Compound wall treatment for RANS computation of complex turbulent flows

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Abstract

We present a generalized treatment of wall boundary conditions for RANS computation of turbulent flows and heat transfer, which combines the integration up to the wall with generalized wall functions that include non-equilibrium effects. Wall boundary conditions can thus be defined irrespective of whether the grid point nearest to the wall lies within the viscous sublayer, in the buffer zone, or in the fully turbulent region. The computations with fine and coarse meshes of flows in a plane channel, behind a backward-facing step and in a round impinging jet using the proposed *compound wall treatment* are in satisfactory agreement with the available experiments and DNS data.

Keywords: RANS turbulence models; Wall functions; Integration to the wall; Compound wall treatment

1. Introduction

Integration of governing equations up to the wall (ItW) with exact wall boundary conditions (BC) has never appealed to industrial CFD. Such an approach requires models that account for wall-vicinity and viscous effects (low-Re-number models), and a very dense computational grid with the first near-wall cell-center located at $y^+ \mathcal{O}(1)$. In addition to excessive computational costs, in complex flows and especially with automatic gridding it is difficult to fulfill this prerequisite in all flow regions. The more popular wall function approach (WF) tolerates much coarser grids, but here the first cell-center ought to lie outside the viscosity affected region, roughly at $y^+ \geq 30$, which is also difficult to ensure in all regions of complex flows. Besides, the conventional wall functions are often inadequate for complex problems of industrial relevance, because they have been derived for simple wall-attached near-equilibrium flows.

The continuous increase in computing power has resulted – among others – in a trend towards using denser computational grids for computing industrial flows. However, because of prohibitive costs, in most cases such grids are still too coarse to satisfy the prerequisites for the ItW. Instead, often the first grid point appears in the buffer layer ($5 \leq y^+ < 30$), making neither ItW nor WF applicable. Esch et al. [1] proposed for such grids a quadratic blending of a near-wall k–ω model with the wall functions.

We propose here a *compound wall treatment* (CWT), which reduces either to ItW when the first near-wall cell is in the viscous sublayer, or to WF when it lies in the fully turbulent region. When the first grid point is in the buffer region, BCs are provided from blending the viscous and fully turbulent limits using exponential blending functions. This blending is based on a generalization of the approach of Kader [2]. It makes the model insensitive to the precise positioning of the first grid point and – within reasonable limits – to the quality of the mesh in the near-wall region. The CWT can be applied in conjunction with any turbulence model with the integration to the wall, but here we will use the robust elliptic relaxation ζ-f model proposed by Hanjalic and Popovac [3].

2. The ζ-f model

The ζ-f model [3] is a variant of Durbin's $\overline{v}^2$-f model [4] in which the eddy viscosity is defined as $\nu_t = C_\mu \zeta k^2/\varepsilon$, with $C_\mu = 0.22$. A transport equation is solved for the time-scale ratio $\zeta = \overline{v}^2/k$ instead for $\overline{v}^2$:

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$$\frac{D\zeta}{Dt}=f-\frac{\zeta}{k}\mathcal{P}+\frac{\partial}{\partial x_k}\left[\left(\nu+\frac{\nu_t}{\sigma_\zeta}\right)\frac{\partial\zeta}{\partial x_k}\right] \tag{1}$$

in conjunction with an equation for the elliptic relaxation function (using the quasilinear pressure-strain model of Speziale et al. [5] (SSG)):

$$L^2\nabla^2 f-f=\frac{1}{\tau}\left(C_1-1+C_2'\frac{\mathcal{P}}{\varepsilon}\right)\left(\zeta-\frac{2}{3}\right) \tag{2}$$

and with equations for the kinetic energy k and its dissipation rate ε.

