Yin–Yang grid and geodynamo simulation

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Abstract

For geophysical simulations in the spherical geometry, a family of overset grids named Yin–Yang is proposed. The Yin–Yang grid is based on the dissection of a spherical surface into two identical (exactly the same shape and size) subregions. Each subregion of a sphere is patched by the low-latitude part of the usual latitude–longitude grid. The two identical component grids are combined in a complemental way to cover a spherical surface with partial overlap on their borders. High performance of the Yin–Yang grid on a massively parallel computer is demonstrated by an application to geodynamo simulation.

Keywords: Overset grid; Yin–Yang grid; Spherical geometry; Geodynamo

1. Introduction

The spherical shell is a key geometry in computational geophysics. Simulations of geodynamo, mantle convection, and global circulation of the atmosphere are all performed in the spherical shell geometry. The latitude–longitude grid, which is defined in the spherical polar coordinates with radius r ($r_i \leq r \leq r_o$), colatitude θ ($0 \leq \theta \leq \pi$), and longitude ϕ ($-\pi \leq \phi \leq \pi$), would be a natural choice for the computational grid, since its orthogonality is desirable from numerical reasons; see Fig. 1.

There are two kinds of numerical problems in the latitude–longitude grid. One is the coordinate singularity *on* the poles ($\theta = 0$, and π) and the other is the grid convergence *near* the poles that imposes severe restriction on the Courant-Friedrichs-Lewy (CFL) condition there. The first problem (coordinate singularity) can be overcome by applying L'Hospital's theorem. The second problem (grid convergence) can be overcome by applying a low-pass filter that effectively generates quasi-uniform grid spacings over a sphere. We were using a finite difference method in the latitude–longitude grid with the L'Hospital's theorem method and an FFT-based spherical filter in our previous geodynamo simulations; see, for example, Kageyama and Sato [1] and Li et al. [2].

We were always troubled with numerical costs and inefficiency caused by the spherical filter in our geodynamo simulations. The spherical filter spoils the locality of our finite difference scheme, which is a strong point on massively parallel computers in contrast to the spectral method. We have noticed that the latitude–longitude grid should be laid aside at least in its original form.

2. Yin–Yang grid

Reviewing the latitude–longitude grid, one would notice that both drawbacks of the grid, i.e., the coordinate singularity and the grid convergence, originate from high latitude regions. The low latitude region near the equator, on the other hand, has rather desirable features for a base grid in the spherical geometry; it is orthogonal and its grid spacings are quasi-uniform. The low latitude part of the latitude–longitude grid – within 90° (between 45°N and 45°S) around the equator – is almost an ideal grid for the spherical geometry; see Fig. 1.

Suppose a sphere with unit radius ($r = 1$). An interesting observation on the low latitude, barrel-like part of the latitude–longitude grid (90° around the equator) is that if one cuts off 1/4 of the longitude, the remaining part of the 'barrel', has an area of

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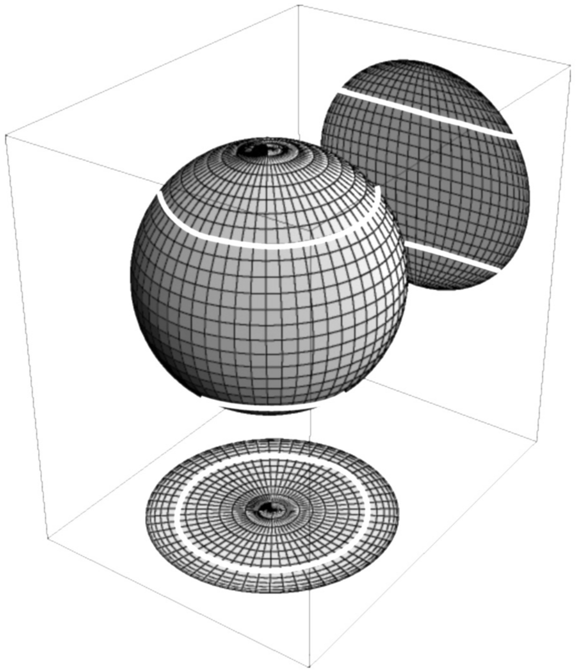

Fig. 1. Latitude–longitude grid. The low-latitude part has favorable features as a base grid system for numerical simulations in the spherical geometry: it is an orthogonal grid with quasi-uniform spacings.

$$\int_{\pi/4}^{3\pi/4} \sin\theta d\theta \int_{-3\pi/4}^{3\pi/4} d\phi \sim 2.12\pi \tag{1}$$

which is roughly a half of the full spherical surface area, 4π.

This observation leads us to decomposition of the spherical surface into two identical parts. A pair of two identical grids, each of which is nothing but the low-latitude, 3/4 part of the 'barrel' of the latitude-longitude grid, are combined to cover the spherical surface; see Fig. 2.

