Wake mode development for flow past low aspect ratio cylinders

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Abstract

The side-forces imparted on a short cylinder with free hemispherical ends are investigated numerically in comparison to the flow past sphere.

A time-dependent wake flow initiates both axial and transverse modes of oscillation in the wake of the cylinder, whereas for a sphere oscillation is only observed in one plane. The relationship between these modes to the first-occurring Hopf mode in the wake of a sphere is considered.

The bifurcations in the cylinder wake are analyzed with the Landau equation. It is found that the transitions occur through supercritical bifurcations, and the bifurcation process is initiated by a transverse mode even at short length ratios.

Keywords: Sphere wake; Cylinder with hemispherical ends; Planar symmetry; Computational fluid dynamics; Spectral-element method; Hopf transition; Reynolds number

1. Introduction

The flow normal to a cylinder with hemispherical ends has met with attention in recent years [1,2]. Interest in the study of this body predominantly stems from the ability of the body to represent a sphere at the limit of small length ratio, while approaching a straight circular cylinder at large length ratios. The length ratio L_R is defined as a ratio of cylinder length to diameter (see Fig. 1).

The Strouhal–Reynolds number profiles of cylinders with length ratios in the range $1 \leq L_R \leq 5$ have been recorded [1,2]. These studies showed that, over this range, the critical Reynolds number for the onset of unsteady flow increases with a decrease in length ratio, suggesting a link in the first-occurring Hopf bifurcation in the wake between a straight circular cylinder ($Re_c \approx 47$ [3,4]) and a sphere ($Re_c \approx 272$ [5,6]). Consistent with studies of the flow past spheres and circular cylinders, a Reynolds number $Re = Ud/\nu$ is defined for the flow past cylinders with hemispherical ends based on the diameter d, the freestream velocity U and kinematic viscosity ν.

A complication for the bifurcation process leading to unsteady flow past cylinders with hemispherical ends relates to the limit of small length ratio: for the wake of a sphere ($L_R = 1$), the first-occurring Hopf bifurcation is preceded by a regular bifurcation to non-axisymmetric flow [6,7]. The existence of a similar regime of steady asymmetric flow has not yet been reported for cylinders with short length ratios.

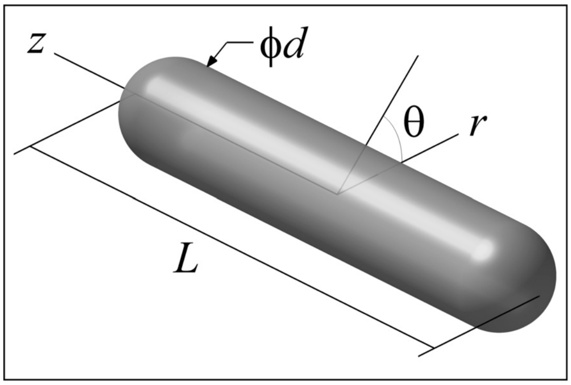

Fig. 1. The coordinate system relative to a cylinder with free hemispherical ends. The direction of flow is across a plane of constant z, and the length ratio is $L_R = L/d$.

In the present study, the flow past a cylinder with $L_R = 2$ is compared to the computed wake behind a sphere. A non-linear bifurcation analysis [3,6,8] is used

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to determine coefficients of the Landau model describing the non-linear evolution of the instability modes in the wake. For details, the aforementioned references should be consulted. Briefly stated, a plot of $\mathrm{d}\log|A|/\mathrm{d}t$ against $|A|^2$ (where $|A|$ is the mode amplitude, here taken from pressure force measurements at the cylinder surface) can be used to determine the linear growth rate of the instability by extrapolating the data to $|A|^2 = 0$, and the slope in the vicinity of $|A|^2 = 0$ determines the potential of the mode to exhibit hysteresis. A hysteretic (i.e. discontinuous or subcritical) bifurcation is predicted by a positive gradient, and a non-hysteretic (i.e. continuous or supercritical) bifurcation is predicted by a negative gradient.

2. Numerical formulation

The flow is computed using a cylindrical-polar formulation of a spectral-element method that has been successful in computing the low-Reynolds-number flows past both spheres [6] and rings [8,9]. Details of the mesh formulation and grid independence studies may be found in [10,11], but a summary is included here.

The numerical formulation is somewhat unique in that a Fourier expansion is employed to discretize the flow about the symmetry axis of the body, but instead of computing the flow in a direction parallel to the symmetry axis as is standard for sphere computations [6,7], the flow is computed normal to the symmetry axis (and the cylinder).

