Numerical investigation of flow instability in rotating cavities

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Abstract

In this paper we consider instability of incompressible fluid flow in a rotating cavity. Identification and characterisation of mechanisms related to the laminar-turbulent process in rotating cavities should improve the prediction methods and lead to new more effective control strategies, of considerable importance in practical flow. Both direct numerical methods and linear theory are used to investigate flow in the rotating cavity. Direct numerical simulation is based on an effective pseudo spectral Chebyshev-Fourier method for solving 3D Navier-Stokes equations. Linear theory considers the complete rotor/stator and rotor/rotor flows. The spatio/temporal development of the instability structures that appear in the first stage of laminar-turbulent transition is investigated. The instability structures are interpreted in light of the type I and type II instabilities.

Keywords: Instability; Laminar-turbulent transition; Absolute instability; Direct numerical simulation; Rotating geometry; Spectral collocation method

1. Introduction

Flows in rotating cavities are very important from both theoretical and practical points of view. Typical industrial configurations are cavities between compressors and turbine disks. Flow around a single rotating disk, rotor/stator flow and forced radial flow between rotating disks are the main configurations investigated numerically and experimentally. The laminar-turbulent transition process in the flow around a rotating disk or in the rotating cavity is related to the type I and type II generic linear instabilities. The type I instability is due to the presence of an inflection point in the boundary layer velocity profile. The mechanism for the type II instability is related to the combined effects of Coriolis and viscous forces.

In spite of intensive work and numerous papers devoted to the instabilities associated with single rotating disk flow [1,2,3], and associated with differentially rotating disk flow [4,5,6,7,8,9], no full understanding of the laminar-turbulent transition flow has been achieved and many problems remained unsolved. In 1995, Lingwood [1] discovered numerically, using LSA, that the flow around a rotating disk is absolutely unstable. It was also demonstrated using linear theory that both boundary layers in the rotor/stator cavity are absolutely unstable and the critical Reynolds numbers of the absolutely unstable areas were given [4,7,8]. In the present paper we focus attention on the spatio/temporal development of the instability structures that appear in the rotating cavity. Computations are performed using direct numerical simulation and results are discussed in light of our linear results. Investigations are performed for different cavities, i.e. cylindrical, annular with and without throughflow.

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2. Mathematical model and numerical method

We consider incompressible flow in two geometrical configurations: (a) annular cavity radially confined by a shaft and a shroud, and (b) annular cavity with the forced flow (Figs 1(a) and 1(b)). The radius of the inner end-wall of the cylindrical cavity, $R_0 = 0$. The rotor rotates at uniform angular velocity $\Omega = \Omega\, e_z$, e_z being the unit vector. The flow is controlled by three physical parameters, which are the rotational Reynolds number based on the outer radius, $Re = R_1^2\Omega/\nu$, the aspect ratio $L = (R_1 - R_0)/2h$, and the curvature parameter $Rm = (R_1 + R_0)/(R_1 - R_0)$. The governing equations are the incompressible 3D Navier-Stokes equations.

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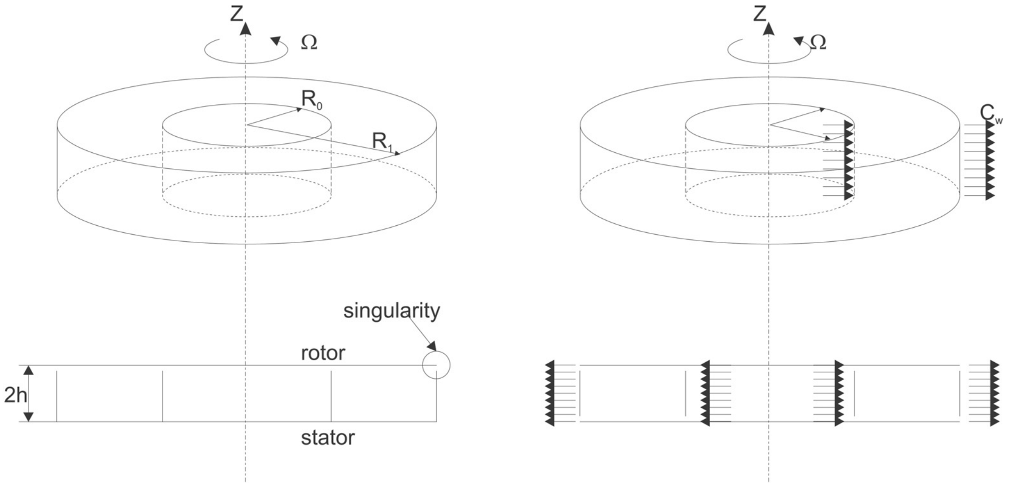

Fig. 1. Schematic picture of the annular rotating cavity without (a) and with (b) throughflow.

The numerical solution is based on a pseudospectral collocation Chebyshev-Fourier Galerkin approximation [5,6]. The time scheme is semi-implicit and second-order accurate. It corresponds to a combination of the second-order backward differentiation formula for the viscous diffusion term and the Adams-Bashforth scheme for the non-linear terms. The method uses a projection scheme to maintain the incompressibility constraint. The boundary conditions are as follows: no-slip boundary conditions at any rigid walls u = w = v = 0. For the azimuthal velocity component the boundary conditions on the rotating walls are given by: (Rm + r)/(Rm + 1).

We proceed as follows: The rotation of the rotor is increased step-by-step with small increments $\Delta Re = 1000$. The solution obtained for smaller Re is then used as an initial condition for higher Reynolds numbers.

