A brief review on mathematical models for electrospinning

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Abstract

A brief view of mathematical models of electrospinning is given; the shortcomings of each model are illustrated. An accurate model describing the electrospinning procedure is suggested by taking into account the effect of non-Ohmic-like resistance of the fiber and polymer concentration.

Keywords: Electrospinning; Nanofiber; Mathematical model

1. Introduction

Electrospinning [1,2,3] is a process that produces nanofibers of polymers; the electrospun nanofibers can find wide applications in many areas, such as air and water filtration, and agricultural nanotechnology, to mention a few. The procedure involves applying a very high voltage to a capillary tube and pumping a polymer solution through it. Nanofibers of polymers collect as a nonwoven fabric on a grounded plate below the capillary tube. The basic logic underlying the mechanical characteristics of the electrospinning jet, however, has remained elusive. In particular, the equation for the current balance is difficult to establish.

2. One-dimensional steady model

The one-dimensional steady model for the electrospinning jet [4,5,6,7] can be written as

$$\pi r^2 u\rho = Q \tag{1}$$

$$2\pi r\sigma u + \pi r^2 kE = I \tag{2}$$

$$u\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{2\sigma E}{\rho r} + \frac{1}{r^2}\frac{\partial \tau}{\partial z} \tag{3}$$

where Q is the mass flow rate, u is the velocity, ρ is density, E is the applied voltage, I is the current, p is the internal pressure of the fluid, τ is the viscous force, σ surface density of the charge, and r is the radius of the jet at axial coordinate z.

3. Spivak-Dzenis model

Spivak et al. [8,9] established a model of a steady-state jet in the electrospinning process:
Equation of mass balance:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{4}$$

Linear momentum balance:

$$\rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \nabla T^m + \nabla T^e \tag{5}$$

Electric charge balance:

$$\nabla \cdot \boldsymbol{J} = 0 \tag{6}$$

The right-hand side of Eq. (6) is the sum of the viscous and electric forces.

4. Wan-Guo-Pan model

The Wan-Guo-Pan model [10] considers the couple effects of thermal, electricity, and hydrodynamics effects. A complete set of balance laws governing the general thermo-electro-hydrodynamics flows were derived by Ko et al. [11] and Chen [12]. It consists of modified Maxwell's equations governing an electrical field in a moving fluid, modified Navier-Stokes equations governing heat and fluid flow under the influence of an

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electric field, and constitutive equations describing the behavior of the fluid. The governing equations are [10]:

$$\frac{\partial q_e}{\partial t} + \nabla \cdot \boldsymbol{J} = 0 \tag{7}$$

$$\rho \frac{D\boldsymbol{u}}{Dt} = \nabla \cdot \boldsymbol{t} + \rho \boldsymbol{f} + q_e \boldsymbol{E} + (\nabla \boldsymbol{E}) \cdot \boldsymbol{P} \tag{8}$$

$$\rho c_p \frac{DT}{Dt} = Q_h + \nabla \cdot \boldsymbol{q} + \boldsymbol{J} \cdot \boldsymbol{E} + \boldsymbol{E} \cdot \frac{D\boldsymbol{P}}{Dt} \tag{9}$$

The current is composed of three parts: (1) the Ohmic bulk conduction current, $J_c = \pi r^2 kE$; (2) the surface convection current, $J_s = 2\pi r \sigma u$; and (3) the current caused by a temperature gradient, $J_T = \pi r^2 \sigma_T \partial T/\partial z$.

The disadvantage of this model is that no thermal effect is considered in Eq. (8), which can be modified as

$$\rho \frac{D\boldsymbol{u}}{Dt} = \nabla \cdot \boldsymbol{t} + \rho \boldsymbol{f} + q_e \boldsymbol{E} + (\nabla \boldsymbol{E}) \cdot \boldsymbol{P} + \zeta \nabla T \tag{10}$$

5. Allometric model

We know from Ohm's law that current flows down a voltage gradient in proportion to the resistance in the circuit. Current is therefore expressed as

$$I = \frac{E}{R} = gE \tag{11}$$

where I is the current, E is the voltage, R is the resistance, and g is the conductance. The resistance, R, in Eq. (11) is expressed in the form

$$R = \frac{kL}{A} \tag{12}$$

where A is the area of the conductor, L is its length, and k is a resistance parameter.

