Numerical analysis of two distinct types of vortex dislocation in wake-type flows with different spanwise nonuniformities

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Abstract

Two distinct types of vortex dislocation generated in wake-type flows with different spanwise nonuniformities are numerically studied by direct numerical simulation (DNS). A local spanwise disturbance to the velocity of the oncoming flow leads to a generation of consecutive twisted vortex dislocations in the middle downstream, which is mainly caused by phase differences among vortex-shedding cells. A stepped variation of velocity in the oncoming flow yields a periodic vortex splitting-reconnection type of dislocation, which is mainly caused by frequency differences among the shedding cells. Dynamics and main features of these dislocations, especially the vortex linkages over the dislocations, are clearly described by tracing the substantial modification of vorticity lines and other key flow quantities. Local irregularities in the variations of velocity, vorticity and frequency, and transition behavior of the wake flows are analyzed by wavelet analysis and the proper orthogonal decomposition (POD) method.

Keywords: Vortex dislocation; Wake-type flow; Direct numerical simulation

1. Introduction

It has been confirmed that for low Reynolds numbers the vortex dislocation is a fundamental characteristic of the three-dimensionality of bluff body wakes. The occurrence of vortex dislocation plays an important role in the flow transition to turbulence. Depending on different three-dimensional flow conditions, either in the geometry of the cylinder or in the oncoming flow, the vortex dislocation may present various fascinating modes or patterns. Flow visualization and measurement in previous works have provided much information on the generation and evolution of vortex dislocation. Some initial analyses on the mechanism based on inviscid vortex dynamics have been also carried out. Details of presentations of previous works mentioned above can be found in a sophisticated review paper of Williamson [1]. Recently there are few numerical studies on the forced vortex dislocations in wake-type flow [2] and the natural vortex dislocations in the wake transition [3]. The generation and the influence of the dislocation on the flow transition were reported. However, the physical understanding and the description are far from complete. Moreover, concerning the influence of the nonuniformity in the oncoming flow on vortex dislocation, we at present only have a little knowledge.

The purpose of the present work is to study, by the DNS approach, two distinct types of vortex dislocation generated in wake-type flows, where two different kinds of spanwise nonuniformities are imposed in the oncoming flow. Dynamics and features of vortex dislocations occurring in real viscous flows will be reported. Vortex linkage over the dislocations, especially on vortex splitting and reconnection between vortex rows will be described in detail by analyzing the substantial modifications of vorticity fields. The influence of nonuniformity in the oncoming flow on vortex dislocations is discussed. Transition behavior of the flow caused by the dislocation is analyzed by means of wavelet analysis and by proper orthogonal decomposition (POD), as well as using properties of the low-dimensional dynamical system, which is constituted by POD modes based on the DNS results.

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2. Numerical method

In the present calculations, the oncoming wake-type flow is represented as

$$U(y,z) = 1.0 - a(z)(2.0 - \cosh(by^2))\exp(-(cy)^2) \qquad (1)$$

which is a kind of time average, streamwise velocity profile in a cylinder wake where the flow is most unstable. The parameters b and c are determined by referring to DNS results on cylinder wake flow [4], and experimental measurements [5], while $a(z)$ is introduced to express the nonuniformity in momentum defect along the span. Here we consider two different types. For case one, $a(z) = 1.1 + 0.4\exp(-z^2)$, $b = 1.1$, and $c = 1.2$. The local disturbance to the oncoming flow is imposed near the center of the span and will exponentially decay with the increase of the distances. For case two, $a(z) = 1.1$, $b = 0.9$, $c = 0.9818$ for $|z| < 2.5$, and $a(z) = 1.1$, $b = 1.1$, $c = 1.2$ for $2.5 \leq |z| \leq 15$, respectively. In this case a stepped variation of the oncoming flows is introduced. It is expected that the evolution of these two flows will lead to different types of vortex street flow and may be similar, to some extent, to those of cylinder wakes with diameter variation. It is helpful to study the mechanism of vortex dislocation.

