RANS computations of artery flows

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Abstract

Pulsatile blood flow in stenotic vessels is investigated using a Reynolds-averaged Navier–Stokes framework. Two simplified models for arterial stenoses were analyzed: a two-dimensional channel with a one-sided semicircular constriction and a smooth axisymmetric stenosis. Although these flows are at a relatively low Reynolds number, the constriction induces separation that causes the flow to transition to turbulence for most large arteries. The prediction of such flows represents a challenge for RANS turbulence models and warrants a closer look at their low-Re modifications.

Keywords: RANS; Turbulence modeling; Low Reynolds; Artery flows; Atherosclerosis

1. Introduction

Atherosclerotic constrictions in arteries or arterial stenoses are found predominantly in larger arteries such as carotid artery that supplies blood to the brain or the coronary artery that supplies blood to the cardiac muscles. Although the Reynolds number for such flows is still relatively low (from few hundreds to few thousands), the flow does transition to turbulence downstream of the stenosis, as discussed in Ku [1]. A recent DNS study by Mittal et al. [2] presents detailed statistics at various downstream locations and clearly shows that the flow is indeed turbulent for Reynolds numbers higher than 1000.

Reynolds-averaged Navier–Stokes analysis is a powerful approach in the analysis of flow in arterial stenoses. Low computational time compared to LES or DNS is a significant factor, especially when the intent is to couple the flow simulation with an optimal design procedure for interventional devices such as stents (eventually allowing surgeons to quickly tailor a stent to the particular geometry of a patient artery). However, the capacity of existing RANS turbulence models to predict correctly the flow in arterial stenoses is somewhat questionable. The flow is pulsatile at low Reynolds numbers, requiring an unsteady RANS simulation with low-Re modifications of turbulence models.

The low-Re modifications are often designed to capture transition to turbulence for flow over a flat plate, see Durbin et al. [3] and Wilcox [4]. Recent computations with low-Re models for recirculating flows [5] indicate that these low-Re modifications fail to predict flows that transition to turbulence under a different scenario. Therefore, a more thorough analysis of RANS turbulence models used for computations in arterial stenoses is warranted.

In this paper, two test cases based on a simplified arterial stenosis shown in Fig. 1 are presented. The first test case is a DNS computation by Mittal et al. [2] where a one-sided semicircular constriction is placed in a two-dimensional channel. The second test case is a smooth axisymmetric stenosis analyzed experimentally by Ahmed et al. [6]. For both test cases, Reynolds and Womersley numbers were chosen in the range consistent with typical values for blood flow in the larger arteries of the human cardiovascular system [1].

2. RANS turbulence models

Two most commonly used two-equation eddy-viscosity turbulence models (k-ϵ and k-ω models) are analyzed here. The standard high-Re k-ε model equations are:

$$\partial_t k + u \cdot \nabla k = P_k - \varepsilon + \nabla \cdot [(\nu + \sigma_k \nu_t)\nabla k] \tag{1}$$

$$\partial_t \varepsilon + u \cdot \nabla \varepsilon = (C_{\varepsilon 1} P_k - C_{\varepsilon 2}\varepsilon)/T + \nabla \cdot [(\nu + \sigma_\varepsilon \nu_t)\nabla \varepsilon] \tag{2}$$