The ζ-f model is aimed at integration up to the wall and thus contains the necessary near-wall modifications: the non-viscous wall blocking *via* the elliptic-relaxation equation (2), and viscous effects through the viscous diffusion and Kolmogorof time and length scales as lower scale bounds [4]. Because of more convenient BCs for f, the ζ-f model proved to be more robust and less sensitive to near-wall gridding than its parent $\overline{v^2}$-f model, tolerating the first grid point at $y^+ < \approx 3$. It is noted that in the limit of isotropic turbulence $\zeta \rightarrow 2/3$ and the ζ-f model reduces to the conventional k-ε model. More details on the model and its performances are provided in [3].

3. Wall functions for fully turbulent flows

When the first grid point is in the fully turbulent region, it is inevitable to use wall functions to provide the wall shear stress and other variables in the first near-wall cell, and the CWT should automatically ensure this. However, the standard WF are known to fail in non-equilibrium flows. We followed the arguments of Craft et al. [6] and derived more general wall functions for the velocity and temperature, based on a single assumption that the non-dimensional eddy viscosity varies linearly with the distance from the wall (though, unlike Craft et al., without including explicitly the viscous sub-layer thickness). This makes it possible to integrate the simplified momentum equation, which after some transformations yields the following expression for the wall shear stress in the fully turbulent region:

$$\tau_w^t=\frac{\rho\kappa c_\mu^{1/4}k_p^{1/2}U_p}{\ln\left(Ey_p^+\right)}\psi_p \tag{3}$$

$$\text{where}\qquad \psi_p=1+\frac{C_{tan}y_p}{\rho\kappa c_\mu^{1/4}k_p^{1/2}U_p}\qquad y_p^+=\frac{c_\mu^{1/4}k_p^{1/2}y_p}{\nu} \tag{4}$$

$c_\mu = 0.07$, 'p' denotes the wall nearest cell-center, and ψ_p includes *via* C_{tan} the tangential pressure gradient and convection, thus accounting for local non-equilibrium effects.

4. Compound wall treatment (CWT)

The quantities for which the boundary conditions ought to be specified in the CWT for the ζ-f model are: wall shear stress τ_w, wall heat flux q_w, production $\mathcal{P}$ and dissipation ε of the turbulence kinetic energy k, and the elliptic function f. The blending of the wall-limiting values and their fully turbulent counterparts can be accomplished by taking a quadratic mean as proposed by Esch et al.[1]:

$$\phi_p=\sqrt{\phi_v^2+\phi_t^2} \tag{5}$$

where ϕ denotes the variable considered, 'v' the viscous and 't' the fully turbulent value of that variable. This expression has no physical justification, and produces inaccurate values for ϕ in the buffer region. One can generalize Eq. (5) by using $\phi_\mathrm{p}=(\phi_\mathrm{v}^\mathrm{n}+\phi_\mathrm{t}^\mathrm{n})^{1/\mathrm{n}}$ to obtain better approximation in the buffer layer, but our experience with $n = 4$ produced only marginal improvement. One can use even a simpler expression:

$$\phi_p=max\left(\phi_v,\phi_t\right) \tag{6}$$

In all cases, good approximation was obtained when the first grid point is very close to the wall, where effectively the ItW is in play. When the first point is in the fully turbulent region, typical WF quality of results are obtained. However, all the above models fail when the first grid point is in the buffer region. This is illustrated in Fig. 1(a) for the wall shear stress in a plane channel flow at $Re_\tau = 800$, using the DNS data of [7]. In the region $5 < y^+ < 80$ both expressions show a large departure from the constant $\tau_\mathrm{w}^+ = 1$, which should be insured irrespective of the location of the first grid point. We considered the blending of the wall-limiting and fully turbulent properties in the manner of Kader [2], who proposed unique temperature profile throughout the whole wall boundary layer:

$$\Theta^+=Pr\,y^+e^{-\Gamma}+\left[\alpha ln\left(y^+\right)+\beta\left(Pr\right)\right]e^{-1/\Gamma} \tag{7}$$

where $\alpha = 2.12$ and $\beta(Pr) = (3.85Pr^{1/3} - 1.3)^2 + 2.12ln(Pr)$, and the coefficient Γ is a function of the normalized distance to the wall y^+:

$$\Gamma=\frac{0.01\left(Pr\,y^+\right)^4}{1+5Pr^3y^+} \tag{8}$$

Although Kader [2] offered no physical justification for Eq. (7), one can argue that $e^{-\Gamma}$ represents a solution of a simplified 1-D elliptic blending equation. For the mean velocity we can adopt the same expressions (Eqs. (7) and (8)) by putting $Pr = 1$. We now expand the same blending principle to other properties for which continuous boundary conditions are required:

