# Image-based computational hemodynamics with the Lattice Boltzmann method

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# Abstract

An overview is presented of our work on the D3Q19 Lattice-BGK model for the simulation of pulsatile blood flow. First, a set of benchmark results for pulsatile flow in a tube in the relevant range of Reynolds and Womersley numbers is presented, leading to the conclusion that the Lattice Boltzmann method is a robust, accurate and efficient flow solver for this specific application. Moreover, an example of simulation of systolic flow in the lower abdominal aorta of a volunteer is presented. The L-BGK simulations show all complex fluid flow phenomena as reported in earlier studies, such as regions of low and oscillatory shear stress, back-flow regions, and vortex rings.

Keywords: Computational hemodynamics; Lattice Boltzmann method; Abdominal aorta

### 1. Introduction

Hemodynamics, the study of blood flow, is a very challenging field involving a non-Newtonian pulsatile fluid flow in a complex branching and curving structure with elastic walls [1,2]. Over the last decade, non-invasive medical imaging techniques allowed to obtain high quality three-dimensional images of the blood vessel structures of individuals. This boosted the field of image-based computational hemodynamics, where one currently is capable of simulating blood flow in all the major arteries in the body [3,4]. These simulations typically involve state-of-the-art finite-element simulations of the Navier–Stokes equations.

Over the past years we have investigated to what extent the Lattice Boltzmann method [5] would be capable of accurate and efficient simulations of blood flow. We specifically concentrated on the lower abdominal aorta, but are currently extending our studies to other parts of the arterial tree (e.g. the Carotid artery).

In this paper we present a short review of our main results of Lattice Boltzmann-based computational hemodynamics and an example of blood flow simulations in the lower abdominal aorta.

#### 2. Lattice BGK model for pulsatile blood flow

First, as in most computational hemodynamics studies, we will make two important assumptions: (1) blood is a Newtonian fluid and (2) arteries are rigid structures. The first assumption is generally acknowledged to be valid in the large arteries [1,2,4,6]. This is because of the relative high shear stress in those arteries that renders a constant viscosity of 4 centipoise. As to the second assumption, during a systolic cycle the diameter of the larger arteries may vary 5-10% (see, e.g [6]). However, in typical hemodynamics simulations the computational mesh is obtained from Magnetic Resonance Imaging (MRI), which typically results in errors in the position of the arterial wall of 1-8% [7]. Therefore, given this accuracy, the influence of elasticity of the wall can be considered a secondary effect. However, simulations do suggest that the movement of the wall may have some influence on the resulting flow fields [4], and for that reason we have carried out an initial study of pulsatile flow in elastic tubes using the Lattice Boltzmann method [8]. However, in this paper we keep the arterial walls rigid.

We use the three-dimensional 19-velocity (D3Q19) Lattice-BGK model (L-BGK [5]). This specific instance of the Lattice Boltzmann family has proven to be a very robust and reliable flow solver. The main problem with L-BGK lies in the possible onset of instabilities if the single dimensionless relaxation time  $\tau$  appearing in the collision

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term approaches the limiting value 0.5. In typical simulations, where we want to keep the size of the computational mesh manageable, we need to push the relaxation time close to its limiting value in order to obtain wanted high Re numbers. Therefore, in order to test if L-BGK is applicable, we carried out benchmark tests of pulsatile flow in straight tubes with Reynolds and Womersley numbers in the order of Re = 1000 and Wo = 16, as in the lower abdominal aorta. A representative result of such a benchmark test is shown in Fig. 1.

We have done extended benchmark testing of L-BGK both for pure Womersley flow (i.e. applying a harmonic pressure gradient) and for systolic flow (with a pressure pulse mimicking real pressure pulses in humans), see [9,10,11,12]. In all cases we can conclude that we are capable of reproducing the theoretical flow fields and shear stresses<sup>1</sup> with accuracies of a few percent.

Having assured ourselves that L-BGK is a robust solver for computational hemodynamics, we are now in the process of optimizing it in terms of execution time. We have proposed Ma-number annealing, which allows in principle a drastic improvement in execution time for time-periodic flows [14]. We have demonstrated reductions of computing time by a factor of at least three for benchmark flows, and are currently testing the method for flow in realistic arteries. Another issue is that in our L-BGK simulations we have to choose four parameters (spatial and temporal discretization, Mach number, relaxation time), under the constraint of a specified Reand Wo-number. An optimal choice for these parameters, in the sense of minimal execution time for a wanted level of accuracy, is not immediately obvious. We have undertaken a first study of this issue [15,16], but believe that optimally choosing simulation parameters is still not fully resolved for Lattice Boltzmann simulations.

# 3. A case study: pulsatile flow in the lower abdominal aorta

As a case study we present results of flow in the human abdominal aorta (see Fig. 2). The model is reconstructed from magnetic resonance angiography of a volunteer. The pressure gradient at the entrance of the aorta is averaged from flow-rate data obtained from the literature [e.g. 17]. Resulting flow fields in a small part of the model are shown in Fig. 3. The flow fields all show complex fluid flow phenomena as reported in earlier studies, such as regions of low and oscillatory shear stress, back-flow regions, and vortex rings. More details on this case study can be found in [9,18].

# 4. Conclusions

We have demonstrated the usability of the Lattice Boltzmann method (LBM) in the field of computational hemodynamics. LBM may have a few advantages over traditional methods: we obtain the shear stresses directly from the simulated quantities (the distribution functions), so we do not need to numerically differentiate the velocity fields and therefore we do not have the need for very fine meshes close to the arterial walls [3]. Next, we only use Cartesian meshes, which are easy to generate semi-automatically from raw MRI data. Whether LBM is capable of producing results with the same accuracy at the same speed as traditional Navier-Stokes solvers needs to be investigated for this specific application, but given the huge body of results of LBM in other areas we are confident that LBM will be a competitive and viable alternative in the field of computational hemodynamics.



Fig. 1. Velocity profiles during a full period for Womersley flow in a straight tube, with Re = 1250 and Wo = 7.73. The solid lines are theory, the dots are our L-BGK simulations.



Fig. 2. The lower abdominal aorta bifurcation, reconstructed from magnetic resonance angiography, as used in our simulations.



Fig. 3. Resulting flow fields in the lower abdominal aorta during peak systole.

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# Note

<sup>1</sup>Note that the shear stress in L-BGK simulations is obtained immediately from distribution functions, so there is no need for numerical differentiation of the velocity field [10,13]. This is a big advantage of the Lattice Boltzmann method.

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