Receptivity of a low Reynolds number Bickley jet to harmonic vortical excitation

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Abstract

The receptivity of a Bickley jet to a time-harmonic symmetric (S-class) and anti-symmetric (AS-class) vortical excitation is reported. Unlike wall-bounded flows, the eigen-spectrum of jets reveals the presence of multiple dominant modes. The S-class displays the presence of upstream propagating disturbances. It is reasoned that, due to the limited streamwise extent of the domain, experiments and computations on round jets do not always correlate with the linear stability properties. For DNS, a new compact scheme (OUCS4), introduced in [1], along with RK_4 time stepping is used. A new filtering procedure is advocated in the radial direction, which removes the numerical instability at the core (due to a mathematical singularity) and allows us to study the receptivity of round jets to different classes of excitations.

Keywords: Bickley jet; CAA; DNS; DRP; Eigen-spectrum; Filtering techniques; Viscous instability

1. Introduction

Our present interest is to study the viscous instability of jets by using the eigenvalues (along with their directionality of propagation), as cataloged in [2]. In addition to usual classification (into S-class or varicose mode and the AS-class or the helical mode), such instabilities have also been labeled as shear layer and preferred modes [3]. Danaila et al. [4] have reported from their DNS results that the disturbances switch from helical to varicose mode, when Re increases from 200 to 500. It is experimentally noted for plane jets [5], that the flow is strictly laminar when Re is less than 10. When Re exceeds 50, irregular turbulent fluctuations develop. In actual flow, simultaneous presence of multiple modes decides the jet flow evolution. Hence, a 3-D DNS for a round jet is undertaken to relate the coherent structures with the eigenfunctions of the Bickley jet shown in Fig. 1 for anti-symmetric excitation [2].

2. Governing equations and auxiliary conditions

Three-dimensional, unsteady, compressible, non-dimensionalized Navier-Stokes (NS) equations in conservation form are solved. These equations in the generalized curvilinear coordinate system are given as follows:

$$\frac{1}{J}\frac{\partial Q}{\partial t}+\frac{\partial}{\partial \xi}\left(\frac{F-F_\nu}{J}\right)+\frac{\partial}{\partial \eta}\left(\frac{G-G_\nu}{J}\right)+\frac{\partial}{\partial \zeta}\left(\frac{H-H_\nu}{J}\right)=0 \quad (1)$$

where ξ, η and ζ are the generalized coordinates in the computational plane and J is the Jacobian of transformation from the physical to the computational plane. The ideal gas equation is used to relate pressure, density and temperature. The molecular viscosity is calculated using the Sutherland's law, with the Sutherland's constant chosen as 110 K and the Prandtl number taken as 0.7. The radiation and outflow boundary conditions on the lateral and the outflow boundary respectively, are the same as given in [6]. At the inflow, all values are equal to the jet centerline values, except the axial velocity, which is taken as the Bickley jet profile parallel to the jet axis and given by:

$$U(z)=\cosh\left(a^*\sqrt{((y-y_s)^2+(x-x_s)^2)}\right)^{-2} \quad (2)$$

with $a = 0.88136$.

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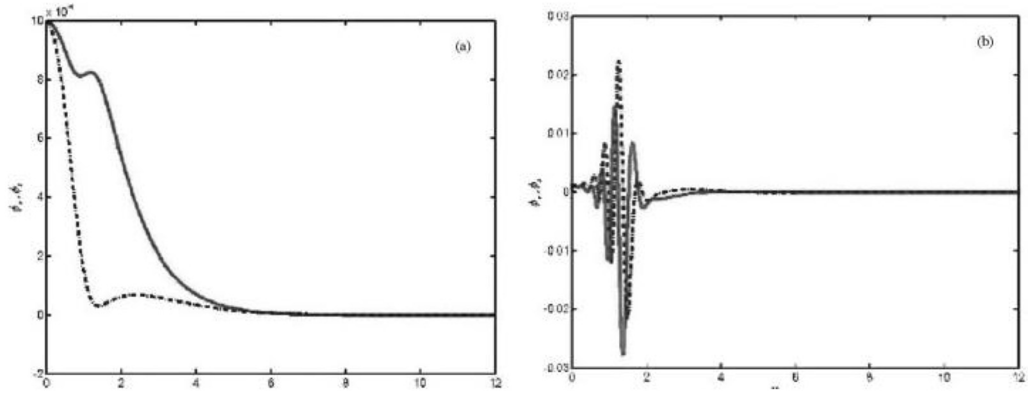

Fig. 1. Eigenvectors for AS- class disturbance field (Re = 500 and $\omega_0 = 0.5$) for (a) an unstable mode and (b) a very stable mode. Solid lines are for real part and chain-dotted lines are for imaginary part.

