Mixing in a convecting viscous fluid: applications to the Earth's mantle

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Abstract

We investigate the effect of viscosity and thermal conductivity variations on mixing by thermal convection with the goal of providing constraints on the timescales of stirring in the Earth's mantle, which undergoes convection on geologic timescales. Geophysical observations and high-pressure experiments indicate substantial increases in viscosity and thermal conductivity in the lower mantle, which could slow the rate of heat and mass transfer due to mantle convection and help maintain long-lived geochemical reservoirs in the lower mantle. In the extreme case, heat transport by convection might be negligibly small. However, the increase in viscosity will be somewhat offset by a reduction in viscosity due to increased temperature; moreover, there are large uncertainties in the estimates of these properties. We explore the possible dynamics effects of a substantial increase in viscosity and thermal conductivity, using finite-element models of mantle convection with depth-dependent and temperature-dependent material properties. Tracers are introduced to determine whether isolated reservoirs can be maintained in the lower mantle in this scenario. We observe a wide range of phenomena ranging from rapid mixing to formation of isolated regions. The preservation or destruction of isolated blobs is controlled by the oscillations of downwellings and upwellings.

Keywords: Mixing; Mantle convection; Computational fluid dynamics

1. Introduction

Under the elevated temperatures and pressures of the Earth's deep interior, rock responds to stress by slow, creeping flow. On geological timescales, this flow drives plate tectonics, earthquakes, and mountain building. The process is poorly characterized because of the difficulty of making observations. One constraint from the composition of mantle-derived basalt requires that reservoirs of material persist for billions of years. Mantle heterogeneity is observed on all possible scales from the global scale (thousands of kilometers) to the sub-meter scale. Heterogeneities are introduced at subduction zones, and are destroyed by mixing. A successful model of mantle convection must preserve heterogeneities for long enough to satisfy the geochemical constraints and must account for the range of scales of heterogeneities.

Several models have been proposed to satisfy these constraints (Schubert et al. [1] provide an overview). The mantle may be composed of layers of sufficiently different density that they do not rapidly mix (e.g. [2]). The mantle may contain blobs of material of slightly different composition from the matrix (e.g. [3]), that may be preserved [4]. The mantle may resemble a 'marble cake' that develops a spectrum of heterogeneities as crustal material is produced at the surface, recycled into the mantle at subduction zones, and stretched, thinned, and folded by convection [5].

2. Method

We consider convection in an incompressible, 2-D box, designed as a simplified representation of convection in the Earth's mantle. Conservation of mass for an incompressible fluid requires that

$$\nabla \cdot u = 0 \tag{1}$$

where u is the velocity. The flow is driven by

$$\nabla^2 u - \nabla P + \mathrm{Ra} T \hat{k} = 0 \tag{2}$$

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where P is pressure and T is temperature. The viscosity μ is a function of temperature and pressure

$$\mu = \mu_u \exp\left[\frac{E^* + V^* z}{T - T_*} - \frac{E^* + V^* z_*}{T_1 - T_*}\right] \text{ for } z > z_{1/2} \tag{3}$$

$$\mu = \mu_l \exp\left[\frac{E^* + V^* z}{T - T_*} - \frac{E^* + V^* z_*}{T_1 - T_*}\right] \text{ for } z < z_{1/2} \tag{4}$$

where E^* is an activation energy, V^* an activation volume, T_* and z_* are a reference temperature and reference length, respectively, T_1 is the temperature at the base of the box, and μ_u and μ_l are reference viscosities in the upper and lower parts of the mantle, respectively. Ra is the Rayleigh number

$$\mathrm{Ra} = \frac{\rho g \alpha (T_1 - T_0) d^3}{\kappa_u \mu_u} \tag{5}$$

The advection–diffusion equation controls the temperature:

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \frac{\kappa}{\kappa_u} \nabla^2 T \tag{6}$$

The thermal conductivity κ can differ between the upper and lower layers.

In the Earth's mantle, viscosity and thermal conductivity vary in a complex way with depth. However, we are interested here in the fundamental processes involved in mixing and stirring. For simplicity, therefore, we located the transition between regions of contrasting viscosity and thermal conductivity at the middle of the box ($z_{1/2}$). All equations above have been nondimensionalized using the depth of the box as the characteristic length scale, the temperature contrast across the box as the characteristic temperature scale, and the conductive timescale as the characteristics time.

We use the finite element method [6], into which tracer particles have been introduced [7]. The latter studied a similar system to the one presented here, but did not include temperature-dependent viscosity or variable thermal conductivity.

We show three models with different properties in the lower half of the box (Table 1). The bottom and top of the box have fixed hot and cold temperatures, respectively, and are traction-free. We ran each model to thermal equilibrium before introducing particles; this eliminates the influence of initial conditions on the results. Figure 1(a) shows initial locations of particles.

