Simulation of polymer flow in a rotating die-slot

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Abstract

The present work deals with the net structures, which are produced by replacing the static die (spinneret) with two concentric dies rotating in opposite directions in a melt-extrusion process. The effect of die rotation on the shape of extrudate is investigated by analysing the polymer flow inside the die slot using the computational fluid dynamics (CFD) package. Mathematical models of velocity profiles and shear stresses are also developed for polymeric flows between rotating dies.

Keywords: Power law; Non-Newtonian; Computational fluid dynamics; Die slot; Net

1. Introduction

Polymers are the most rapidly growing materials in terms of applications and innovations in the processing technology. In general, there are wide ranges of applications of polymers from fibres to films; however, one of the major applications of polymers is in the production of net structures. The simplest way of producing the net structure is by replacing the static die or a spinneret, normally used for filament production, by two counter-rotating dies in a melt-extrusion process. The process of producing the net structures in a single stage by rotating die system was termed the Netlon Process [1]. There are slots on each die, and filaments are produced when these slots are off set from each other and a joint is formed when the slots are adjacent to each other as shown in Fig. 1.

In a melt extrusion process, the shape of the extrudate is mainly dependent upon the corresponding shape of the die slot, pressure drop and flow rate of the polymer inside the die slot [2]. In case of rotating die system, the extrudate shape is also influenced by the rotational speed of the die. Therefore, the objective of the present work is to simulate and analyse the polymer flow at the exit of the rotating die slot in order to predict the shape of the extrudate.

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2. Mathematical model of velocity profiles and shear stresses for polymeric fluid between rotating dies

Initially, the flow between the rotating dies was analysed without considering the slots on them. Therefore, the system was analogous to the rotating cylinders and a tangential flow problem was considered using the well-known power law model [3,4]. This was carried out by considering the tangential component of velocity and neglecting the components of velocity in axial and radial directions. The mathematical model of velocity profiles was carried out for Newtonian fluids [5]: however, this can be modified for non-Newtonian fluids by applying the power-law method as described below.

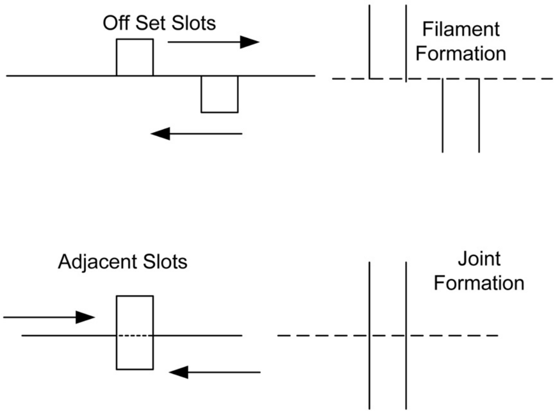

Fig. 1. Filament and joint formation.

Assumptions:

(1) The polymer flow inside the dies is incompressible, i.e. the fluid density is constant between the rotating dies.
(2) The flow is laminar, i.e. the path lines of the fluid particles are smooth.
(3) The temperature of the fluid is constant between the dies.

Definitions:

ω_o – Angular velocity of outer die
ω_i – Angular velocity of inner die
k – Ratio of the diameters of the two dies
kR – Radius of inner die ($k < 1$)
R – Radius of outer die
L – Length of die
υ_θ – Tangential velocity
r – Radial distance from z-axis
γ – Shear rate in the tangential direction
$\tau_{r\theta}$ – Shear stress at the radial position in tangential direction
τ – Stress on the polymer due to the rotating cylinders
m,n – Power law constants

Since the rotating dies were considered, the cylindrical co-ordinate system was used in the method. Consider small element of radius 'δr' in the form of thin cylindrical shell as shown in Fig. 2. The resultant torque acting on the system is zero, as it has no angular acceleration. Therefore,

$$\tau(2\pi rL)r - (\tau + d\tau)2\pi L(r + \delta r)^2 = 0 \tag{1}$$

Neglecting smaller quantities, i.e. δr^2 and $\delta r d\tau$, in the above equation and dividing both sides by $r\tau$:

$$\frac{2\delta r}{r} + \frac{d\tau}{\tau} = 0 \tag{2}$$

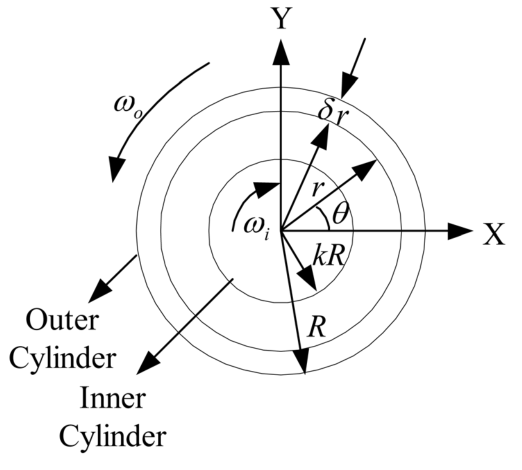

Fig. 2. Rotating cylinders with cylindrical co-ordinate system.