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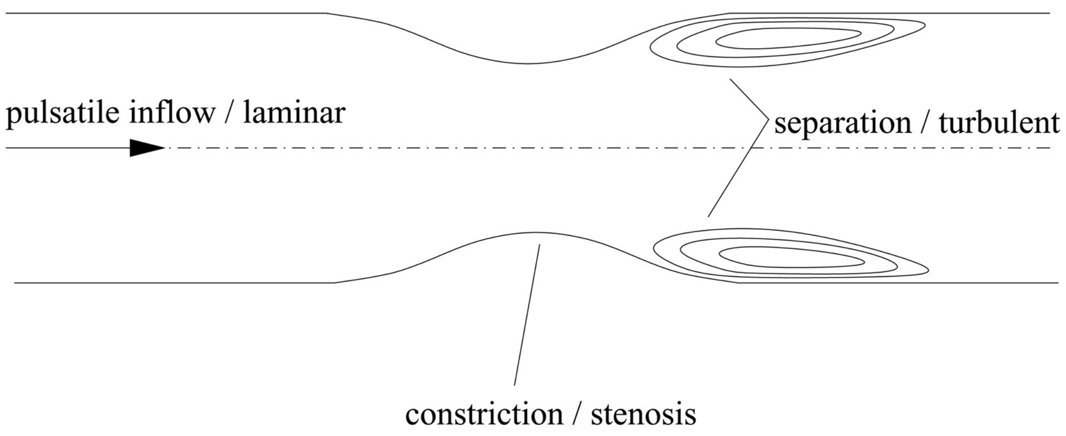

Fig. 1. Schematic of a simplified stenosis.

The equations for the high-Re k-ω model are [4]:

$$\partial_t k + u \cdot \nabla k = P_k - C_\mu \omega k + \nabla \cdot [(\nu + \sigma_k \nu_t)\nabla k] \tag{3}$$

$$\partial_t \omega + u \cdot \nabla \omega = \left(C_{\omega 1} P_k - C_{\omega 2} C_\mu k\omega\right) / \left(C_\mu^2\, k\, T\right) + \nabla \cdot [(\nu + \sigma_\omega \nu_t)\nabla \omega] \tag{4}$$

The production term in Eqs. (1)–(4) is $P_k = 2v_t|S|^2$ with $S_{ij} = 1/2\ (\partial_j u_i + \partial_i u_j)$.

Turbulent viscosity is defined as $\nu_t = C_\mu k T$ with $T = k/\varepsilon$ and $T = 1/(C_\mu \omega)$ for the k-ε and k-ω models, respectively. An upper bound for turbulence time-scale is imposed, $\mathrm{T} \leq 0.6/\left(\sqrt{6}\mathrm{C}_\mu|\mathrm{S}|\right)$ [4]. The closure coefficients are, for the k-ε model:

$$C_\mu = 0.09, \quad C_{\varepsilon 1} = 1.44, \quad C_{\varepsilon 2} = 1.92, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1.3 \tag{5}$$

and for the k-ω model:

$$C_\mu = 0.09, \quad C_{\omega 1} = \gamma C_\mu, \quad \gamma = 5/9, \quad C_{\omega 2} = 0.075, \quad \sigma_k = 0.5, \quad \sigma_\omega = 0.5 \tag{6}$$

In contrast to the k-ε model, the k-ω model can be integrated up to the wall. For the k-ε model, an additional near-wall layer model is needed (this can be achieved by using a two-layer k-ε model or by modifying the standard k-ε model to include low Reynolds effects near the wall).

The concept of unsteady RANS sometimes causes confusion. There is no inconsistency between representing turbulent mixing by a turbulence model, while computing an unsteady mean flow. In the presence of coherent, periodic unsteadiness (such as pulsating inflow), the energy spectrum will look like Fig. 2. Mixing due to the broadband portion of the spectrum is represented by the turbulence model. The spike is due to mean flow unsteadiness. It is a source of additional mixing – mixing that is not due to turbulence, but rather, to vortices in the mean flow.

