Fully developed two-phase liquid–liquid flow in a finned duct

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Abstract

Numerical simulation has been made of incompressible two-layer stratified flow containing immiscible fluids in a duct whose cross-sectional area varies periodically in the streamwise direction. The results correspond to oil–water flow of fixed volumetric flow rates. Solutions were obtained for a range of Froud number. Two configurations of the duct were considered: duct with fins attached to the bottom plate and duct with fins attached to the top plate. These configurations represent an idealized model of a certain type of locking and regulating device. The results demonstrate the effect of the fin location on total volumetric flow rate.

Keywords: Fully developed flow; Complex geometry; Multiphase flow; Interface; Periodic boundary conditions

1. Introduction

Flows possessing multiple distinct immiscible fluids are ubiquitous in natural and industrial processes. One of the most important illustrations is production logging. Most oil wells produce a mixture of oil, water and gas. Diagnosis and control of production of unwanted fluids is a growing concern in the industry. Simulation of interfacial flow problems, via numerical solution of appropriate partial differential equations, is the principal part of a fundamental understanding of such flows.

In the present research stratified flow possessing immiscible fluids is studied (where the less dense phase, usually oil, flows above the more dense phase, usually water, with a defined interface). The chosen duct configuration represents an idealized model of a certain type of locking and regulating mechanism. The objective is the description of the flow field and moving interface and the understanding of the mechanism that governs the flow. Stratified flows have been the topic of numerous numerical and experimental investigations due to their applications [1,2,3].

2. Formulation of the problem

Consider a two-layer stratified flow in a duct whose cross-sectional area varies periodically in the streamwise direction. The more dense fluid flows along the bottom of the duct (fluid h), and lighter fluid (fluid l) forms the top layer. Fluids are assumed to be incompressible and immiscible. Two configurations of the duct are considered: duct with fins attached to the bottom plate and duct with fins attached to the top plate. The identification of the periodicity characteristics enables the flow field analysis to be confined to a single isolated module. The duct configuration and the solution domain selected are depicted in Fig.1 for case 1.

3. Mathematical formulation

The interface position must be determined as a part of the overall flow solution. When two fluids cannot

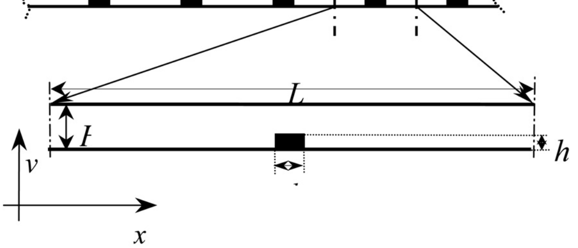

Fig. 1. Flow configuration.

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possess mixed states, it is often useful to represent them in terms of volume fractions. The interface is treated by introducing a function C(X,Y) that is defined to be equal to 1 at any point occupied by fluid l and zero elsewhere. When averaged over the cells of a computing mesh, the average value of C in a cell is equal to the fractional volume of the cell occupied by fluid l. The C function is utilized to determine which cells contain an interface and where one or the other fluid is located in those cells. The algorithm was developed to find the value of C in each cell.

The mathematical formulas for the considered task are based on the Navier-Stokes system of equations:

$$\tilde{\rho}\mathrm{U}\frac{\partial \mathrm{U}}{\partial \mathrm{X}}+\tilde{\rho}\mathrm{V}\frac{\partial \mathrm{U}}{\partial \mathrm{Y}}=-\frac{\partial \tilde{\mathrm{P}}}{\partial \mathrm{X}}+\frac{4}{3}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial \mathrm{X}}\left(\tilde{\mu}\frac{\partial \mathrm{U}}{\partial \mathrm{X}}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial \mathrm{Y}}\left(\tilde{\mu}\frac{\partial \mathrm{U}}{\partial \mathrm{Y}}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial \mathrm{Y}}\left(\tilde{\mu}\frac{\partial \mathrm{V}}{\partial \mathrm{X}}\right)-\frac{2}{3}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial \mathrm{X}}\left(\tilde{\mu}\frac{\partial \mathrm{V}}{\partial \mathrm{Y}}\right)$$

$$\tilde{\rho}\mathrm{U}\frac{\partial \mathrm{V}}{\partial \mathrm{X}}+\tilde{\rho}\mathrm{V}\frac{\partial \mathrm{V}}{\partial \mathrm{Y}}=-\frac{\partial \tilde{\mathrm{P}}}{\partial \mathrm{Y}}+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial \mathrm{X}}\left(\tilde{\mu}\frac{\partial \mathrm{V}}{\partial \mathrm{X}}\right)+\frac{4}{3}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial \mathrm{Y}}\left(\tilde{\mu}\frac{\partial \mathrm{V}}{\partial \mathrm{Y}}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial \mathrm{X}}\left(\tilde{\mu}\frac{\partial \mathrm{U}}{\partial \mathrm{Y}}\right)-\frac{2}{3}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial \mathrm{Y}}\left(\tilde{\mu}\frac{\partial \mathrm{U}}{\partial \mathrm{X}}\right)-\frac{1}{\mathrm{Fr}}\tilde{\rho}$$