3. Numerical methods

The governing equations (1) are solved using OUCS4 (for spatial discretization) and RK_4 temporal discretization [1]. The time step Δt is taken as 2.5×10^{-3}. The radial and the stream-wise extent of the computational domain is 10R and 12R respectively, which is chosen using 71, 32 and 52 points in the r-, θ- and z-directions, respectively. The grid in the θ-direction is equi-angular, while in the r- and z-directions it is stretched in an arithmetic progression. To avoid spurious reflections, an 8th-order filter is used in the axial (z-) and the azimuthal (θ-) directions. A spectral filter $F(k_r)$, which is the same transfer function of the 1st derivative of OUCS4 for interior points, is used in the radial direction after every 100 time steps. The filter is shown in Fig. 2. This function windows the Fourier transform of the numerical solution (Q) (where $Q(r, \theta, z) = \int \tilde{Q}(k_r, \theta, z) e^{ik_r r} dr$) that exhibits the presence of high wave number components at the jet core due to the numerical instability introduced by a mathematical singularity at $r = 0$. If the unknowns at the core are obtained by the interpolation formula given in [7], then the solution develops a trough at the core. Such a flow profile suffers an inflexional instability – an inviscid mechanism. This instability triggers a spurious transition. Instead of that, if one filters the solution by the above-mentioned filter in the spectral plane and the filtered solution is then inverse-transformed to obtain the physical variable, then no such spurious inflexional instability occurs.

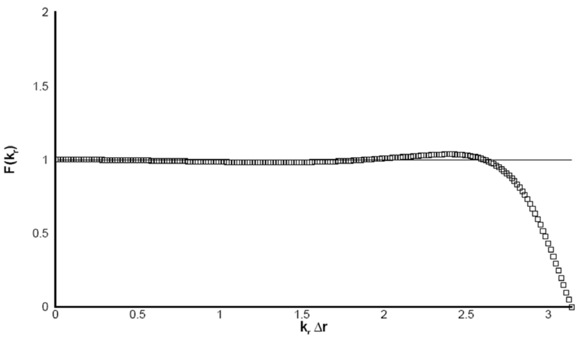

Fig. 2. The spectral filter used in the radial direction for all the physical variables.

4. Results and discussion

4.1. Eigen-spectrum of a Bickley jet

Sengupta et al. [2] report the behaviour of the eigen-spectrum for two classes of disturbance. For the anti-symmetric disturbances (AS-class), the displayed mode in Fig. 1(a) is violently unstable with spatial growth rate given by $\alpha_i = -0.1744721$ and whose energy propagates with the group velocity, $V_g = 0.7321$ in the downstream direction. In contrast, the displayed eigenvector in Fig. 1(b) has the wave property given by $\alpha_r = 1.2340$, $\alpha_i = 1.411685$ and $V_g = 0.5440$. This mode is oscillatory across the middle of the shear layer, while it damps in the downstream direction.

In contrast, the symmetric disturbances are less unstable, with some modes moving upstream that only exist for low frequencies [2].

4.2. DNS of a round jet

A three-dimensional DNS of a Bickley jet ($M_\infty = 0.5$, Re = 500) is reported. The OUCS4 scheme for spatial derivatives in the r- and z-directions, and the DRP scheme of Tam et al. [8] is used in the θ-direction. Two