Low-Re modifications of the k-ε model [4,5] are based on the asymptotic consistency near the solid wall. A damping function f_μ is introduced in the definition of eddy viscosity: $\nu_t = f_\mu C_\mu k T$. However, it is not sufficient to damp the eddy-viscosity. Most low-Re modifications also modify the ε-equation, introducing a new variable

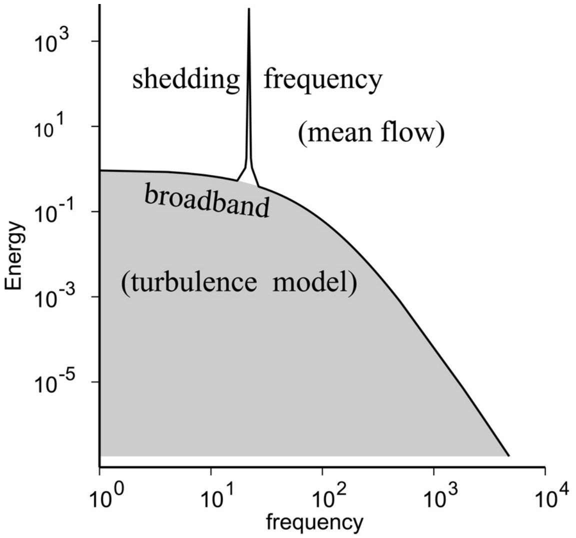

Fig. 2. Energy spectrum with broadband and coherent frequencies.

$\tilde{\varepsilon} = \varepsilon - \varepsilon_0$ with $\varepsilon_0 = \varepsilon|_{y=0}$, $T = k/\tilde{\varepsilon}$ and, where f_1, f_2 and E are additional damping functions,

$$\partial_t \tilde{\varepsilon} + u \cdot \nabla \tilde{\varepsilon} = (f_1 C_{\varepsilon 1} P_k - f_2 C_{\varepsilon 2} \tilde{\varepsilon})/T + E + \nabla \cdot [(\nu + \sigma_\varepsilon \nu_t) \nabla \tilde{\varepsilon}] \quad (7)$$

For example, the Launder-Sharma k-ε model has

$$f_\mu = e^{-3.4/(1+Re_T/50)^2}, \quad f_1 = 1, \quad f_2 = 1 - 0.3 e^{-\mathrm{Re}_T^2},$$
$$\varepsilon_0 = \nu \left(\frac{\partial k}{\partial y}\right)^2 / 2k, \quad E = 2\nu\nu_T \left(\frac{\partial^2 u}{\partial y^2}\right)^2 \quad (8)$$

with turbulence Reynolds number $\mathrm{Re}_T = k^2/(\nu\varepsilon)$.

For the k-ω model, a low-Re modification was derived from the analysis of transition to turbulence for the flow over a flat plate. The high-Re k-ω model transitions to turbulence at Reynolds numbers that are an order of magnitude lower than observed in experiments. By trying to delay the transition and match the correct critical Reynolds number (and achieve asymptotic consistency near the wall) a low-Re modification was proposed by Wilcox [5]. Turbulent viscosity is modified as $\nu_t = \alpha^* C_\mu k T$ with $T = 1/(C_\mu \omega)$ and coefficients α^*, $C_\mu{}^*$ and γ^* are modified as functions of the turbulence Reynolds number $\mathrm{Re}_T = k/(\nu\omega)$:

$$\alpha^* = \frac{\alpha_0^* + \mathrm{Re}_T/R_k}{1 + \mathrm{Re}_T/R_k}, \quad \alpha_0^* = C_{\omega 2}/3,$$
$$\gamma^* = \gamma \frac{\alpha_0 + \mathrm{Re}_T/R_\omega}{1 + \mathrm{Re}_T/R_\omega} \frac{1}{\alpha^*}, \quad \alpha_0 = 1/10, \quad (9)$$

$$C_\mu^* = C_\mu \frac{5/18 + (\mathrm{Re}_T/R_\beta)^4}{1 + (\mathrm{Re}_T/R_\beta)^4}, \quad \text{with} \quad R_k = 8,$$
$$R_k = 6, \quad R_\omega = 2.7 \quad (10)$$

3. Numerical results

An advanced version of the research solver NSIKE by Medic et al. [7] was used for the computations. Furthermore, the commercially available package FLUENT [8] was also used to compare and verify the results. NSIKE is based on the projection method with the stabilization of convection terms via the PSI residual distribution scheme and the Poisson problem for the pressure is solved using conjugate gradient technique. FLUENT solves the discretized equations in a segregated manner using the PISO algorithm suitable for unsteady simulations.