Table 1
Model details

Parameter	Value	
μ_u	1	
Ra	10^7	
E^*	2.0	
V^*	0.5	
T_*	0.25	
z_*	0.1	
	μ_l / μ_u	κ_l / κ_u
Model 1 (baseline)	1	1
Model 2	50	50
Model 3	100	100
Boundary conditions		
T_1	1	
T_0	0	

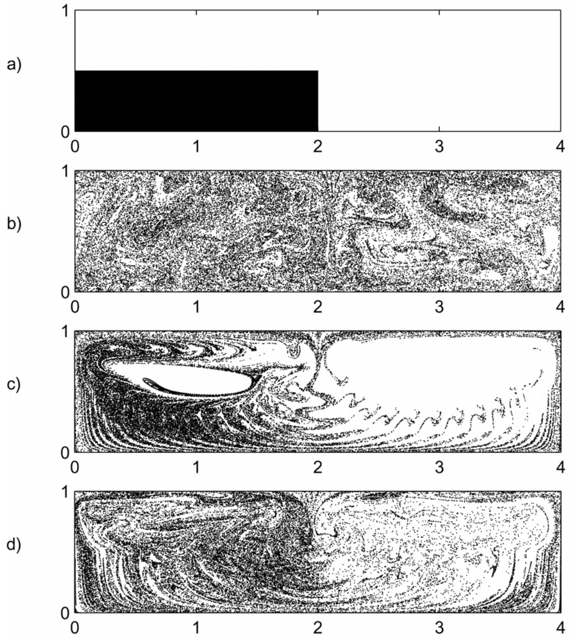

Fig. 1. Mixing in a basally heated mantle, with temperature-dependent viscosity and an increase in viscosity and thermal conductivity at a depth of 0.5. (a) The tracer particles were introduced into the lower right-hand quadrant of the box. (b) The baseline model, with no viscosity and thermal conductivity increase at depth, exhibits relatively rapid homogenization. (c) Viscosity and thermal conductivity increase by a factor of 50 in the lower half. (d) Viscosity and thermal conductivity increases by a factor of 100 in the lower half.

3. Results

In all three models, the flow organizes itself into two large, but variable, cells. A baseline case (Fig. 1(b); Model 1) has no 'jump' in viscosity and thermal conductivity in the lower mantle ($\mu_l = \mu_u$ and $\kappa_l = \kappa_u$). As expected, mixing is rapid. The predominant pattern of convection is a single downwelling at the center and upwelling plumes along the sides. Both the cold and hot instabilities oscillate, allowing exchange of particles from one side to the other. Although stirring is not complete in the snapshot shown, the remaining small blobs were soon destroyed in a subsequent extension of the run.

Introducing a jump in viscosity and thermal conductivity with depth (Fig. 2) stabilizes the flow and changes the mixing phenomenology significantly (Fig. 1(c), $\mu_l = 50\mu_u$ and $\kappa_l = 50\kappa_u$). The center downwelling oscillates less than in the baseline model. This strong downwelling also buckles and folds as it encounters the increased resistance of the lower layer. This buckling is one mechanism for moving tracers from the left to the right of the box. The oscillation of the downwelling also transports particles from cell to cell, much like the 'turnstile lobes' documented by Camassa et al. [8]. However, a pair of isolated regions ('owl eyes') develops within each major convective cell, with little or no mass transport across these boundaries. These cells are analogous to ones seen in studies of the kinematics of mixing in cavity flows and journal bearing flows [9]. One difference is that the model presented here is due to convection driven by heat, rather than by oscillating boundary conditions.

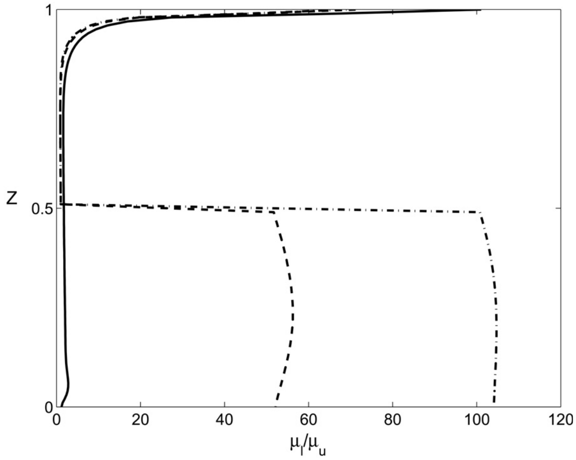

Fig. 2. Laterally averaged viscosity profiles for the three models. The hot and cold thermal boundary layers are in evidence, as is the jump in viscosity at the midpoint.

Increasing the viscosity and thermal conductivity contrast between the upper and lower halves disrupts the owl eye structure and increases the overall rate of stirring (Fig. 1(d)). The high viscosity and thermal conductivity in the lower layer makes flow there more sluggish, reducing heat transport and increasing the temperature in the lower half. As a result, the temperature contrast across the upper layer is more pronounced, and convection in the upper layer becomes more vigorous, rapidly stirring the particles in the upper mantle. However, the internal jump in viscosity and thermal conductivity does not suppress transport across the interface between the two layers. Instead, packets of well-stirred material are carried into the lower layer, where they spread out and are eventually carried back into the upper layer. As a result, the global stirring is efficient, although locally the flow may be quite slow.

4. Conclusions

These models are designed to assess the effects of depth dependent thermal conductivity and depth and temperature dependent viscosity on mixing. Although increasing the viscosity and thermal conductivity with depth slows convection and mixing, the effects are complex and depend strongly on the specific parameters used. Formation of isolated islands can occur if the flow is stabilized sufficiently. Such islands are unlikely to persist to the Earth's mantle, where changes in the configuration of the tectonic plates at the surface would disrupt the flow beneath. We note that in many cases mixing can be locally rapid, even when large-scale heterogeneities persist. This is consistent with the observation that global heterogeneities exist in the Earth's mantle [10]. The rate of mixing is generally dominated by the regions in which mixing is rapid, with exponential separation of tracer particles. This is consistent with prior models of chaotic mixing in the mantle (see, for example, [11,12]).

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