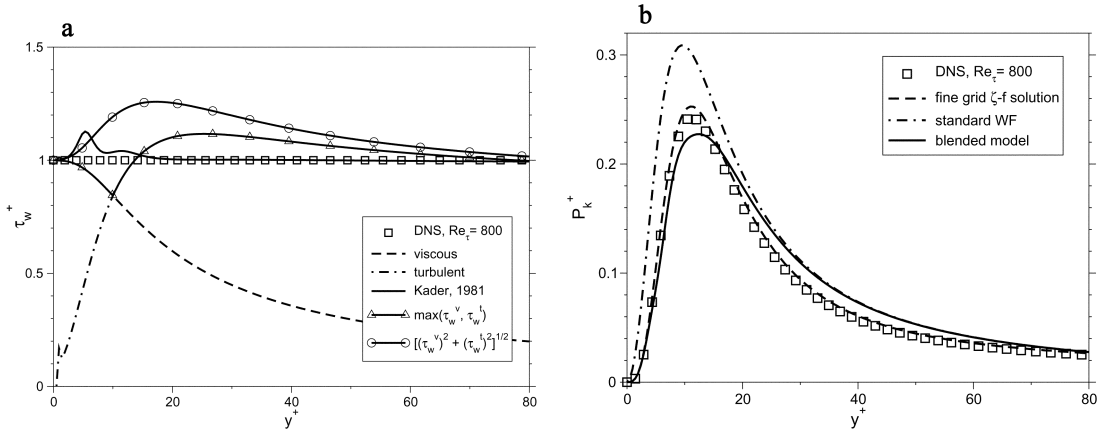

Fig. 1. Blended expressions for τ_w (a) and the adopted blended model of $\mathcal{P}$, Eq. (10) (b). Symbols: DNS of Tanahashi et al. [7].

$$\phi_p = \phi_v e^{-\Gamma} + \phi_t e^{-1/\Gamma} \tag{9}$$

Figure 1(a) shows τ_w^+ for a plane channel obtained from Eq. (9) with $\tau_v = \nu(\partial U/\partial y)$ and τ_t from Eq. (3) evaluated in the same way as from the standard WF for various distances from the wall y and the corresponding velocity U. Apart from a small deviation for $4 < y^+ < 7$ (due to a minor deficiency in Eq. (9)), the resulting τ_w^+ is in very good agreement with the DNS data and superior to Eqs. (5) and (6).

4.1. Kinetic energy production

The common practice in standard WF is to impose the value of $\mathcal{P}$ from the local equilibrium conditions (logarithmic velocity profile and constant shear stress), i.e. $\mathcal{P} = u_\tau^3/(\kappa y)$. This is correct only in the fully turbulent region in equilibrium flows, as shown Fig. 1(b) for the plane channel flow (DNS $Re_\tau = 800$, [7]). We can derive an expression for $\mathcal{P}$ by feeding in the analytical $(\partial U/\partial y)$ derived from Eq. (7) (for $Pr = 1$) in combination with the near-wall and fully turbulent expressions for the turbulent stress. Alternatively, we can simply apply Eq. (9) to blend the ItW and WF values of $\mathcal{P}$:

$$\mathcal{P}_p = -\overline{uv}\frac{\partial U}{\partial y} = \quad C_\mu \zeta_p \frac{k_p^2}{\varepsilon_p}\left(\frac{\partial U}{\partial y}\right)_p^2 e^{-\Gamma} + \frac{c_\mu^{3/4} k_p^{3/2}}{\psi_p \kappa y_p} e^{-1/\Gamma} \tag{10}$$

Note that $C_\mu = 0.22$ and $C_\mu = 0.07$. Figure 1(b) shows $\mathcal{P}$ from Eq. (10), compared with the ItW and WF approaches.