Our numerical method for obtaining high numerical accuracy and wave number resolution in this DNS work employs the compact finite-difference Fourier spectral hybrid method for solving the three-dimensional Navier–Stokes equations. A detailed presentation of these equations and the numerical method can be found in Ling and Xiong [2]. In the method the periodic boundary condition in spanwise direction is assumed. The m-th Fourier component of the Navier–Stokes equations is

$$\frac{\partial \vec{u}_m}{\partial t} + F_m\left[\left(\vec{u} \bullet \nabla\right)\vec{u}\right] = -\nabla_m p_m + \frac{1}{Re}\nabla_m^2 \vec{u}_m \qquad (2)$$

For time discretization of the equation, following the work of Karniadakis and Triantafyllou [4], a third-order mixed explicit–implicit scheme is used. The solution procedure for these m-th harmonic equations is split into three substeps. To evaluate the nonlinear terms in the split equations, the pseudo-spectral method is adopted, and the fifth-order upwind compact scheme is used to approximate the terms. For solving the Helmholtz equations for pressure and velocity, a nine-point compact version of a fourth-order central compact scheme is used for nonhomogeneous term calculations. For the boundary conditions, nonreflecting-type outflow boundary conditions are used. The Reynolds number is defined as $\mathrm{Re} = U_0 D/\nu = 200$, where U_0, D represent an infinite potential flow velocity and characteristic length, respectively. The computation domain is 100, 30, 30(D) in the streamwise, transverse and spanwise directions, respectively. The cutoff of the truncated Fourier series is $N = 64$, and the grid points in the x–y plane are 202×62. The numerical code used in the present work has been verified first and is available.

3. Results

The present DNS results show that spatio-temporal evolution of the oncoming flows leads to two different types of vortex street flow with two distinct types of vortex dislocation generated in the middle of the downstream flows. Isosurfaces of vorticity, vorticity component contours, and fluctuating velocity distributions describe the whole picture of the dislocation and its development in space. For case one, the vortex dislocation occurs consecutively based on the background shedding flow. The dislocation is characterized by an undulated spanwise vortex connected by streamwise and vertical vortex branches as shown in Fig. 1. Behaviors of representative vorticity lines emanating from different

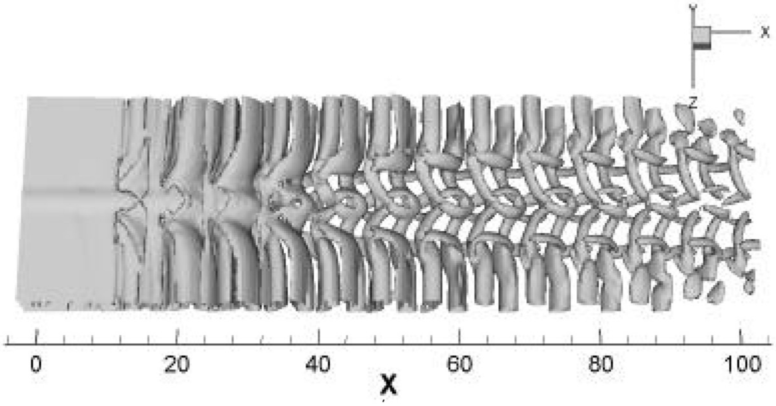

Fig. 1 Consecutive twisted vortex dislocation ($|\omega| = 0.12$, $t = 260$).

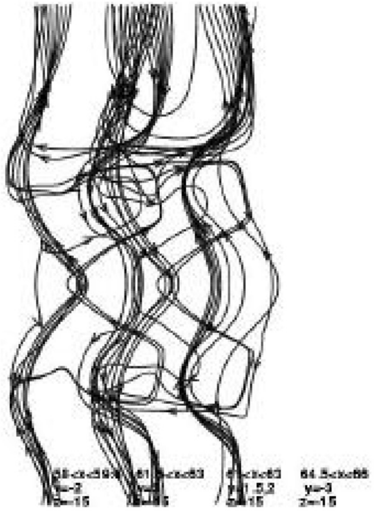

Fig. 2 Vortex linkages in vortex dislocations (top view).