Integrating Eq. (2) from both sides gives

$$\tau = \frac{A_1}{r^2} \tag{3}$$

where A_1 is the constant of integration.

Since stress on the polymer flow is a result of shear stress imparted in the tangential direction, then

$$\tau = \tau_{r\theta} \tag{4}$$

Moreover, the coefficient of viscosity (η) of a polymeric fluid, i.e. non-Newtonian fluid, can be expressed in terms of the following power law [3,4]:

$$\eta = m\gamma^{n-1} \tag{5}$$

Also,

$$\tau = -\eta\gamma = \tau_{r\theta} \tag{6}$$

$$\tau_{r\theta} = -m\gamma^n = -m[r\frac{d}{dr}(\frac{\upsilon_\theta}{r})]^n \tag{7}$$

Substituting the value of $\tau_{r\theta}$ in Eq. (3), the following equation is obtained:

$$-m[r\frac{d}{dr}(\frac{\upsilon_\theta}{r})]^n = \frac{A_1}{r^2} \tag{8}$$

Integrating both sides with respect to r and the following expression is obtained:

$$\frac{\upsilon_\theta}{r} = \frac{(A_1)^{\frac{1}{n}} r^{-(\frac{2}{n})}}{(m)^{\frac{1}{n}}(\frac{2}{n})} + A_4 \tag{9}$$

where A_4 is the constant of integration.

We assume that the inner die is rotating in the clockwise direction (i.e. positive direction) and the outer die is rotating in the anticlockwise direction (i.e. negative direction). Therefore, the boundary conditions to evaluate the tangential velocity analytically are as follows:

$v_\theta = (kR)\omega_i$ at $r = kR v_\theta = -R\omega_o$ at $r = R$

Substituting the above boundary conditions in Eq. (9) gives

$$\frac{\upsilon_\theta}{r} = [\frac{\omega_i + \omega_o}{(R)^{-(\frac{2}{n})}(k^{-(\frac{2}{n})} - 1)}]r^{-(\frac{2}{n})} - \omega_o - [\frac{\omega_i + \omega_o}{(k^{-(\frac{2}{n})} - 1)}]$$

Therefore,

$$\upsilon_\theta = [\frac{\omega_i + \omega_o}{(R)^{-(\frac{2}{n})}(k^{-(\frac{2}{n})} - 1)}]r^{-(\frac{2}{n})+1} - \omega_o r - [\frac{\omega_i + \omega_o}{(k^{-(\frac{2}{n})} - 1)}]r \tag{10}$$

Substituting the value of ν_θ in Eq. (7) to obtain the expressions of shear rates and stresses gives

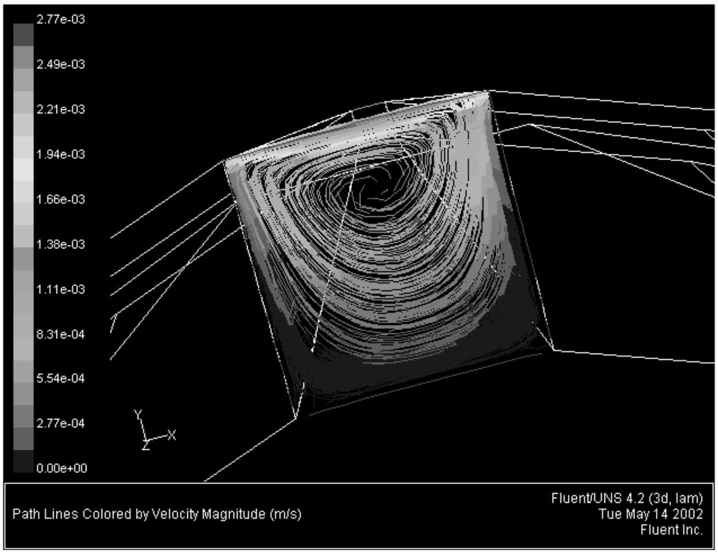

Fig. 3. Velocity profile of polymer flow (HDPE) at 1.5 rpm speed in a square slot.