$$\frac{\partial \tilde{\rho}\mathrm{U}}{\partial \mathrm{X}}+\frac{\partial \tilde{\rho}\mathrm{V}}{\partial \mathrm{Y}}=0$$

where viscosity and density are the known functions of coordinates:

$$\rho=\rho_{\mathrm{h}}(1-\mathrm{C})+\rho_{\mathrm{l}}\mathrm{C}$$

where ρ_h = density of fluid h, and ρ_l = density of fluid l, and

$$\mu=\frac{\mu_{\mathrm{l}}\mu_{\mathrm{h}}}{\mu_{\mathrm{l}}(1-\mathrm{C})+\mu_{\mathrm{h}}\mathrm{C}}$$

where μ_h = viscosity of fluid h, and μ_l = viscosity of fluid l.

Dimensionless variables are defined by the following formulas:

$$\mathrm{X}=\frac{\mathrm{x}}{\mathrm{h}},\quad \mathrm{Y}=\frac{\mathrm{y}}{\mathrm{h}},\quad \mathrm{U}=\frac{\mathrm{u}}{\mathrm{v}^*},\quad \mathrm{V}=\frac{\mathrm{v}}{\mathrm{v}^*},$$

$$\tilde{\mathrm{P}}=\frac{\mathrm{P}}{\rho^*\mathrm{v}^{*2}}\quad \tilde{\mu}=\frac{\mu}{\mu^*},\quad \tilde{\rho}=\frac{\rho}{\rho^*},$$

$$\rho^*=\frac{\rho_{\mathrm{l}}+\rho_{\mathrm{h}}}{2},\quad \mu^*=\frac{\mu_{\mathrm{l}}+\mu_{\mathrm{h}}}{2},\quad v^*=\sqrt{\frac{\Delta \mathrm{P}}{\rho^*}}$$

The velocity components exhibit periodic behavior:

$$\mathrm{U}(0,\mathrm{Y})=\mathrm{U}(\mathrm{L}^*,\mathrm{Y})$$

$$\mathrm{V}(0,\mathrm{Y})=\mathrm{V}(\mathrm{L}^*,\mathrm{Y})$$

There is, however, another type of periodicity condition for the pressure:

$$\tilde{\mathrm{P}}(\mathrm{L}^*,\mathrm{Y})=\tilde{\mathrm{P}}(0,\mathrm{Y})-1$$

Slip boundary conditions are used at the duct wall:

$$\mathrm{U}(\mathrm{X},0)=\mathrm{U}(\mathrm{X},1)=0$$

$$\mathrm{V}(\mathrm{X},0)=\mathrm{V}(\mathrm{X},1)=0$$

Dimensionless parameters of the task are Reynolds number, $\mathrm{Re} = v^*\rho^*H/\mu^*$; Froude number, $\mathrm{Fr} = v^{*2}/gh$; and volumetric flow rate, $K_l = \int_A^1 UdY/\int_0^1 UdY$, where A is the cross-sectional area occupied by fluid h.

4. Solution method

The governing equations, together with the appropriate boundary conditions, are solved numerically by way of a control volume method and the SIMPLER algorithm. A numerical scheme for solving the generalized fully developed regime is described in [4]. It employs the cyclic tridiagonal matrix algorithm for the solution of the difference equations. The scheme used for present research is analogous but more effective. The algorithm used to define the interface is similar to the well-known VOF algorithm [5]. The solutions were obtained using a grid having 500×50 nodal points, respectively, in the X- and Y-directions.

Numerical calculations were performed for the following values of dimensionless variables: H = 1, L = 10, l = 0.6, h = 0.3, Re = 110, $\mathrm{Fr} = 0.012 \div 10^{20}$, $K_l = 0.6$. Quantities of viscosity and density correspond to oil and water: $\mu_l = 1.22$, $\mu_h = 0.78$, $\rho_l = 0.88$, $\rho_h = 1.12$.

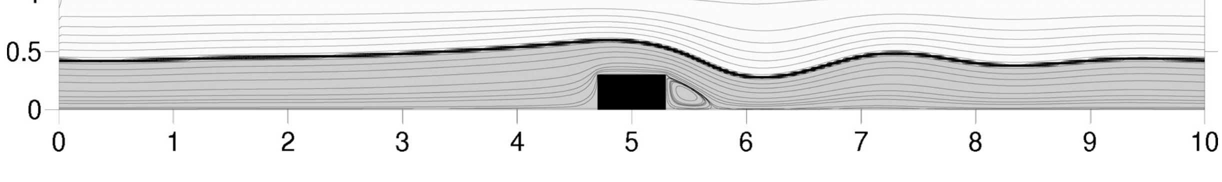

Fig. 2. Flow field and interface location, Fr = 0.06.

