simulation of the experiment

The average temperature of the emitter was 724 K, while the average temperature of the receiver as predicted by the FEM is plotted for different meshes, in Fig.

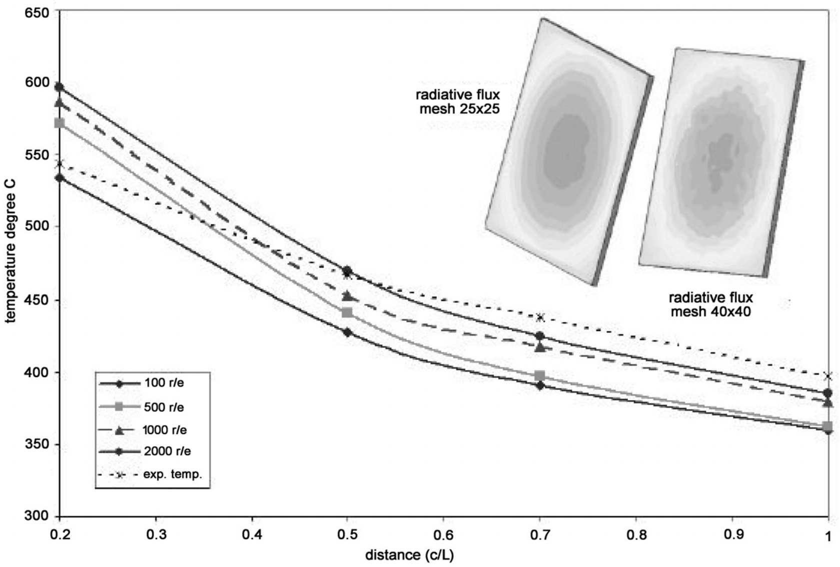

Fig. 3. Average temperatures of the receiver for the mesh 40×40.

1 ($10 \times 10 \times 4$ elements), Fig. 2 ($25 \times 25 \times 4$ elements) and Fig. 3 ($40 \times 40 \times 4$ elements). A different number of rays (100–20,000) are used in each case to calculate the view factors. Convective heat transfer coefficients, with values in the order of 12–15 W/m^2K, were applied as a constant on external surfaces. These values were corroborated with film-theory hand calculations for natural convection, and also with values determined by the commercial software Fluent 6.

It can be seen from Figs 1, 2 and 3 that the overall difference between calculated and measured temperatures is less than 10%, over all meshes and ray numbers used. The highest differences occur at smallest distances of separation, and it is thought that either reflectivity of the surfaces (not currently taken into consideration), view factor accuracy, inadequate thermo-physical properties or a combination of each, may be playing a role in this region.

6. Conclusions

The Monte-Carlo method has successfully been used to calculate view factor values for a three-dimensional heat transfer analysis with combined radiation, conduction and convection.

It is critical to reach a balance between the number of elements and the number of rays used by the Monte-Carlo method, as the CPU time scales exponentially with the number of rays. However, too few rays will not be sufficient to accurately resolve the radiative flux distributions at a local element level. Nevertheless, even for a small number of rays and elements, agreement with experiment is relatively good (within 10%), considering the potentially larger effect of variations in material property values and surface boundary conditions.

Using these types of results, it may be possible to establish an automated method of balancing the number of rays and elements, and future work will also concentrate upon establishing and including the effect of reflectivity in the thermal radiative simulation.

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