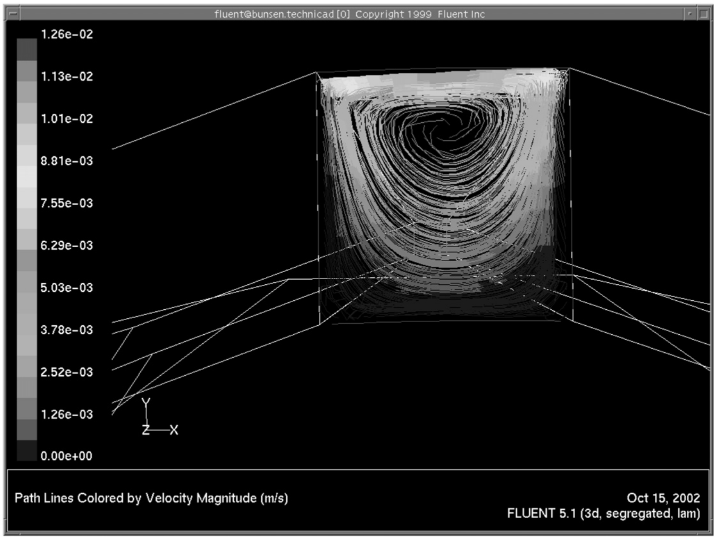

Fig. 4. Velocity profile of polymer flow (HDPE) at 4.5 rpm speed in a square slot.

$$\gamma = [\frac{\omega_i + \omega_o}{(R)^{-(\frac{2}{n})}(k^{-(\frac{2}{n})} - 1)}(\frac{-2}{n})]r^{-(\frac{2}{n})} \tag{11}$$

$$\tau_{r\theta} = -m\gamma^n = -m[\{\frac{\omega_i + \omega_o}{(R)^{-(\frac{2}{n})}(k^{-(\frac{2}{n})} - 1)}(\frac{-2}{n})\}r^{-(\frac{2}{n})}]^n \tag{12}$$

Furthermore, these dies have slots and the polymer flow within these slots can be analysed in three directions, i.e. tangential, axial and radial.

3. Analysis of polymer flow in a rotating die slot

In the Netlon process [1], the polymer fluid flows in the die slot, which is sheared in tangential and radial directions (i.e. by die rotation) and in an axial direction (i.e. by take-up speed and pressure drop). However, the shear in the axial direction can be neglected since the net structures were immediately quenched by water. This shows that the polymer molecules are set in a rigid structure and the take-up speed (axial) is not able to

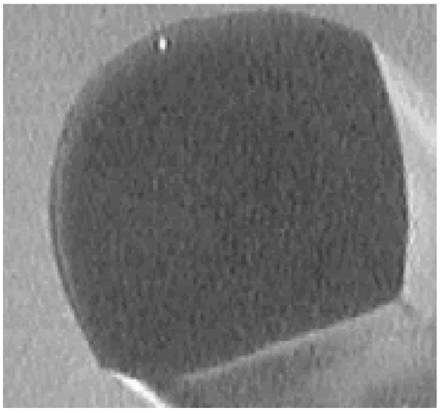

Fig. 5. Extrudate shape at die speed of 4.5 rpm.

induce its shear on the polymer molecules. Hence, shear caused by the take-up speed cannot propagate into the die slot.

Therefore, the effect of die rotation was quantified by analysing the polymer flow in the tangential and radial directions using the computational fluid dynamics (CFD) package. In general, there are a defined number of slots on each die, but it is assumed that the flow rate in all the slots would be same and hence polymer flow can be analysed for a single slot. The polymer flow at the exit of a square die slot (1.67 mm depth and 1.67 mm width) was simulated using Fluent 5.1, a commercial computation fluid dynamics package [6]. The effect of die rotation on the polymer flow emerging from the die slot was analysed at two different speeds, 1.5 rpm and 4.5 rpm, such that only one of the walls of the die slot was rotating and the other three were stationary. The high-density polyethylene (HDPE) and the density and viscosity of 0.96 g/cm^3 and 150 kg/m-s at 190°C, respectively, were considered in our analysis. During the analysis, the following assumptions were made:

- The cells used for generating the meshes for the fluid and geometry objects were hexahedral and quadrilateral, respectively.
- The fluid flow is incompressible and laminar.
- The direction of the rotation of the die slot is towards positive X-axis.

As shown in Figs 3 and 4, velocity of the particles is high near the rotating wall and negligible on the other three stationary walls. The apparent viscosity of the polymer particles is reduced near the rotating wall, since the shear rate is high [7] and hence particles tend to achieve minimum energy by forming a circular shape at such positions. This leads to a semi-circular shape of the extrudate after emerging from a square die slot, as shown in Fig. 5. On further increasing the rotational speed of the die, the circularity of the extrudate will increase and can result in a fully circular-shaped extrudate.

4. Conclusions

The mathematical models of the velocity profiles and stresses of the polymer flow between the rotating dies were formulated based upon the power law [3,4]. The computational fluid dynamics (CFD) package was used for the analysis of polymer flow within a square die slot, which has revealed that there is a high shear rate near the rotating walls, eventually changing the extrudate shape. This analysis can be used for optimising the design of the die slots to produce the desired extrudate shape.

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