3. Selected results and discussion

The investigation has been carried out for the two types of cavities shown in Figs 1(a) and (b). For the annular rotor/stator cavity radially confined by shaft and shroud, calculations have been performed for the aspect ratio L = 5 and the curvature parameters Rm = 5, 3 and 1.5 and for different end-wall boundary conditions. For the annular cavity with throughflow, calculations have been performed for L = 3.37 and Rm = 5.

Let us consider first the flow in the rotor/stator cavity. The base flow consists of two disjoint boundary layers above each disk, with fluid pumped radially outward along the rotating disk and radially inward over the stationary disk. Our linear results obtained for the cylindrical cavity have shown that the stationary disk boundary layer is far more unstable than the rotating one, with the following critical local Reynolds numbers, $Re_\delta = \sqrt{r^{*2}\Omega/\nu}$: stationary disk boundary $Re_{\delta crtypeI} = 47.5$ and $Re_{\delta tcrtypeII} = 34.7$; rotating disk: $Re_{\delta crtypeI} = 278.6$ and $Re_{\delta crtypeII} = 90.23$. Our investigation has shown that almost the whole convectively unstable area in the stationary disk boundary layer is absolutely unstable at $Re_{\delta cra} = 48.5$. For the rotating disk boundary layer we received $Re_{\delta cra} = 562$.

In the annular cavity as in the cylindrical one the stationary disk boundary layer turned out to be far more unstable than the rotating one. However, we have found that the end-wall boundary conditions have a large influence on the instability characteristics of the flow and that the most unstable is the flow with shaft attached to the rotor and shroud attached to the stator. Results obtained for this configuration are analysed below. In the stationary disk boundary layer, for the lower rotational Reynolds number Re, we observed both cylindrical waves, interpreted as type II instability, and strongly 3D spiral vortices, which are interpreted as type I instability. Example solutions are presented in Figs 2(a) and (b), where the iso-lines of the azimuthal velocity component obtained for Rm = 3, L = 5 and Re = 26,500 in the azimuthal section of the stationary disk boundary layer ($z = -0.95$) and in the meridional section are shown. In the stationary disk boundary layer (Fig. 2(a)) we observe two pairs of rings, which dominate at the lower local Reynolds number $Re_\delta = \sqrt{r^{*2}\Omega/\nu}$, and nineteen spiral vortices, which dominate at higher Re_δ. For higher rotational Reynolds numbers only spiral vortices are observed in our graphics. In the rotating disk boundary layer (Fig. 2(b), Re = 26,500, Rm = 5, L = 3) we observe one 2D

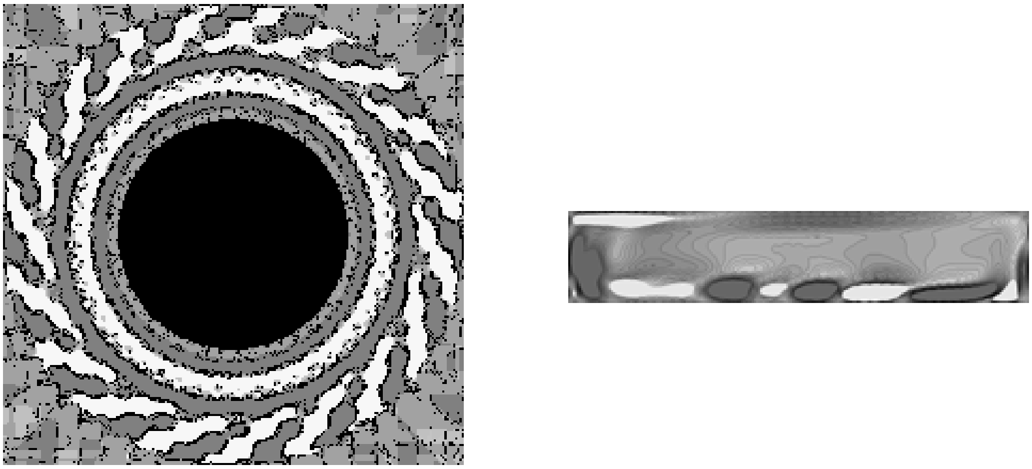

Fig. 2. The iso-lines of the azimuthal velocity component (Rm = 3, L = 5, Re = 26,500) obtained (a) in the azimuthal section (z = −0.95) and (b) in the (r^*/h,z) plane ($\varphi = 0$).

disturbance of the large wave number $\lambda^*/h \sim 8$. The type II disturbances coming from the stationary disk boundary layer are transported up, by the rotating inner end-wall, to the rotating disk boundary layer and disturb it. These disturbances are then convected by the main flow in the rotating disk boundary layer towards the shroud. The spatio/temporal analysis of disturbances in the stationary disk boundary layer has shown that the type I and type II disturbances propagate in the opposite direction. The type II disturbances are convected downstream towards the shaft, whereas the type I vortices propagate upstream towards the shroud. In the animation of the flow the upstream propagation of the type I disturbances are clearly visible, which indicates that flow can be absolutely unstable.

We have found that transition to unsteadiness is subcritical for smaller Rm (Rm = 1.5) and supercritical for higher Rm (Rm = 3, 5). For the cylindrical cavity (Rm = 1 and L = 5) we observed oscillatory transition [6].

We have investigated the spatio/temporal behaviour of the disturbances of the flow in an annular cavity with throughflow. For all considered mass flow rates, disturbances were convected downstream. Our investigations have also shown the large influence of the function used to approximate the radial forced profile on the instability structure.

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