4.2. Energy dissipation rate

Likewise, we can derive the blended expression for the dissipation rate:

$$\varepsilon_p = \frac{2\nu k_p}{y_p^2} e^{-\Gamma'} + \frac{c_\mu^{3/4} k_p^{3/2}}{\kappa y_p} e^{-1/\Gamma'} \tag{11}$$

Because of a specific and strong variation of ε in the near-wall region, we modified the exponent of the damping function into $\Gamma' = 0.001 y^{+4}/(1 + y^+)$. It is noted, however, that for the turbulent region ε_t is tied to $\mathcal{P}_t$ and that even the simpler model (6) performs reasonably well.

4.3. Elliptic relaxation function

For this function, none of the blending formulations (5), (6) or (9) is adequate for the CWT, for two reasons. First, the wall-limit ('viscous') value of $f\,(f_v = -2\nu\zeta_p/y_p^2)$ and its homogeneous value $(f_h = (C_1 - 1 + C_2'\mathcal{P}/\varepsilon)(\zeta - 2/3)/\tau)$ have opposite signs. Second, while f_v ranges from its wall (negative) value to zero, f_h ranges from (positive) infinity to its homogeneous value (tends to zero). Thus there is a huge difference between them in the buffer region, and no blending (except the full elliptic solution) can give a realistic solution there.

Therefore, the simplest CWT for f is to manage its boundary condition in the same manner as for the standard ζ-f model with integration to the wall, i.e. to impose the wall value f_w obtained from the budget of Eq. (1):

$$f_w = \frac{-2\nu\zeta_p}{y_p^2} \tag{12}$$

This definition is correct if the near-wall cell is in the viscous sublayer, and it gives zero value for f_w (it drops fast to zero, because of the power two in the denominator) away from the wall, which is incorrect. But since Eq. (12) sets only BC at the wall, the f equation (2) will

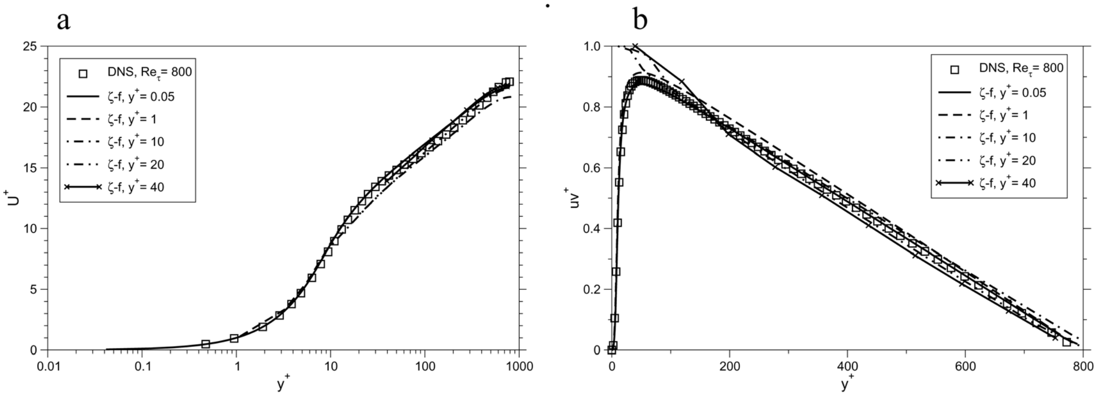

Fig. 2. Channel flow computations with different grids with the first near-wall y^+ from 0.05 to 40, $Re_\tau = 800$ [7], Velocity profiles U^+ (a); Turbulent shear stress $\overline{uv}^+$ (b) Symbols: DNS data of Tanahashi et al. [7].

