Numerical analysis of 3-D lid-driven cavity flows with different spanwise aspect ratios

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Abstract

The incompressible vortical flows in three-dimensional lid-driven square cavities with different spanwise aspect ratios are studied using a combined compact finite difference (CCD) scheme with high accuracy and high resolution. The results show that the bifurcation of the flow structures at the Reynolds number $Re = 850$ occurs at the spanwise aspect ratio between 4 and 5. For the cavities with longer spans, the tori of streamlines localized near the symmetric plane are presented, but, on the other hand, the global flows are observed along the central axis of the cavity.

Keywords: Incompressible flow; Lid-driven cavity; Spanwise aspect ratio; Cell structure; Secondary vortical flow; Torus of streamline; Combined compact difference

1. Introduction

The vortical flow in the lid-driven cavity has been extensively studied as a fundamental model for separated flows, for the vortex dynamics in closed systems with simple geometry and for a simplified version of many manufacturing devices [1]. There are two aspect ratios in the three-dimensional flows in finite domains: $\Gamma = d/h$ and $\Lambda = l/h$ where d, h and l are the width, the height and the span length of the cavity respectively. We consider the square cavity ($\Gamma = 1$) with different spanwise aspect ratio Λ's, and study the side boundary effect on the overall flow structure and the Taylor-Görtler-like vortices in the secondary flow. Albensoeder et al. [2] recently studied the three-dimensional flow in lid-driven cavities with different aspect ratios by a linear stability analysis of two-dimensional flows and experiments. They reported that the cell structures near the symmetric plane are observed in the cavity flow with $\Gamma = 1$ and $\Lambda = 6.55$ at the Reynolds number $Re = Ud/\nu = 850$, where U is the speed of the sliding upper wall and ν is the kinematic viscosity. However, they observed only the flow pattern near the wall. The whole structure is not clear. In this paper, we focus on the global flow structure in the cavity. Since numerical simulations of these vortical flows with different spatial scales need high accuracy and high resolution, we use a spectral-like combined compact finite difference (CCD) scheme proposed by the authors [3].

2. Numerical method

First we explain the spectral-like combined compact finite difference briefly. Consider a function $f(x)$ defined on the interval where N grid points are located with a uniform spacing h. Let f_i, f'_i, f''_i and f'''_i be the values of the function and its first, second and third derivatives at i-th grid point x_i respectively. The CCD scheme evaluates these derivatives implicitly from the following relations:

$$f'_i = a_1(f_{i+1} - f_{i-1}) + a_2(f'_{i+1} + f'_{i-1}) + a_3(f''_{i+1} - f''_{i-1}) + a_4(f'''_{i+1} + f'''_{i-1}) \quad (1)$$

$$f''_i = b_1(f_{i+1} + f_{i-1} - 2f_i) + b_2(f'_{i+1} - f'_{i-1}) + b_3(f''_{i+1} + f''_{i-1}) + b_4(f'''_{i+1} - f'''_{i-1}) \quad (2)$$

$$f'''_i = c_1(f_{i+1} - f_{i-1}) + c_2(f'_{i+1} + f'_{i-1}) + c_3(f''_{i+1} - f''_{i-1}) + c_4(f'''_{i+1} + f'''_{i-1}) \quad (3)$$

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The accuracy of the scheme is determined by the parameters a_j, b_j and c_j. By relaxing the requirement for the highest accuracy, we have free parameters. The values of the free parameters are chosen to minimize errors in the wave number space for most waves except very short waves [3]. This CCD scheme has eighth-order accuracy and spectral-like resolution for the first derivative. The Poisson solver which is necessary for incompressible flow simulations generally takes a large portion of the entire computation time. Since CCD schemes implicitly evaluate the derivatives, it is hard to apply a simple iterative method such as SOR to the CCD Poisson equation solver. We developed an algorithm which uses an ADI method to solve the Poisson equation. This algorithm can be parallelized easily and gives high efficiency.