Careful attention was paid to mesh distribution to ensure that adequate resolution was obtained using the 'crossflow' mesh formulation. The flow past a sphere was computed with the present family of meshes, and the pressure and viscous drag components were predicted to within 1% accuracy when compared with accurate computations from previous studies (e.g. [12]).

3. Comparing sphere and short cylinder wakes

The wakes behind both a sphere ($L_R = 1$) and a cylinder ($L_R = 2$) were computed at $Re = 300$, which is well above the critical Reynolds number for transition to unsteady flow in the wake of a sphere, as well as being greater than the approximate critical Reynolds number observed for a cylinder with $L_R = 2$ in experiment ($Re_c \approx 155$).

As expected, the wake behind a sphere developed the familiar unsteady hairpin vortex pattern [12,13]. It is pertinent to note that as testament to the well-resolved flow in the vicinity of the body surface, the orientation of the periodic wake was not aligned with the major axes of the computational grid.

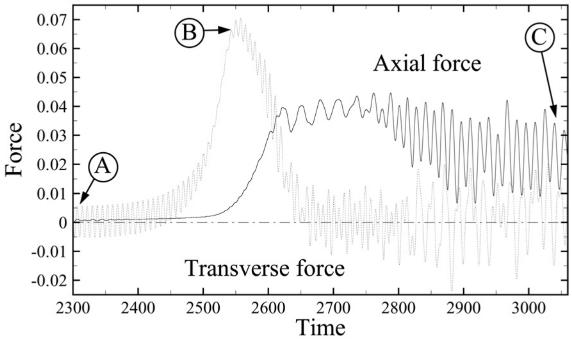

Fig. 2. A plot of the time-history of the evolution of unsteady axial and transverse modes for a cylinder with $L_R = 2$ at $Re = 225$. The approximate times corresponding to the isosurface plots of Fig. 4(a–c) are labeled 'A', 'B' and 'C', respectively.

The development of unsteady flow in the wake of the cylinder was both complicated and interesting. The side-forces acting in both the axial (parallel to the cylinder axis) and transverse (normal to the cylinder and flow) directions were monitored. The plot in Fig. 2 shows the evolution of asymmetric modes in both directions, following the prior development of a periodic zero-mean unsteady oscillation in the transverse direction, which is described in the next section.

4. The first-occurring Hopf bifurcation

In the wake behind a straight circular cylinder, a Hopf bifurcation develops in the transverse direction corresponding to the development of the von Kármán wake at $Re \approx 47$. In experiment [1] the transition to unsteady flow for a cylinder with $L_R = 2$ was observed at $Re \approx 160$, whereas the present computations were performed at $Re = 225$ to hasten the development of the Hopf instability.

The variation in the fluid force on the cylinder was recorded as a measure of the amplitude of the Hopf mode in the wake. The transverse oscillation was initially symmetrical about the cylinder, but almost immediately upon the initial saturation this symmetry was broken, and the mode developed a non-zero mean. To determine the non-linear bifurcation characteristics, the time history of the transverse mode amplitude was used to generate the plot shown in Fig. 3. The plot verifies that the initial transverse bifurcation occurs through a continuous supercritical bifurcation, with a linear growth rate of $\sigma_T \approx 0.036U/d$.

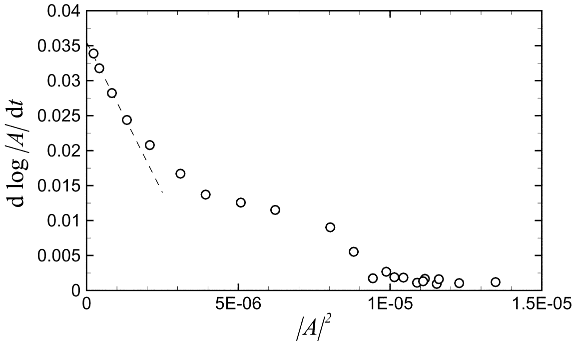

Fig. 3. The variation in growth rate with $|A|^2$ for the transverse Hopf mode for a cylinder with $L_R = 2$ at $Re = 225$. The negative slope in the vicinity of the ordinate axis verifies that the transition is supercritical. A polynomial extrapolation to $|A|^2 = 0$ yields a linear growth rate of $\sigma_T \approx 0.036U/d$.

5. Loss of planar symmetry in the wake

As the plot in Fig. 2 shows, the development of transverse asymmetry in the mean flow develops rapidly, and in addition an axial asymmetry also evolves. This axial mode breaks the planar symmetry of the wake, similar to behavior observed in the wake of a sphere [14,15] above Reynolds numbers $Re \approx 350$. The cylinder Reynolds number is lower here ($Re = 225$), but this discrepancy is explained if the cylinder length is used instead of the diameter to define the body length scale.