Generally, dissection of a computational domain generates internal borders between the subregions. In the overset grid approach, see Chesshire et al. [3], the subdomains are permitted to partially overlap one another on their borders. The overset grid is also called an overlaid grid, or composite overlapping grid, or Chimera grid as proposed by Steger et al. [4]. Interpolations are applied on the boundary of each component grid to set the boundary values as internal boundary conditions.

We have proposed, in Kageyama et al. [5], a family of spherical overset grids and named them 'Yin–Yang grids'. A Yin–Yang grid is composed of two identical

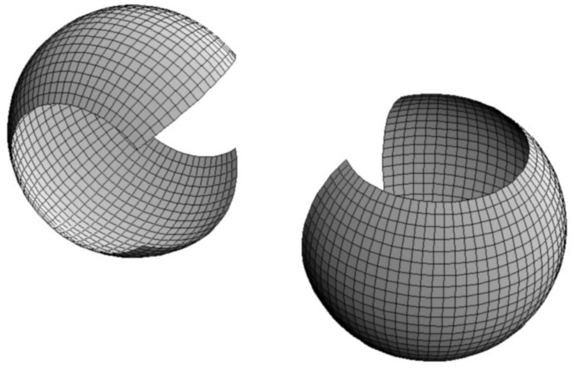

Fig. 2. A pair of component grids. A component grid is a low-latitude part of the latitude–longitude grid, defined by 90° around the equator in latitude and 270° in longitude. The two component grids are identical and they are combined in a complemental way to cover a spherical surface.

component grids that are combined in a complemental way to cover a spherical surface with partial overlap on their borders. An example of the Yin–Yang grid is shown in Fig. 3, whose component grids are in Fig. 2.

The partially overlapped border between the two component grids – they are called the Yin grid and the Yang grid – reminds us of a baseball; see Fig. 4(a). When one cuts along the seam of the baseball, the two pieces of material that are shown in Fig. 4(b) are obtained. Note that the pieces are identical in shape and size. The complemental combination of the two pieces in the baseball can be regarded as a three-dimensional variation of the Chinese philosophical symbol (yin-yang) of complementarity that is shown in Fig. 4(c).

There are many variations of the Yin–Yang grids since there are infinite patterns of dissection that cut a

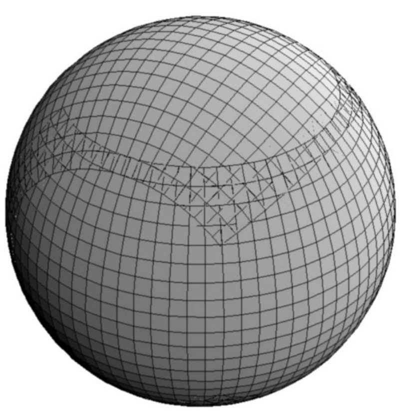

Fig. 3. A Yin–Yang grid. The two component grids – Yin grid and Yang grid – shown in Fig. 2 are combined to cover the spherical surface with partial overlap.

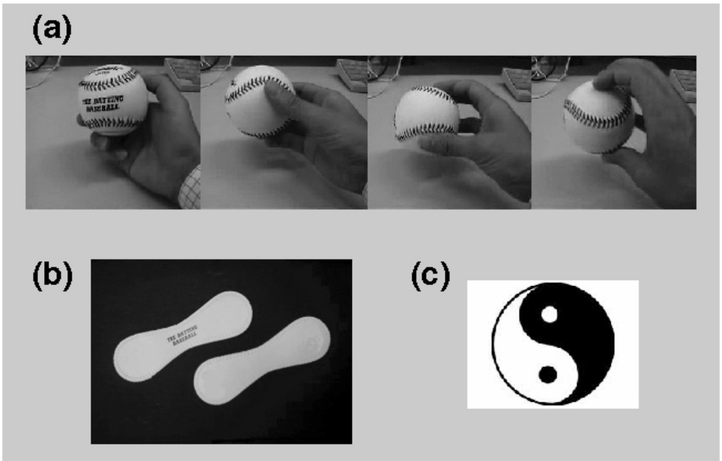

Fig. 4.(a) A baseball. It has a closed seam. (b) Two identical parts obtained by cutting along the seam of the ball. They are combined in a complemental way to cover the spherical surface. (c) Chinese yin-yang symbol of complementarity. The baseball could be regarded as its three-dimensional extension.

spherical surface into two identical parts. From a numerical point of view, the most basic type among them is that shown in Fig. 3 since its component grids are rectangles in the computational space (see Fig. 2). Another example of the Yin–Yang grid is shown in Fig. 5. In this case, the overlapped area between the two component grids is minimized. Note that the two component grids are identical in this Yin–Yang grid, too.

Since the Yin-subregion and the Yang-subregion are identical in shape, geometry, grid, and metric tensors, all subroutines designed for, say, the Yin grid, can be used for the Yang grid with no modification. This fact makes the Yin–Yang-based computer code very concise and efficient. Another advantage of the Yin–Yang grid resides in the fact that the component grid (Yin or Yang) is nothing but a (part of the) latitude–longitude grid. We can directly deal with the equations to be solved with the vector form in the usual spherical polar coordinates. The analytical forms of metric tensors such as for Laplacian in spherical coordinates are well known. Since we can directly code the basic equations as they are formulated in spherical coordinates, we can make use of various resources of mathematical formulas, program libraries, and tools that have been developed in spherical polar coordinates.