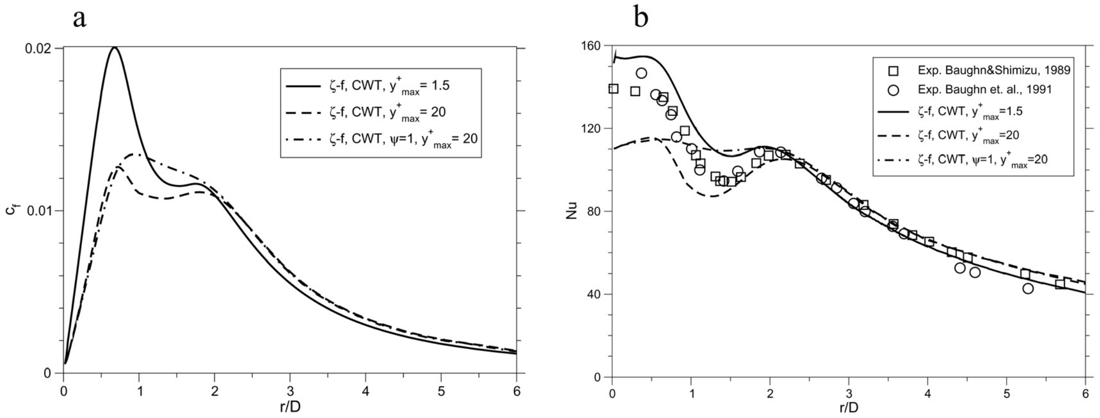

Fig. 3. Friction factor C_f (a) and Nusselt number Nu (b) in a round impinging jet, $Re = 23{,}000$, computed on a fine and coarse grid with maximum wall-nearest $y^+ = 1{,}5$ (ItW) and 20 (CWT), respectively. Symbols: experiments of Baughn et al. [8,9].

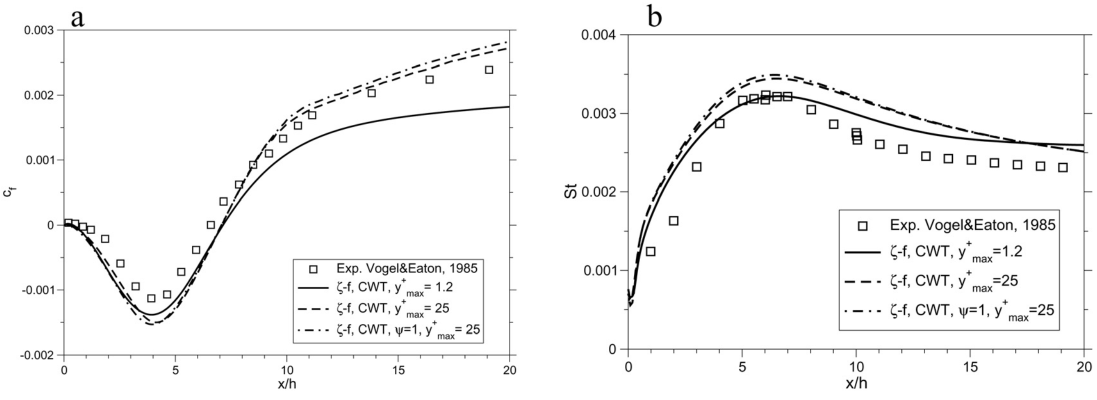

Fig. 4. Friction factor C_f (a) and Nusselt number Nu (b) behind a backward-facing step, $Re = 28{,}000$, computed on a fine and coarse grid with maximum wall-nearest $y^+ = 1{,}2$ (ItW) and 25 (CWT), respectively. Symbols: experiments of Vogel et al. [10].

produce some approximate solution for the near-wall cell center, due to in-domain flow conditions. This is acceptable because the wall blocking effect (which f should describe) fades away in the far-field.

5. Some illustrations

As an illustration of the CWT performance, we present some results of the computations of several generic flows including heat transfer with computational grids that place the wall-nearest grid in the buffer region, compared fine-grid ItW solutions. Figures 2 to 4 show respectively some results for a plane channel flow, a round impinging jet and a separating flow behind a backward facing step. In all cases the CWT approach agreed reasonably well with the ItW approach

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