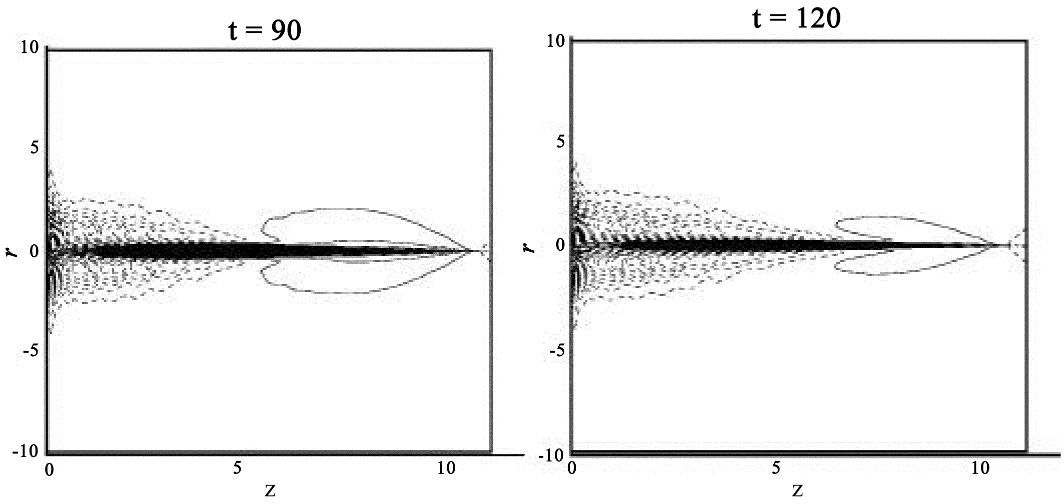

Fig. 3. Disturbance axial velocity component for the symmetric vortical excitation at the indicated times, for Re = 500 and $\omega_0 = 0.2$ in the (r–z) plane.

classes of time-harmonic vortical excitation (with $\omega_0 = 0.2$, implying that the excitation repeats itself after every time interval of 10π) are considered at the jet inflow plane. For the S-class, a vortical disturbance (u_{ds}, v_{ds}) (the form of these disturbances are the same as given in [8], with a half-width equal to the jet-width) is imposed. For the AS-class, an anti-symmetric vortical pulse (given by $u_{das} = u_{ds} \sin(2\theta)$, $v_{das} = v_{ds} \sin(2\theta)$) is used at the inflow plane.

In Fig. 3, the axial velocity distribution in the (r–z)-plane is shown for the symmetric vortical excitation case at the indicated times. For the S-class excitation, the flow is highly stable. For this excitation, the flow is nearly symmetric as well as time-periodic – as seen in

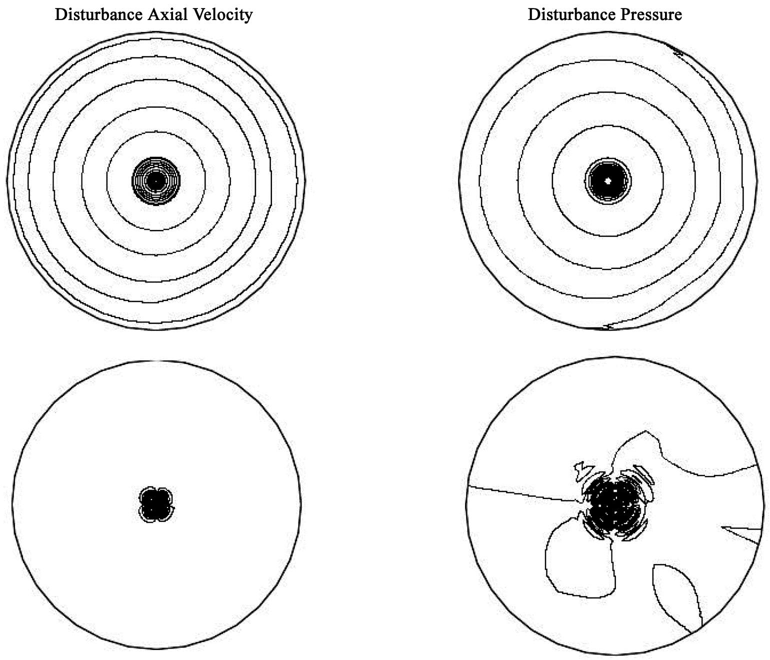

Fig. 4. Disturbance axial velocity and disturbance pressure for S- (top) and AS- (bottom) vortical excitation in the (r–θ) plane at $T = 120$, $z = 4R$, Re = 500 and $\omega_0 = 0.2$.

Fig. 3, while the pressure and the axial velocity plots shown in Fig. 4 for the AS-class excitation show loss of coherence away from the core due to spatial instability.

5. Conclusions

A time-harmonic case of vortical excitation at the inflow of a round jet at Reynolds number 500 and jet centerline Mach number of 0.5 has been computed by solving the full 3-D Navier-Stokes equation, using a new compact scheme for spatial discretization [6] along with RK_4 time-stepping. A new filtering technique is advocated in the radial direction to remove the mathematical singularity that gives rise to spurious inflexional instability at the jet core. The computed flow fields due to symmetric and anti-symmetric vortical excitation at the jet exit plane display flow that is stable for the former and unstable for the latter.

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