Two simplified models for arterial stenoses were analyzed: a two-dimensional channel with a one-sided semicircular constriction [2] and a smooth axisymmetric stenosis [6]. The shape of a real arterial stenosis is somewhat arbitrary. However, Ahmed et al. [6] have pointed out that the shape created by arterial plaques is essentially circular and that contours of actual stenoses tend to be relatively smooth.

Computational grids for both cases were hybrid, consisting of an outer unstructured grid with a layer of structured grid near solid walls for better resolution of boundary layers. Grid (and time-step) independence studies were conducted for both test cases and the final grids used in the computations consisted of approximately 50·000 nodes and 30·0000 cells with y_1^+ values lower than 1.

The test case presented in Mittal et al. [2] consists of a one-sided semicircular constriction with 50% blockage located on the upper channel wall at $x/H = 0$ (see Fig. 3(a)). The pulsating velocity profile at the inflow corresponds to Womersley solution for laminar pulsatile flow in a channel. The flow was computed at peak Reynolds number of $\mathrm{Re} = V_{\max} H/\nu = 2000$ and Womersley number $\mathrm{Wo} = H/2\ (2\pi f/\nu)^{1/2} = 8.6$. The time-averaged skin friction coefficient on the upper wall is presented in Fig. 3. The k-ω model predictions for the recirculation region are better than for the k-ε model. Interestingly, the low-Re variant of the k-ω model does not seem to perform any better than its high-Re counterpart. The smooth axisymmetric stenosis in Ahmed et al. [6] consists of a pipe with an outer wall constriction defined as a cosine function of length $2D$ (D being the diameter of the vessel) with the throat located at $x/D = 0$ (see Fig. 4(a)). The flow was computed for a 75% stenosis (by area) at Reynolds number $\mathrm{Re} = U_0 D/\nu = 600$ and Womersley number $\mathrm{Wo} = D/2(2\pi f/\nu)^{1/2} = 7.5$. The inflow profile for the velocity is varying sinusoidally in time: $U = U_0 + U_m \sin(2\pi f t)$ and the inflow is considered laminar. The time-averaged skin friction coefficient C_f is compared for various models in Fig. 4. Note that the experimental data for C_f is instantaneous corresponding to peak inflow velocity U_{max} (the time-averaged recirculation is likely to be a bit shorter). As for the previous test case, the k-ω model predicts a recirculation region that is longer than the k-ε model. The low-Re k-ω model predicts an even longer recirculation with flow that is almost laminar.

4. Conclusions

The results for both test cases show that flow predictions differ downstream of the constriction throat. As observed in several earlier computations, the k-ω model predicts larger recirculation regions than k-ε model. Interestingly, for both test cases, the low-Re k-ω modifications do not improve the flow predictions; they actually deteriorate the results. However, the low-Reynolds effects certainly play an important role in these flows and further improvements in modeling are needed.