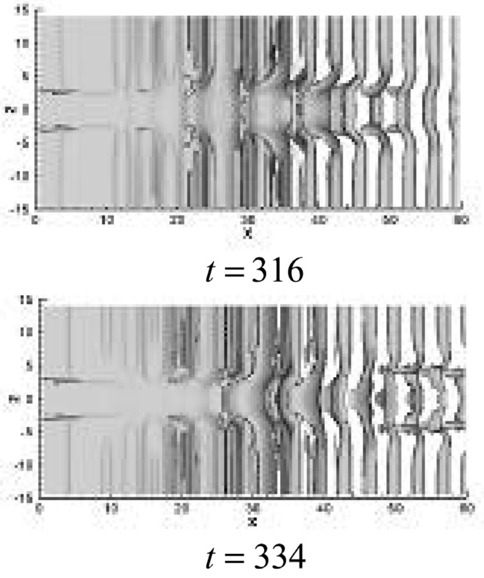

Fig. 3 Periodic vortex splitting-reconnection dislocation ($|\omega| = 0.15$).

positions in the main vortex row are closely investigated. Substantial modification of these lines from spanwise to streamwise and vertical directions clearly illustrates the scenario of the formation of the vortex dislocations. The vortex linkages in the dislocation are realized by some vorticity lines, mainly located in the outer region of a main vortex, splitting from the main vortex, and turning toward the streamwise and vertical direction and then joining, forward and backward, neighboring main vortices rows with opposite directions. Again, some other vorticity lines wind through the neighboring spanwise vortex. The reconnection is taken by a cross-vortex street mode. These features are illustrated in Fig. 2. For this case, the vortex dislocation is mainly caused by the phase differences among vortex-shedding cells. The variations of vorticity lines and other key quantities exhibit strongly the flow irregularity produced by the dislocations. For case two, the vortex dislocation occurs periodically near the steps of velocity in the span and is caused mainly by the frequency differences among shedding cells. The main features are an undulating spanwise vortex row with vortices splitting from it. The split spanwise vorticity lines turn their direction upstream, becoming a streamwise vortex branch and then turning further, joining a neighboring main vortex. Several of these kinds of vortex patterns constitute a spot-like vortex dislocation and travel downstream periodically. The flow pattern with the dislocations is shown in Fig. 3. Two basic frequencies and their difference just correspond, respectively, to the shedding frequencies at the central region of the span and away from it, and the frequency of the spot-like dislocation. The characteristic of frequency in the time-frequency domain given by local wavelet analysis shows the irregularity in the flow; a sample is shown in Fig. 4, which is closely related to the onset of local turbulence. Using DNS results, POD analysis can be applied [6]. This leads to 10 modes that show the main features of the flow with vortex dislocations. Using 20 modes to constitute a low-dimensional system, the behavior in coefficient phase space of different order modes describe clearly the flow transition from two-dimensional to three-dimensional, and also the local irregularities.

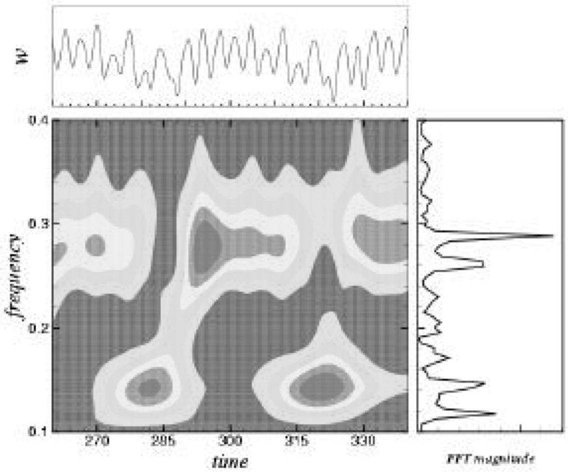

Fig. 4 A sample of time-frequency analysis of spanwise velocity at $x = 45$, $y = 0.5$, $z = 2$.

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