The cylinder geometry constrains the orientation of vortex loop formation, allowing the growth of the planar symmetry-breaking mode to be analyzed (as distinct from the wake of a sphere, in which the direction of side-forces is somewhat arbitrary [16]). The asymmetry evolves as a regular bifurcation, but becomes unsteady

(a)

(b)

(c)

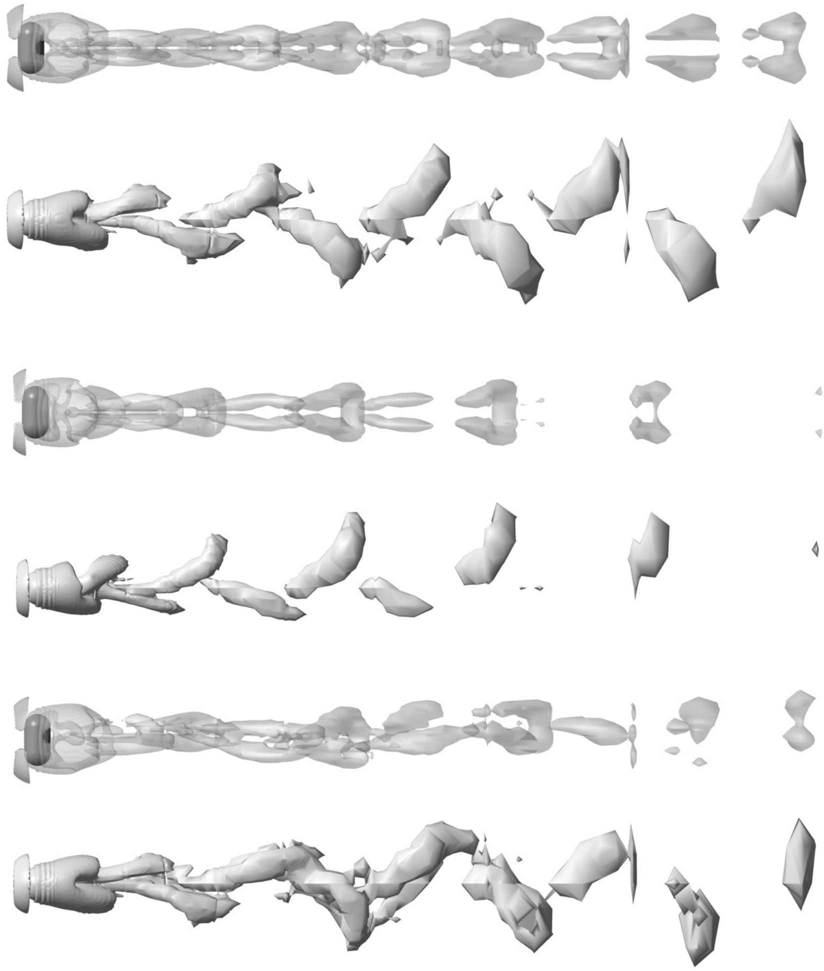

Fig. 4. Isosurface plots showing the alteration in wake structure as the transverse and axial Hopf bifurcations evolve in the flow. The wake is shown (a) prior to the development of a non-zero mean transverse force oscillation, (b) at the point of maximum mean transverse force, and (c) with both axial and transverse Hopf modes saturated.

upon saturation. A similar analysis to that in the preceding section verified that this regular mode also occurs through a supercritical bifurcation. It is interesting to note that once the modes reach an equilibrium, the axial mode has a non-zero mean, while the transverse mode returns to an approximately zero mean.

In Fig. 4, isosurface plots of the wake are shown at each of the major transition points in the bifurcation process.

6. Conclusions

The bifurcation process leading to an unsteady wake with broken planar symmetry has been analyzed for a cylinder with free hemispherical ends and a length ratio of $L_R = 2$ at $Re = 225$. The saturated wake is similar in character to the wake of a sphere within the range $350 < Re < 500$, and the mode initiating the breakdown of planar symmetry was found to develop through a supercritical bifurcation.

As the first-occurring symmetry-breaking bifurcation with $L_R = 2$ is a transverse Hopf bifurcation as for a straight circular cylinder, smaller length ratios could be studied to determine if a steady asymmetric bifurcation is observed prior to a Hopf bifurcation, as is the case in the wake of a sphere.

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