For three-dimensional spherical shell geometry, the Yin–Yang grid for a spherical surface are piled up in the radial direction.

3. Application of the Yin–Yang grid to geodynamo simulation

We have applied the Yin–Yang grid to a geodynamo simulation in which the time development of the magnetohydrodynamic (MHD) equations is followed in the spherical shell geometry by a finite difference method with the basic type Yin–Yang grid shown in Fig. 3. The physical model is not changed from our previous one. See, for example, Li et al. [2] for details.

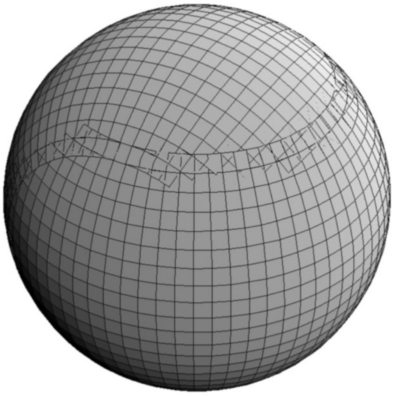

Fig. 5. Another form of Yin–Yang grid. The overlapped area between two identical component grids is minimized in this case.

We consider a spherical shell vessel bounded by two concentric spheres. The inner sphere of radius $r = r_i$ denotes the inner core and the outer sphere of $r = r_o$ denotes the core-mantle boundary. An electrically conducting fluid is confined in this shell region. Both the inner and outer spherical boundaries rotate with a constant angular velocity Ω. We use a rotating frame of reference with the same angular velocity. There is a central gravity force in the direction of the center of the spheres. The temperatures of both the inner and outer spheres are fixed: hot (inner) and cold (outer). When the temperature difference is sufficiently large, a convection motion starts when a random temperature perturbation is imposed at the beginning of the calculation. At the same time an infinitesimally small, random 'seed' of the magnetic field is given.

The spatial derivatives of the compressible MHD equations with uniform viscosity, thermal conductivity, and resistivity are discretized by the second-order central finite difference in spherical coordinates: (r, θ, ϕ). The fourth-order Runge-Kutta method is used for the temporal integration.

We have developed a new geodynamo simulation code by converting our previous geodynamo code, which was based on the traditional latitude–longitude grid. We found that the code conversion from our previous latitude–longitude-based code into the new Yin–Yang-based code is straightforward and rather easy. This is because most of the Yin–Yang code shares source lines with the latitude–longitude code. Our previous geodynamo code was basically a finite-difference MHD solver on spherical coordinates with a full span of colatitude ($0 \leq \theta \leq \pi$) and longitude ($-\pi < \phi \leq \pi$); on the other hand, the Yin–Yang grid code is also a finite-difference MHD solver on the spherical coordinates, but with just the smaller span of colatitude ($\pi/4 \leq \theta \leq 3\pi/4$) and longitude ($-3\pi/4 \leq \phi \leq 3\pi/4$). The major difference is the new boundary condition (interpolation) for the communication between the Yin grid and the Yang grid.

Since the Yin grid and the Yang grid are identical, dividing the whole computational domain into the Yin part and the Yang part is natural for parallel processing. Further domain decomposition within each part is applied in both the latitudinal and longitudinal directions. We apply vectorization in the radial dimension of the three-dimensional arrays for physical variables.

The best performance achieved so far by our Yin–Yang dynamo code is 15.2 Tflops with 4096 processors of the Earth Simulator. This is 46% of the theoretical peak performance. The total grid size of 511 (radial) $\times$ 514 (latitudinal) $\times$ 1538 (longitudinal) $\times$ 2 (Yin and Yang). The average vector length is 251.6, and the vector operation ratio is 99%. Such a high performance of the code demonstrates an excellent potential of the Yin–Yang grid for simulations in the spherical shell geometry. Details of the performance of the Yin–Yang dynamo code are reported in Kageyama et at. [6].

4. Summary

For geophysical simulations, grid design in the spherical shell geometry plays a key role. We have proposed a new spherical overset grid named the Yin–Yang grid. The Yin–Yang grid is composed of two component grids that have the same shape and size. They are combined in a complemental way to cover a spherical surface with partical overlap on their borders. Each component grid is a low-latitude part of the usual latitude–longitude grid. The grid spacing over the spherical surface is quasi-uniform.

We have applied the Yin–Yang grid to geodynamo simulation with good performance (15.2 Tflops, 46% of peak performance) on the Earth Simulator. The Yin–Yang grid was also applied to a mantle convection simulation in a spherical shell geometry; see Yoshida et al. [7]. The Yin–Yang grid has also been applied to simulations of the global circulation of the atmosphere, ocean, and their coupled system; see Takahashi et al. [8], Komine et al. [9], Ohdaira et al. [10], and Hirai et al. [11].

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