The dimensionless incompressible Navier–Stokes equations in conservation form

$$\frac{\partial \mathbf{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\mathbf{u}_i \mathbf{u}_j) = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\nabla^2 \mathbf{u}_i \quad (4)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (5)$$

are numerically solved by the MAC method and the fourth-order Runge-Kutta method for the time integration, where $\mathbf{u}$ is the velocity and p is the pressure. The derivatives in the equations are evaluated by the CCD scheme, in which the first and the second derivatives have eighth-order and sixth-order accuracy, respectively, except for the boundary. At the boundary, the first and the second derivatives have sixth-order accuracy. The boundary conditions for the velocity and the pressure: $\partial_n \mathbf{u}_n = 0$ and $Re \nabla p = \nabla^2 \mathbf{u}$ as well as the no-slip condition of the velocity are imposed in the numerical calculation, where $\mathbf{u}_n$ and ∂_n are the normal component of the velocity and the normal derivative on the wall, respectively. In the case of the cubic cavity ($\Lambda = 1$) at Re = 100 to 400, the numerical results obtained with this new scheme using a small uniform grid system (33×33×33) are in good agreement with the previous results [4]. In this study the 100×100×100 Λ uniform grid system is used in the calculation, in which Λ is a spanwise aspect ratio.

3. Results

We consider the flows in a square cavity ($0 < x < 1$, $0 < y < \Lambda$, $0 < z < 1$) with a sliding upper wall with the velocity $U = 1$ in x-direction. The flows are studied in the cavities with spanwise aspect ratios $\Lambda = 1, 3, 4, 5$ and 6.55 at the Reynolds number $Re = 850$. In all cases, we get steady state solutions. In the previous study of the cavity flow with $\Lambda = 1$ [4], we also got a steady flow structure at $400 < Re < 1000$, which consists of a main

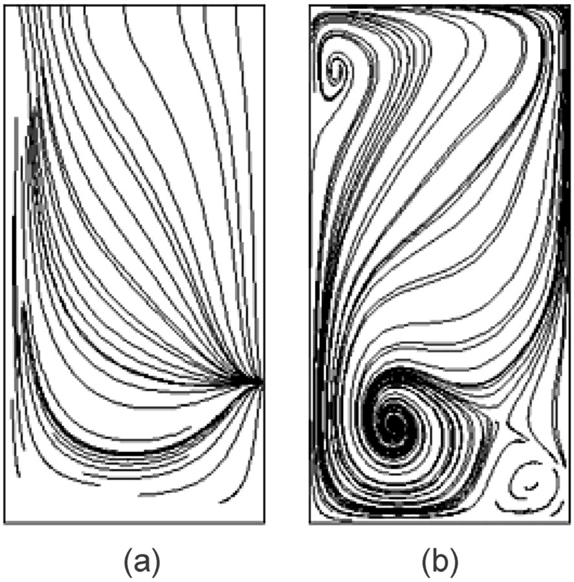

Fig. 1. Streamlines drawn from the projected velocities on the vertical left half plane of (a) $x = 0.01$ (b) $x = 0.5$ in the cavity with $\Lambda = 1$.

flow and secondary flows near the walls, and we observed chaotic streamlines in the whole flow region. In Fig. 1, we show the streamlines drawn from the projected velocity on the vertical left half plane (a) near the upstream end wall $x = 0.01$ and (b) at $x = 0.5$ in the case of $\Gamma = 1$. Note that the upper lid moves in x-direction. In Figure 1(a), we can observe main upward flow to the side wall and secondary flow along the bottom and side walls. On the other hand, we find a main upward flow region to the mid-plane, a Taylor-Görtler-like vortex near the bottom and a streamwise vortex of the upper corner region in Fig. 1(b). The flow structures of $\Lambda = 3$ and 4 are similar to that of $\Lambda = 1$. The streamlines drawn from the projected velocity on the vertical left half plane at $x = 0.01$ in the cavity with $\Lambda = 6.55$ are shown in Fig. 2. We find cell structures in the central region and the other cell structure near the side wall in Fig. 2. These results agree with the experimental result [2]. This change of the flow structure occurs between $\Lambda = 4$ and $\Lambda = 5$. These patterns of long span cavities are strongly related to the secondary flow of the overall flow structure. We show parts of two streamlines at $\Lambda = 6.55$ in Figs. 3 and 4. The streamline in Fig. 3 is localized near the symmetric plane and it makes a torus along the lid and the walls. It implies that there is a closed streamline [4] inside the torus. On the other hand, the streamline in Fig. 4 is global and chaotic. It shows