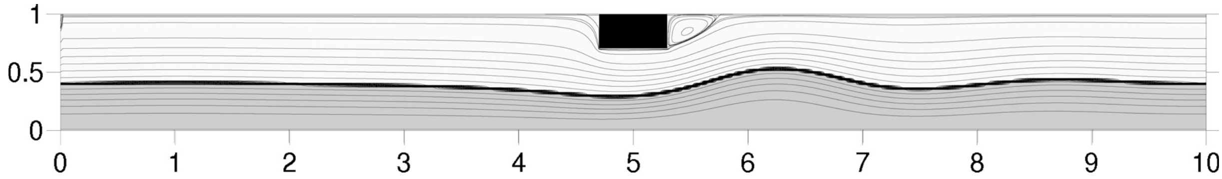

Fig. 3. Flow field and interface location, Fr = 0.06 (case 2).

Table 1
Total volumetric flow rate for the different Froud numbers

Fr		0.012	0.06	0.1	0.12	0.5	1e20
G	up	0.596184	0.674497	0.714933	0.717441	0.714652	0.711988
	down	0.491913	0.617382	0.697568	0.704579	0.705317	0.700876

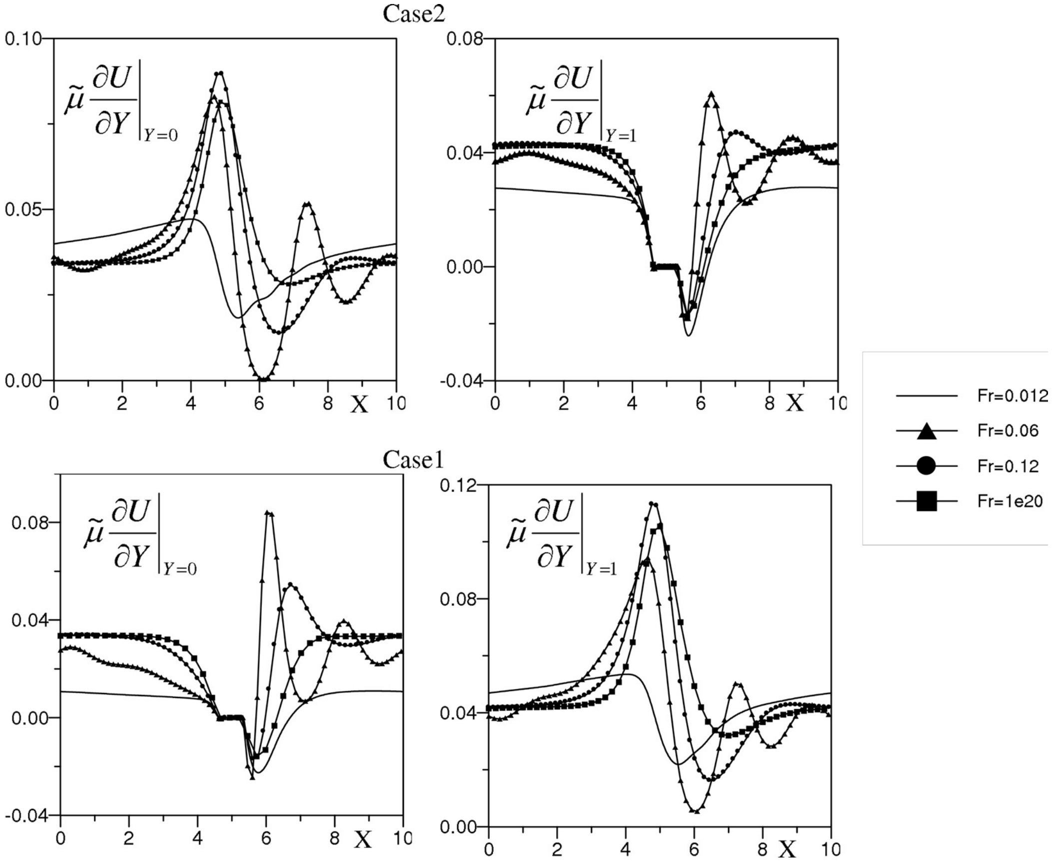

Fig. 4. Dimensionless friction at the walls of the channel.

5. Results and discussion

The presence of the fins causes the appearance of recirculation zones. Portrayals of the flow field via streamline maps and interface configuration are shown in Figs 2 and 3, respectively, for cases 1 and 2.

The total volumetric flow rates for the different Froud numbers are presented in Table 1. As can be seen from the data, the volumetric flow rate for case 2 is always greater than for case 1. One of the explanations of this phenomenon is based on terms of developing region. For the chosen parameters of the fluids, the Reynolds number for fluid l is later than for fluid h. Thus, the developing region, in which the velocity distribution adjusts to the duct geometry and the wall friction, is later for case 2 than for case 1. As is well known, the maximal pressure losses occur in this region. Hence, the volumetric flow rate for case 2 is always greater than for case 1 due to the developing region for case 2 being shorter than for case 1. As can also be seen from Table 1, the dependence of the volumetric flow rate on the Froud number has a non-monotonous behavior.

Dimensionless friction at the walls is presented in Fig. 4 for both cases and for different Froud numbers. One of the practical important points of the result obtained is the following. Power is proportional to volumetric flow rate and pressure drop. Since calculations were performed for a constant pressure drop, less power is needed to circulate the same volume of fluids when fins are situated at the top wall.

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