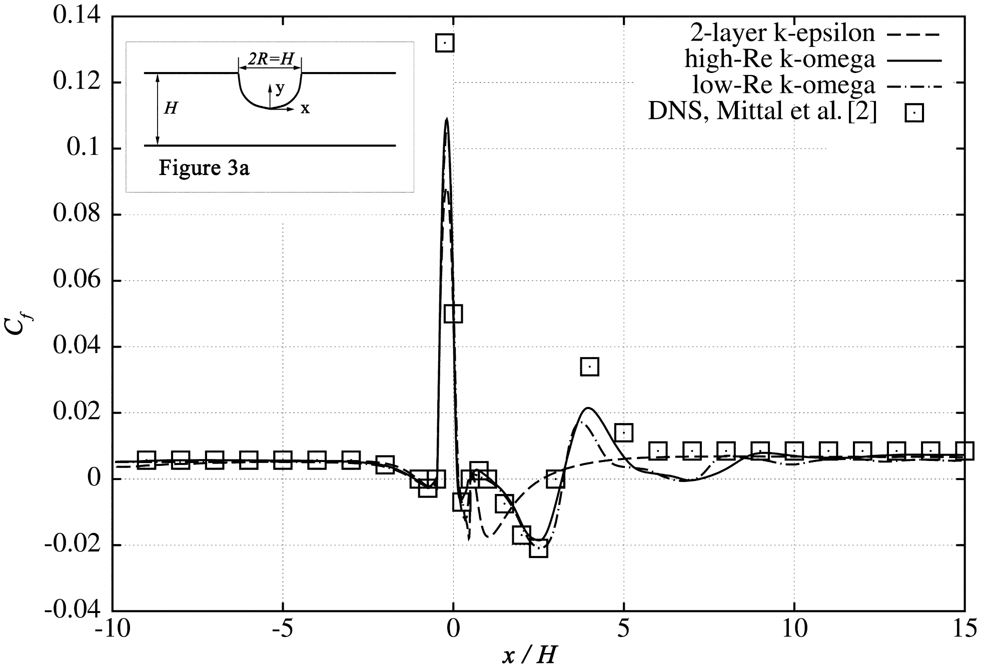

Fig. 3. Time-averaged skin friction coefficient C_f at the upper wall.

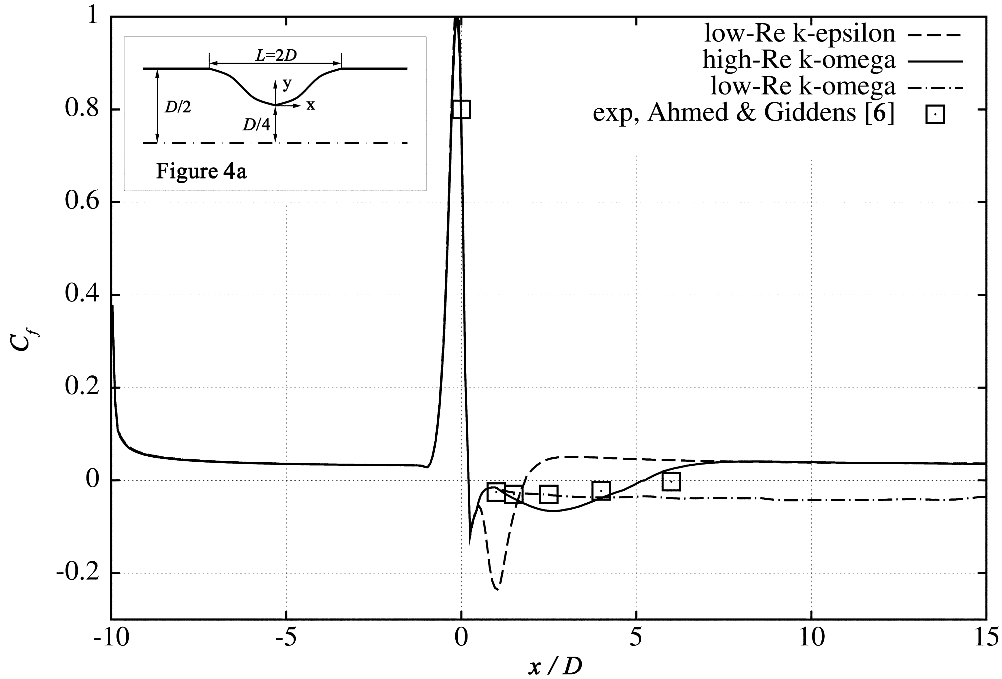

Fig. 4. Time-averaged skin friction coefficient C_f.

Successful RANS computations for artery flows with stationary walls will allow for more realistic simulations with compliant walls in the future.

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