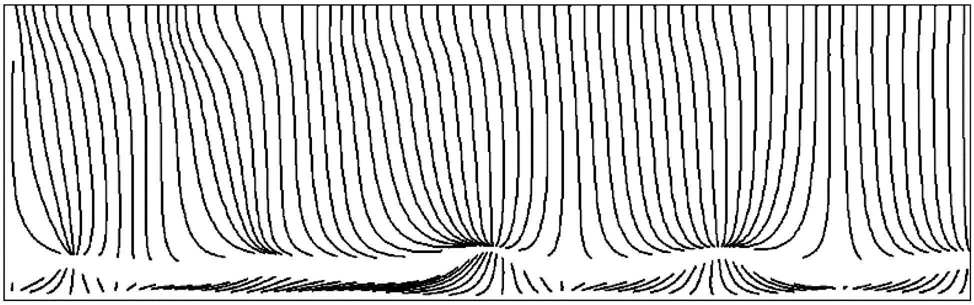

Fig. 2. Streamlines drawn from the projected velocities on the vertical left half plane of $x = 0.01$ in the cavity with $\Lambda = 6.55$.

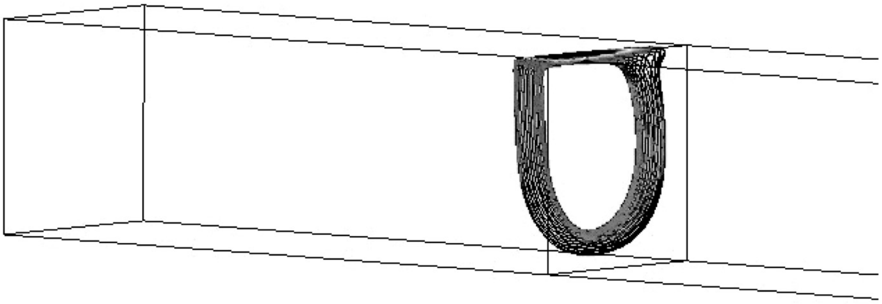

Fig. 3. A typical localized streamline in the cavity with $\Lambda = 6.55$.

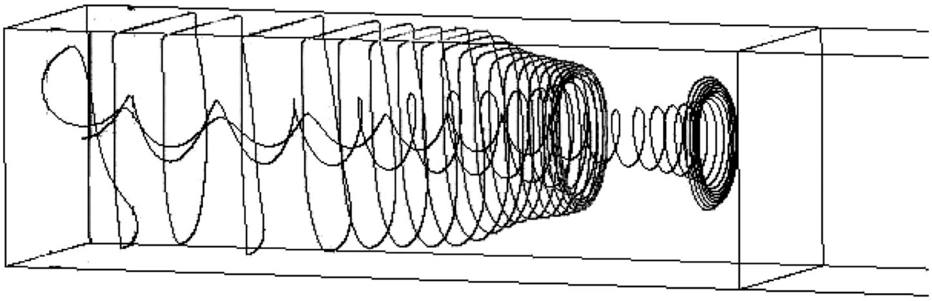

Fig. 4. A part of a typical chaotic streamline in the cavity with $\Lambda = 6.55$.

that there is a strong secondary flow from the side wall to the symmetric plane.

4. Concluding remarks

The results show that the spectral-like CCD scheme is effective and useful for vortical flow calculations. The cell structures related to closed streamlines near the wall are found in the lid-driven cavity flow over $\Lambda \sim 5$ at $Re = 850$. On the other hand, a global flow is observed along the central axis of the square, even when Λ is large.

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