Actually, Eq. (11) is valid only for metal conductors where there are plenty of electrons in the conductor. However, in the electrospinning jet, the current is not caused by electrons, so Eq. (11) should be modified in order to accurately describe the polymer conduction. In [13], an allometric scaling law between the conductance and the radius of the jet is proposed in the form

$$g \sim r^{\alpha} \tag{13}$$

where α is a scaling exponent. Allometric scaling laws are widely seen in nature, see [14,15,16,17] and references cited therein.

When $\alpha = 2$, it becomes a metal-like conductor, so for the Ohmic bulk conduction current (see Fig. 1), we have $I_c = \pi r^2 kE$, where k is the dimensionless conductivity of the fluid.

When $\alpha = 1$, no free ions or electrons exists in the bulk, the current is caused by surface charge distributed along the surface which is in motion (see Fig. 2). So for the surface convection current, we have $I_s = 2\pi r \sigma u$. The conduction of an actual electronically charged jet lies between Ohmic bulk conduction and surface convection (see Fig. 3), so the value of α lies between 1 and 2.

We assume that the scaling relationship between the conductance and polymer concentration has the form (see the experimental data in [18])

$$g \sim c^{\beta} \tag{14}$$

where c is the polymer concentration, and β is a scaling exponent. So the conductance for electrospinning jet can be expressed as

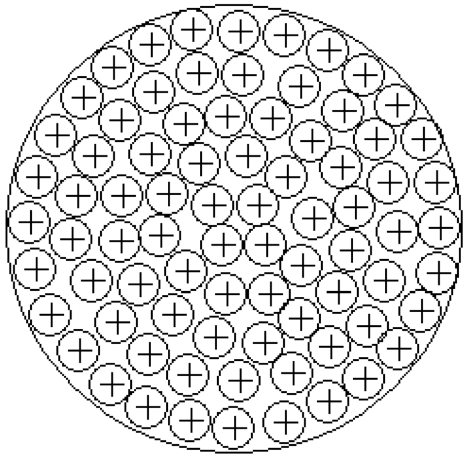

Fig. 1. Conductance of an ideal electronically charged jet: $g_c \sim r^2$, where r is the radius of the conductor.

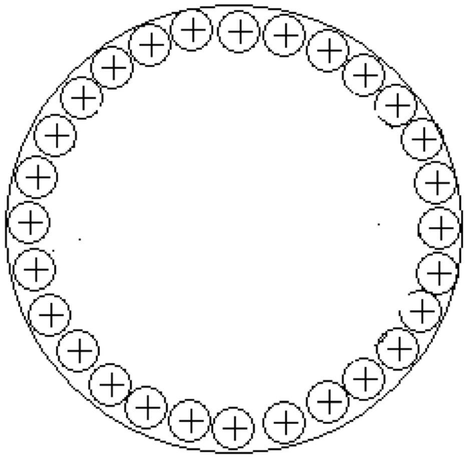

Fig. 2. Conductance of an ideal surface: $g_s \sim r$.

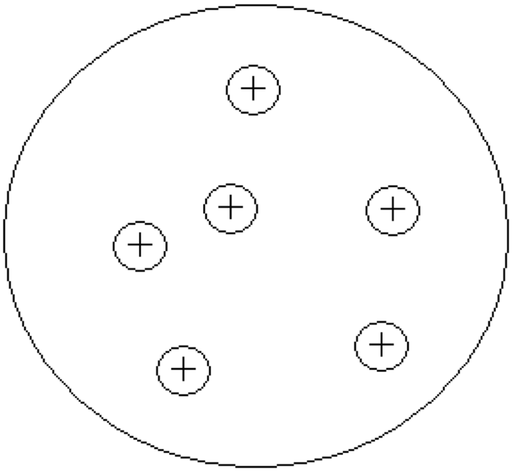

Fig. 3. Conductance of an actual electronically charged jet.

$$g = \lambda c^{\beta} r^{\alpha} \tag{15}$$

where λ is a constant.

So the current balance in the jet can be expressed as follows:

$$2\pi r \sigma u + \lambda c^{\beta} r^{\alpha} E = I. \tag{16}$$

This equation gives the implications of the polymer concentration and non-metal conductive effect on the electrospinning process.

6. Conclusion

The allometric model might initiate a revolution in the understanding of dynamic phenomena in the electrospinning procedure. Though Eq. (16) is more reasonable, its experimental verification is still needed.

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